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AN EXAMINATION OF INVESTIGATIONS BY DR MAURICE FISHBERG AND PROFESSOR FRANZ BOAS DEALING WITH MEASUREMENTS OF JEWS IN NEW YORK.

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(1) *Introduction.* Questions relating to the stability of physical characters are of prime importance in anthropology. It is evident that any investigations concerning individual or racial heredity are made undesirably complicated if the characters used are subject to appreciable modification owing to the direct influence of changing environmental conditions. Physical anthropologists have generally assumed that most of their measurements, and particularly the cephalic index and others of the head or cranium, are not affected markedly, if at all, by environment. A considerable amount of indirect evidence may be brought forward in support of this view, such as that relating to geographical distributions, and to the comparison of samples believed to represent the same racial stock at different periods. Evidence of this kind suggests that the types of the characters in question (for a particular population) may become modified in the course of several generations, owing to a slow process of selection, but that otherwise they tend to remain remarkably stable. These are but inferences, however, and it must be admitted that the results of a single investigation bearing directly on the question might make it necessary for the physical anthropologist to modify, or abandon, his general assumptions. There is, in fact, only one investigation having sufficient scope and authority which has threatened to effect this purpose. In the well-known writings of Professor Boas dealing with measurements on considerable numbers of individuals obtained in New York for the United States Immigrant Commission it was claimed, as early as 1910, that conclusive evidence was provided to show that the head form is modified quite markedly, and in one generation, by the immediate influence of a new environment. His reports have been continually referred to in the later anthropological literature: some writers are willing to accept the conclusions reached and others consider them invalid, but in neither case has there usually been any adequate discussion of the original material. The problem presented by Professor Boas's data is a somewhat complicated one, and a proper examination of it has to be rather lengthy and to be concerned with detailed points. He attached greatest importance to the evidence of the Jewish series, and the others are not considered in the present paper.

Our purpose has been not to discuss his analysis step by step, but to use what we believe to be more direct methods, and these, if valid, lead to essentially different conclusions. Dr Maurice Fishberg's earlier measurements of immigrant

Jews in New York are dealt with first, as they provide independent evidence of the nature of the population with which we are concerned.

(2) *Dr Maurice Fishberg's Measurements of Jews in New York.* A paper dealing with these, entitled "Materials for the Physical Anthropology of the Eastern European Jews," was published in 1905*. The nature of the groups of people dealt with and the way in which the data were collected must be clearly appreciated. The following quotations (from pp. 16—17) refer to these matters:

"It appeared to the present writer that an anthropological study of the Jews in various countries by one observer, thus greatly eliminating the effects of the personal equation, may contribute to the solution of some of the obscure problems of the origin of certain physical traits of the Jews....New York City is the best place in the world to obtain anthropometrical measurements of Jews. Of the 600,000 Jews or more living here, more than three-quarters have arrived in the United States within the last thirty years from the various European countries, also from Asia and even Africa. The vast majority are natives, or the descendants of Jews from Russia, Poland, Austria, Hungary, Roumania, and other parts of eastern Europe; some have come from Syria, Palestine, and even from Algiers, Tunis and Morocco....By using ordinary tact I succeeded in obtaining measurements of over 2000 individuals of both sexes, and all over twenty years of age. Over one half of these were applicants for relief in the United Hebrew Charities in this city. They offered no serious objections to the procedure, believing that the measurements were a means of discovering the nature of their ailment, or their physical ability to work. Only the native Jews objected seriously, suspecting that these measurements were the 'Bertillon system,' and, denying guilt of any crime, they usually refused to submit. It is to be regretted that mainly for this reason I succeeded in obtaining measurements of only 124 Jews born in the United States."

It is clear that the people measured cannot be supposed to represent a random sample, or random samples, from any population or populations. The majority of them were immigrants into the United States, and the groups from the different countries represented, even if unselected in any other way, would be unlikely to conform exactly to the types of the parent populations. But there was clearly a social selection from all immigrants, since the majority of the subjects were applicants for charitable relief†.

* *Memoirs of the American Anthropological Association*, Vol. I. pp. 1—146. Also published in *Annals of the New York Academy of Sciences*, Vol. 16 (1905), pp. 155—296. Page references given in the text relate to the former of these volumes. Part of Fishberg's material had been previously treated by him in "Physical Anthropology of the Jews. I, The Cephalic Index," *American Anthropologist*, N.S., Vol. IV. (1902), pp. 684—706; and "II, Pigmentation," *Ibid.*, Vol. V. (1903), pp. 89—106.

† Dr Fishberg was physician to the United Hebrew Charities of New York City at the time he was writing. He refers to the conditions of the applicants for relief there in another work. (*The Jews: a Study of Race and Environment*. London 1911, pp. 367—368.) Only two per cent. of them were born in the United States. It is pointed out that the vast majority of the immigrants left Europe for economic reasons and they are said to have possessed less than fifteen dollars per head on landing. This does not necessarily indicate that they were of the poorest class, since they had been able to meet the cost of transportation.

Finally, there was a selection of those particular individuals who would consent to be measured, and it is noted that this did not apply in the same way to the Jews born in the United States as to the immigrants.

The total sample was divided into sub-groups according to the country or province (government) in which the Jews under consideration were born. It is stated that, according to the Russian census of 1897, 93·9 per cent. of the 5,189,400 Jews in Russia lived in the "Pale of Settlement" which comprised 25 provinces in western and southern Russia and Poland. It was advisable to consider subdivisions of this large area for the purpose required, and the groups distinguished are those from: (*a*) Galicia, (*b*) the ten pre-war Polish provinces, (*c*) Lithuania and White Russia, (*d*) Little Russia, including Bessarabia, (*e*) Hungary, (*f*) Rumania. All the foreign Jews represented probably belonged to the Ashkenazim stock, and those born in New York may be assigned to the same. The number of individuals in each group can be seen from Table I below.

It is unfortunate that the way in which the measurements were taken is not adequately explained in this study: no technique is referred to, and nothing is said about the question of using pressure, or not, in determining head measurements. The data recorded relate to (1) stature; (2) "girth of the chest, during quiet respiration"; (3) maximum length of the head from the glabella; (4) breadth of the head "obtained by searching with the points of the calipers along the temples, over the ears and somewhat posteriorly"; (5) horizontal circumference of the head found with a tape, "for which the chances of error in observation are very great"; (6) height of the face "from the root of the nose to the point beneath the middle of the chin"; (7) breadth of the face, presumably the maximum bizygomatic breadth; (8) height of the nose "from the root to the subnasal spine"; (9) breadth of the nose, i.e. "the distance of the alae without compressing them"; (10) hair, eye and skin colours. Indices derived from these absolute measurements are also considered. The absence of any remarks on the extreme difficulty of defining the point at the root of the nose, used as the terminal of the facial and nasal heights, renders these measurements practically valueless for comparative purposes, but Fishberg's own records for them may be supposed comparable *inter se*. It is extremely probable, though nothing is said on the point, that the order in which the individuals were measured was haphazard as far as their districts of birth were concerned, and, under these circumstances, the effect of personal equation on comparisons between different regional groups may be supposed reduced to a minimum for the particular measurer and the particular definitions of measurements he was following. The chest girth and horizontal circumference of the head are not given for the Jewesses. Otherwise, all the measurements were obtained for all, or nearly all, the subjects of both sexes except in the case of the chest girths for the men. Fishberg writes (p. 50): "Of the Jews recorded in this work [I, 528], only 98 have been examined as to the circumference of the chest. Many submitted to the ordeal of having their heads, etc., measured, but they declined to undress for the purpose of having their chest measured." He neglects, however, the

probable effect of this further selection in discussing the results for the measurement in question. The men who submitted to having their chests measured were, doubtless, better developed and taller, on the average, than the remainder. This is suggested by the data in Table I below, which gives the total for each geographical division and the number for which the chest girth was obtained. The second figures expressed as percentages of the first are 62·6, 54·9, 45·8, 86·4, 49·3, 97·3 and 95·2, and the groups are arranged in order of their mean statures. For groups with a larger average stature a larger proportion of the men were willing to undress. By dividing the average girths by the average statures for the total number of men measured in each group, as Fishberg does, a fallacious conclusion is likely to be suggested. It must be remembered that the chest girths should be considered apart from the other measurements owing to the fact that they were subject to a peculiar selection.

Individual measurements are not given in the memoir discussed. The distributions for the total sample are provided, however, in the case of every character, and the numbers of groups used are adequate. For some of the characters, distributions are also provided for each of the sub-samples representing different geographical divisions. These are actually in the form of percentages, but as the totals are known, the absolute frequencies can easily be calculated. By adding the frequencies for the sub-samples we should obtain the distributions given for the total sample. This comparison can be made in ten cases (five characters for each sex), and for six of these agreement is not found, showing that there must be an error, or errors, in the distributions which cannot be rectified as the individual measurements are not available. In all cases of disagreement, the means and standard deviations were calculated separately for the total distributions printed and for those obtained by adding the distributions for the sub-samples. Differences between these pairs of constants are only found in the fourth figure in the case of means and in the third figure in the case of standard deviations. There may possibly be errors of a larger order in the constants calculated for the distributions of the sub-samples, but their probable errors are also larger. We may hope that by accepting all the distributions as they are printed, the constants deduced from them will not be in error to any appreciable extent, and they were accepted, accordingly, to give the data in our Tables I and II. A few other arithmetical or printer's errors were found in Fishberg's tables, but it is possible to rectify most of these*. One point connected with the presentation of the material is of more consequence. The means of the head diameters are given to the nearest whole mm., while the probable errors of these constants for the Jews are all less than 0·4 mm., and some of them are less than 0·1 mm. The indices derived from these absolute measurements were evidently calculated for each individual to give the distributions, but the mean indices provided were simply obtained by dividing the mean of one component length by the mean of the other. These crude approxi-

* In Fishberg's Table VI (p. 33) the percentage 0·36 for the group 135—136 should be added to the distribution representing Jews from Lithuania and White Russia. The other errors referred to do not affect any statistical constants to an appreciable extent.

TABLE I.

Mean Measurements for Groups of Jews measured by Dr Maurice Fishberg in New York.*

		Galicia	Poland	Lithuania and White Russia	Hungary
Jews	Stature	1623±2·7 (305)	1635±2·3 (315)	1644±2·5 (275)	1657±3·5 (140)
	Chest girth	839·3 (191)	855·3 (173)	849·3 (126)	863·0 (121)
	Circumference of head	551·0± ·68 (305)	554·2± ·67 (315)	556·1± ·71 (275)	557·7±1·00 (140)
	Head length	186·4± ·23 („)	188·7± ·23 („)	189·6± ·24 („)	187·6± ·35 („)
	Head breadth	153·9± ·22 („)	154·2± ·20 („)	153·1± ·22 („)	154·2± ·31 („)
	Cephalic index	82·6± ·12 („)	81·8± ·12 („)	80·8± ·12 („)	82·4± ·18 („)
	Height of face	119 („)	119 („)	119 („)	119 („)
	Breadth of face	132 („)	132 („)	135 („)	136 („)
	Facial index	{90·2} („)	{90·2} („)	{88·1} („)	{87·5} („)
	Height of nose	52 (295)	51 (307)	52 („)	51 („)
	Breadth of nose	36 („)	37 („)	36 („)	36 („)
	Nasal index	69·1± ·33 („)	71·6± ·30 („)	69·1± ·29 („)	71·0± ·46 („)
Jewesses	Stature	1524±3·9 (122)	1523±5·4 (56)	1528±3·7 (100)	1551±6·6 (39)
	Head length	178·2± ·37 („)	180·1± ·55 („)	179·6± ·39 („)	178·1± ·67 („)
	Head breadth	149·1± ·33 („)	147·5± ·44 („)	147·9± ·29 („)	148·4± ·52 („)
	Cephalic index	83·7± ·20 („)	81·9± ·29 („)	82·4± ·21 („)	83·6± ·35 („)
	Height of face	107 („)	110 („)	109 („)	110 („)
	Breadth of face	128 („)	129 („)	126 („)	127 („)
	Facial index	{83·6} („)	{85·3} („)	{86·5} („)	{86·6} („)
	Height of nose	47 (117)	47 („)	48 („)	47 („)
	Breadth of nose	33 („)	33 („)	33 („)	32 („)
	Nasal index	70·7± ·44 („)	70·8± ·57 („)	69·4± ·48 („)	67·9± ·90 („)

		Little Russia	Rumania	United States	Pooled
Jews	Stature	1657±2·6 (219)	1662±3·8 (150)	1679±4·0 (124)	1641±1·1 (1528)
	Chest girth	850·0 (108)	844·1 (146)	880·9 (118)	858·7±1·2 (983)
	Circumference of head	556·2± ·80 (219)	558·1± ·96 (150)	561·8±1·06 (124)	555·5± ·30 (1528)
	Head length	188·4± ·29 („)	186·8± ·31 („)	190·3± ·38 („)	188·2± ·11 („)
	Head breadth	154·2± ·24 („)	154·7± ·26 („)	153·9± ·30 („)	154·0± ·092 („)
	Cephalic index	81·7± ·13 („)	82·6± ·16 („)	81·0± ·18 („)	81·8± ·053 („)
	Height of face	120 („)	120 („)	120 („)	119·5± ·12 („)
	Breadth of face	137 („)	135 („)	137 („)	135·0± ·11 („)
	Facial index	{87·6} („)	{88·9} („)	{87·6} („)	88·1± ·10 („)
	Height of nose	53 („)	52 („)	53 („)	52·2± ·074 (1510)
	Breadth of nose	37 („)	36 („)	36 („)	36·5± ·050 („)
	Nasal index	69·6± ·31 („)	69·3± ·44 („)	68·8± ·48 („)	70·2± ·14 („)
Jewesses	Stature	1550±5·4 (74)	1551±5·4 (44)	—	1534±2·0 (435)
	Head length	179·9± ·48 („)	179·2± ·56 („)	—	179·1± ·20 („)
	Head breadth	147·2± ·41 („)	148·7± ·43 („)	—	148·2± ·16 („)
	Cephalic index	81·9± ·22 („)	83·0± ·27 („)	—	82·8± ·10 („)
	Height of face	110 („)	113 („)	—	109·7± ·21 („)
	Breadth of face	125 („)	128 („)	—	127·1± ·19 („)
	Facial index	{88·0} („)	{88·3} („)	—	86·1± ·18 („)
	Height of nose	48 (67)	52 („)	—	47·4± ·11 (423)
	Breadth of nose	33 („)	36 („)	—	33·0± ·085 („)
	Nasal index	68·9± ·61 („)	68·9± ·65 („)	—	69·6± ·24 („)

* All the means with probable errors were found from Fishberg's distributions except in the case of stature. The means for stature and all the means without probable errors are his values; the values for the facial index in curled brackets being the mean heights divided by the mean breadths.

mations are likely to falsify any conclusions based on the constants. The following table shows some of the largest divergences found among the means for the different sub-samples of New York Jews:

		Galicia	Little Russia	Rumania
Number of individuals		305	219	150
Fishberg's means	Head length (A) ...	186	188	187
	Head breadth (B) ...	155	155	153
	Cephalic index ($100B/A$)	83·3	82·45	81·8
Means found from distributions	Head length (C) ...	186·4 ± ·23	188·4 ± ·29	186·8 ± ·31
	Head breadth (D) ...	153·9 ± ·22	154·2 ± ·24	154·7 ± ·26
	Cephalic index	82·6 ± ·12	81·7 ± ·13	82·6 ± ·16
	" " ($100D/C$)	82·6	81·8	82·8

Fishberg's means might suggest that the Jews from Galicia had a significantly higher cephalic index than those from Rumania, but the more accurate means found from the distributions are actually identical for these two samples. The means obtained from the distributions are evidently the only ones on which any reliance can be placed, and these are the values given with their probable errors in our Table I. The means in the table without probable errors are Fishberg's values, no distributions being provided in these cases. The standard deviations in Table II were all calculated from the distributions: Fishberg gives some standard deviations, and in most of these cases our values do not agree exactly with his.

Some conclusions which can be drawn from Fishberg's revised data in our Table I may now be considered. We require to know whether the means for the different series indicate that they were probably drawn from the same population, or not. In estimating this the characters were dealt with singly and the difference between each pair of means was divided by its probable error. If these ratios are considered individually, then a value of 3·5 will indicate that a greater divergence will only be found once in 55 trials between samples actually drawn from the same population; a ratio of 5·0 indicates that a greater divergence will only be found once in 1342 trials, and one of 10 once in more than 500,000,000 trials. In the case of a single comparison we might accept a ratio greater than 3·5 as evidence that the two samples do not represent the same population, but if a single ratio greater than this limit is found in a set of ratios—such as those found between the mean statures for a number of series—then it may be unnecessary to accept it as evidence of differentiation, since the probability of its occurrence owing to chance will now be greater. All the ratios (Δ/p.e. Δ) between the means for which probable errors are given in Table I, excluding those for the pooled series, are summarised below:

	Jews				Jewesses		
	0—3·5	3·5—5	5—10	10—12	0—3·5	3·5—5	5—10
Stature	7	4	9	1	9	6	—
Circumference of head	13	3	5	—	—	—	—
Head length	5	5	7	—	15	0	—
Head breadth ...	19	2	—	—	14	1	—
Cephalic index ...	8	4	8	1	10	3	2
Nasal index	16	3	2	—	15	0	—

Considering the data for the Jews first, it will be safest to suppose that the head breadths do not distinguish the series, but every one of the other characters shows differences large enough to preclude entirely the possibility that all the samples represent the same population. The statures and cephalic indices distinguish the series more clearly than do the other characters. It may be noted that in Table I the series are arranged in order of the mean statures for the men, and the circumferences of the head provide a very similar order. There is also a very close agreement between the arrangements suggested by the head lengths and cephalic indices, but those for the statures and cephalic indices are markedly different. Data relating to far more characters would clearly be needed before any attempt could be made to determine the racial relationships of the different populations represented. While many of the differences between the means for the Jews are of marked statistical significance, their absolute values are quite small, and if smaller numbers of individuals had been measured it is probable that far fewer of the means would have been differentiated. This point is illustrated by a comparison of the means for the Jewesses. In this case the stature and cephalic index are the only characters which distinguish the series beyond question. The fact that there are no means for Jewesses born in the United States may also account partly for the smaller proportion of significant differences between the female means. The female means for the stature and cephalic index again arrange the series in quite different orders, but the order in which the groups are arranged by the mean statures for the Jews is very similar to that given by the statures for the Jewesses, and the same is true for the cephalic indices. The mean female index is slightly greater than the corresponding male mean in all six cases, and this relation is generally found when samples are taken from the same population.

The standard deviations of Fishberg's series are given in our Table II, and, as was to be expected, far fewer significant differences are found between them than between the means. No differences whatever greater than 3·5 times their probable errors are found in the case of the head length and cephalic index for the Jews. For the head breadth one of the 21 differences may be supposed significant if considered alone, the Galician σ exceeding the Rumanian by 3·9 times the probable error of the difference; for the nasal index two differences are distinguished in the same way, the Galician value of σ exceeding those for the Little Russian (Δ/p.e. $\Delta = 4{\cdot}9$)

TABLE II.

Standard Deviations for Groups of Jews measured by Dr Maurice Fishberg in New York.*

		Galicia	Poland	Lithuania and White Russia	Hungary	Little Russia	Rumania	United States	Pooled
Jews	Stature	71·3 ±1·9	61·7 ±1·7	60·3 ±1·7	60·9 ±2·5	57·4 ±1·9	69·5 ±2·7	65·1 ±2·8	66·0 ± ·81
	Chest girth	—	—	—	—	—	—	—	57·8 ± ·88
	Circumference of head	—	—	—	—	—	—	—	17·5 ± ·21
	Head length	5·88± ·16	6·08± ·16	5·78± ·17	6·18± ·25	6·35± ·20	5·68± ·22	6·23± ·27	6·13± ·075
	Head breadth ...	5·73± ·16	5·24± ·14	5·45± ·16	5·47± ·22	5·23± ·17	4·76± ·19	4·95± ·21	5·36± ·065
	Cephalic index ...	3·04± ·083	3·27± ·088	2·92± ·084	3·24± ·131	2·86± ·092	2·90± ·113	2·91± ·125	3·10± ·038
	Height of face ...	—	—	—	—	—	—	—	6·71± ·082
	Breadth of face ...	—	—	—	—	—	—	—	6·31± ·077
	Facial index	—	—	—	—	—	—	—	5·80± ·071
	Height of nose ...	—	—	—	—	—	—	—	4·26± ·052
	Breadth of nose ...	—	—	—	—	—	—	—	2·90± ·036
	Nasal index	8·50± ·24	7·87± ·24	7·24± ·21	8·13± ·33	6·91± ·22	7·91± ·31	7·96± ·34	8·03± ·13
Jewesses	Stature	63·4 ±2·7	60·0 ±3·8	54·3 ±2·6	61·2 ±4·7	68·5 ±3·8	53·0 ±3·8	—	61·0 ±1·4
	Head length	6·01± ·26	6·09± ·39	5·83± ·28	6·20± ·47	6·06± ·34	5·46± ·39	—	6·05± ·14
	Head breadth ...	5·38± ·23	4·89± ·31	4·25± ·20	4·77± ·36	5·25± ·29	4·27± ·31	—	4·94± ·11
	Cephalic index ...	3·35± ·14	3·23± ·21	3·05± ·15	3·25± ·25	2·83± ·16	2·68± ·19	—	3·20± ·073
	Height of face ...	—	—	—	—	—	—	—	6·41± ·15
	Breadth of face ...	—	—	—	—	—	—	—	5·87± ·13
	Facial index	—	—	—	—	—	—	—	5·68± ·13
	Height of nose ...	—	—	—	—	—	—	—	3·33± ·077
	Breadth of nose ...	—	—	—	—	—	—	—	2·58± ·060
	Nasal index	7·02± ·31	6·35± ·40	7·10± ·34	8·36± ·64	7·44± ·43	6·39± ·46	—	7·17± ·17

* The number of individuals on which each standard deviation is based can be seen from Table I.

and Lithuanian and White Russian series (4·0); and for the stature there are four differences greater than 3·5 times their probable errors, the Galician σ exceeding the Little Russian (5·2), Lithuanian and White Russian (4·3), and Polish (3·8) values, while the Rumanian also exceeds the Little Russian (3·9). Only one difference greater than the same limit is found between the female standard deviations: in the case of the head breadth the Galician constant exceeds the Lithuanian and White Russian (3·9) by an amount which must be supposed significant if this difference is considered alone. If the constants for all the characters are considered together, there is a suggestion that the Galician male is appreciably more variable than the other male series, but no other differences of any consequence can be detected. It may be noted that the Jews born in the United States do not form a sample which can be said to be either more or less variable than those made up by groups of immigrants from different European countries.

Fishberg also investigated the association between stature and occupation in the case of the Jews he measured, giving the following mean values (p. 41):

Indoor workers: tailors, cobblers and factory workers	1620 (720)
Peddlers	1643 (130)
Outdoor workers	1664 (344)
Merchants	1687 (163)
Clerks	1692 (105)
Professional men	1696 (66)

The significance of the differences between these means cannot be estimated as neither standard deviations nor distributions are provided, but there can be no doubt that many of them are markedly significant. A comparison with the data in Table I shows that grouping by social class leads to more dissimilar mean statures than does the grouping based on the country of origin. What is needed, clearly, is a grouping by country and social class in order to discover whether there are differences between the geographical groups dependent on the fact that the proportions representing different occupations are not the same for the different samples. It is at least probable that the sample of Jews born in the United States differed from the other samples in having a different occupational constitution.

Nothing further can be deduced from Fishberg's material, as far as this point is concerned, since it is not presented in a suitable form. The possible association between characters other than stature and occupation is not considered.

One other question investigated by Fishberg may be commented on. He gives the distribution of stature for his total sample of 1528 Jews in the form of a frequency curve. A smooth curve fitted to this would evidently be uni-modal and asymmetrical with the mode to the left of the mean, i.e. nearer to zero stature. Fitting the distribution with a normal curve gives a P_{χ^2} for goodness of fit of ·020 (18 groups). But this material is known to be heterogeneous, since significant differences are found between the means of immigrants from different

countries and different social classes, and the form of the distribution will hence depend on the fortuitous proportions of the sub-samples composing the total sample. The series from Little Russia and Hungary have identical mean statures. Pooling their distributions (359 men) and fitting with a normal curve leads to a P of ·658 (17 groups). Fishberg's conclusion that there is an excess of tall men among the Jews in New York appears to be dependent on the heterogeneous nature of his material, and it is quite unsafe to suppose, as he does, that an asymmetrical stature distribution is a characteristic of Jewish populations in general.

Finally, we may examine the records of pigmentation for the New York Jews. These were determined without the use of colour scales, but the percentages for the different series may be supposed comparable *inter se*. The assumption that the personal equation of the observer will not affect these comparisons is justified if, as we suppose, the individuals from different countries were examined in a haphazard order. Six categories of hair and four of eye colour were distinguished (see Table III below), and it is admitted that no sharp divisions can be made between these. The colour of the iris is said to have been observed at a distance of about one metre, which is a large distance to use for the purpose. Fishberg gives percentage frequencies only (Table XLIV), and from these we have calculated the actual frequencies and the probable errors of the percentages given in our Table III.

A comparison of the percentages for Jews and Jewesses from the same country may be made first. No significant sexual differences are actually found for any category of hair or eye colour. The method given in *Biometrika*, Vol. VIII, pp. 250—254, to measure the probability that two independent distributions of frequency are really samples of the same population was applied. This gives the following values of P for the corresponding male and female distributions, the first referring to hair colour and the second to eye colour: Galicia ·620, ·533; Little Russia ·851, ·277; Hungary ·153, ·478; Lithuania and White Russia ·515, ·477; Poland ·635, ·101; Rumania ·100, ·821. As far as can be told from these characters, there is every reason to believe that the corresponding groups of Jews and Jewesses from any particular country represented came from the same population which showed no sexual differentiation in pigmentation among adults. Fishberg also obtained these records for a larger sample (including the individuals for whom measurements are given) of 2716 Jews and 1519 Jewesses in New York. He concludes that the women had fairer hair than the men, on the average, but the P for the distributions (p. 114) is ·141 (6 groups). There is a better indication that the Jewesses had darker eyes than the Jews (p. 115), since a P is found of ·0001 (4 groups). This conclusion apparently conflicts with the fact that the eye colours for male and female groups from the same country are in close agreement, but the result obtained may be misleading owing to the heterogeneity of the pooled sample. If the proportions from different countries were not the same in the male and female total samples, an erroneous conclusion might be suggested. Fishberg also states that Jewesses have a slightly darker skin than Jews, but

TABLE III. *Hair and Eye Colours for Groups of Jews observed by Dr Maurice Fishberg in New York.*

				Hair colours						Eye colours			
				Black	Brown	Dark Chestnut	Light Chestnut	Blond	Red	"Black"	Brown	Grey	Blue
Galicia	Men	305	Frequency	106	103	38	25	22	11	52	91	84	78
			Percentage	34·8±1·8	33·8±1·8	12·5±1·3	8·2±1·1	7·2±1·0	3·6±0·7	17·0±1·5	29·8±1·8	27·5±1·7	25·6±1·7
	Women	122	Frequency	41	38	13	9	15	6	26	43	28	25
			Percentage	33·6±2·9	31·1±2·8	10·7±1·9	7·4±1·6	12·3±2·0	4·9±1·3	21·3±2·5	35·2±2·9	23·0±2·6	20·5±2·5
Little Russia	Men	219	Frequency	95	71	22	10	15	6	37	74	57	51
			Percentage	43·4±2·3	32·4±2·1	10·0±1·4	4·6±1·0	6·8±1·2	2·7±0·7	16·9±1·7	33·8±2·2	26·0±2·0	23·3±1·9
	Women	74	Frequency	32	27	5	4	3	3	20	24	15	15
			Percentage	43·2±3·9	36·5±3·8	6·8±2·0	5·4±1·8	4·1±1·5	4·1±1·5	27·0±3·5	32·4±3·7	20·3±3·2	20·3±3·2
Hungary	Men	140	Frequency	67	29	12	13	7	12	28	43	38	31
			Percentage	47·9±2·8	20·7±2·3	8·6±1·6	9·3±1·7	5·0±1·2	8·6±1·6	20·0±2·3	30·7±2·6	27·1±2·5	22·1±2·4
	Women	39	Frequency	19	15	2	1	1	1	10	15	9	5
			Percentage	48·7±5·4	38·5±5·3	5·1±2·4	2·6±1·7	2·6±1·7	2·6±1·7	25·6±4·7	38·5±5·3	23·1±4·6	12·8±3·6
Lithuania and White Russia	Men	275	Frequency	132	89	16	14	13	11	62	93	70	50
			Percentage	48·0±2·0	32·4±1·9	5·8±0·9	5·1±0·9	4·7±0·9	4·0±0·8	22·5±1·7	33·8±1·9	25·5±1·8	18·2±1·6
	Women	100	Frequency	44	30	6	4	9	7	29	27	24	20
			Percentage	44·0±3·3	30·0±3·1	6·0±1·6	4·0±1·3	9·0±1·9	7·0±1·7	29·0±3·1	27·0±·30	24·0±2·9	20·0±2·7
Poland	Men	315	Frequency	155	87	28	17	18	10	68	111	67	69
			Percentage	49·2±1·9	27·6±1·7	8·9±1·1	5·4±0·9	5·7±0·9	3·2±0·7	21·6±1·6	35·2±1·8	21·3±1·6	21·9±1·6
	Women	56	Frequency	24	21	5	3	1	2	15	17	18	6
			Percentage	42·9±4·5	37·5±4·4	8·9±2·6	5·4±2·0	1·8±1·2	3·6±1·7	26·8±4·0	30·4±4·1	32·1±4·2	10·7±2·8
Rumania	Men	150	Frequency	74	44	7	11	11	3	28	45	47	30
			Percentage	49·3±2·8	29·3±2·5	4·7±1·2	7·3±1·4	7·3±1·4	2·0±0·8	18·7±2·1	30·0±2·5	31·3±2·6	20·0±2·2
	Women	44	Frequency	16	13	8	3	3	1	8	14	11	11
			Percentage	36·4±4·9	29·5±4·6	18·2±3·9	6·8±2·6	6·8±2·6	2·3±1·5	18·2±3·9	31·8±4·6	25·0±4·4	25·0±4·4
United States	Men	124	Frequency	44	46	15	9	5	5	26	43	30	25
			Percentage	35·5±2·9	37·1±2·9	12·1±2·0	7·3±1·6	4·0±1·2	4·0±1·2	21·0±2·5	34·7±2·9	24·2±2·6	20·2±2·4

among 1084 women he recorded 808 (74·54 ± 1·08 per cent.) with fair skin, and among 1188 men 919 (77·36 ± 0·82 per cent.). The difference between these percentages is only 2·1 times its probable error, so it should not be asserted that there is any sexual differentiation in skin colour.

Pigmentation comparisons were made by us between the samples from different countries in the case of the men only, as there are too few individuals in some of the female groups to lead to any definite conclusions. The *P*'s for all pairs of series are given in Table IV. We may suppose that a value greater than ·02 indicates that the two samples in the comparison are not differentiated*, thus using a limit which

TABLE IV.

Values of P for Hair and Eye Colours between Groups of Jews observed by Dr Maurice Fishberg in New York.

		Lithuania and White Russia	Poland	Rumania	Little Russia	United States	Galicia	Hungary
Lithuania and White Russia	Hair	—	·293	·596	·409	·120	·005	·041
	Eyes		·527	·491	·331	·930	·075	·719
Poland	Hair	·293	—	·557	·754	·161	·017	·075
	Eyes	·527		·139	·417	·905	·095	·554
Rumania	Hair	·596	·557	—	·362	·228	·023	·054
	Eyes	·491	·139		·598	·602	·575	·869
Little Russia	Hair	·409	·754	·362	—	·472	·303	·015
	Eyes	·331	·417	·598		·758	·794	·841
United States	Hair	·120	·161	·228	·472	—	·865	·034
	Eyes	·930	·905	·602	·758		·411	·865
Galicia	Hair	·005	·017	·023	·303	·865	—	·005
	Eyes	·075	·095	·575	·794	·411		·792
Hungary	Hair	·041	·075	·054	·015	·034	·005	—
	Eyes	·719	·554	·869	·841	·865	·792	

is approximately the same as that taken to indicate significance between means and standard deviations considered singly. It may be unnecessary to attach significance to a single value of *P* less than ·02 found in a series of *P*'s. For the eye colours the lowest *P* found is ·075, so there is no evidence that any pair of the populations represented were differentiated by this character. For the hair colours the only values of *P* less than ·02 are between the Jews from Galicia, on the one hand, and those from Lithuania and White Russia, Poland and Hungary, on the other,

* [This seems to be a personal judgment of what is a suitable limit. A more interesting point would be whether the 21 values of *P* for Hair, and the 21 values of *P* for Eyes form each a random series of *P*'s. In the former case 17 of the 21 values are below 0·5; in the latter case 14 of the 21 values are above 0·5. Ed.]

and between the Jews from Hungary and Little Russia. For the last comparison the P is ·015 and it need not be supposed to indicate distinction, but two of the P's with the Galician series are small enough to denote clear differentiation of the populations sampled. Turning to the frequencies in Table III, it may be observed that the Galician Jews and Jewesses have the lowest percentages of individuals with black hair, and the Jewesses have the highest percentage of individuals with blond hair. It must be concluded that they do, in fact, represent a population with slightly fairer hair, on the average, than the others. The difference in hair colouring between the Jews from Hungary and Little Russia is not confirmed by the records for the Jewesses from the same countries, and no significance need be attached to it.

This comparison of the groups of Jews and Jewesses observed by Fishberg in New York suggests that the head breadth and eye colours were practically identical for all the populations represented, as far as can be told; the horizontal circumference of the head, nasal index and hair colours show a few significant differences; the stature, head length and cephalic index show differences which are small absolutely, but many of which are large enough to indicate clear differentiation of the populations sampled. The differences between the statures may be due partly, at least, to some peculiar selection of the material, but it is unlikely that the head lengths and cephalic indices were influenced in any such way. Finally, it may be noted that the Jews born in the United States are only distinguished clearly from all the other groups in having greater statures, but they were probably distinguished also in representing higher social classes, on the average, and social selection would be expected to induce greater differences for stature than for the other characters.

(3) *The Investigation of the United States Immigrant Commission reported on by Professor Franz Boas: (a) The Measurements of Adults.* The report on this investigation was first published in 1910* and it was re-issued in a greatly enlarged form, and with the greater part of the text re-written, in 1912†. We are only concerned here with the data relating to Jewish individuals. The children were measured in schools and "the school measurements were supplemented by extended series of measurement of whole families, the observers making a house-to-house canvass, measuring families of immigrants, with their consent. Naturally, a considerable amount of opposition was offered to observers from time to time...." (1912, p. 82). It is said that "only immigrants and their direct descendants" are included in the study, and those individuals with whom we are now concerned appear to have belonged to families for which both parents were considered to be

* "Changes in Bodily Form of Descendants of Immigrants," *The Immigrant Commission*, 61*st Congress*, 2*nd Session*, *Senate Document No.* 208 (Washington: Government Printing Office), pp. 113.

† *Changes in Bodily Form of Descendants of Immigrants.* By Franz Boas (New York: Columbia University Press), pp. 573. References (1912) in the text are to this publication. In "Materials for the Study of Inheritance in Man," *Columbia University Contributions to Anthropology*, Vol. VI. (1928), Professor Boas has given the individual measurements of all the people measured for the Immigrant Commission in 1909–1910 with some additional material.

of Jewish descent. The totals for this group are 3306 boys and 1049 girls between 4 and 19 years of age, and 799 men and 839 women aged 20 or over, according to Table 18 (p. 84) in the 1912 report.

The measurements relate to stature, head length, head breadth, face (bizygomatic) breadth, weight and grip, and we may omit consideration of the last two. They were obtained by thirteen observers, and data given (1912, pp. 85—92) suggest that their "errors" of personal equation were not large enough to influence appreciably comparisons made between different parts of the total material. A hair colour scale was used, but no standards aided the discrimination of eye and skin colours, and it is admitted that the results for these last two characters are not of great value. Professor Boas consulted with Dr Fishberg (1912, p. 2), and it may be noted that several measurements taken by the latter were omitted, probably because they were felt to be unsatisfactory. One would expect that the measurements common to the two investigations were obtained by following identical techniques, but this is not stated and no adequate definitions are given in either case.

A comparison may be made first between the measurements of the adult (aged 20 and over) Jews given by Fishberg and Boas. The latter writes (1912, p. 99): "The Hebrew material which has been collected by us consists very largely of immigrants from Russia, Austria and Roumania, with a comparatively speaking small number of descendants of west European Jews. It has been shown by Dr Maurice Fishberg that Hebrews from different parts of eastern Europe show appreciable differences in type. For this reason I tried first to subdivide the material collected by us according to the origin of the people from the different parts of Russia, Galicia, Roumania, etc.....The results of this classification, however, were not quite satisfactory on account of the frequent cases of mixture, intermarriages between Polish and Russian Jews from various parts of the Russian Empire being quite numerous." These remarks are somewhat obscure. Birth-places were recorded, and if an immigrant was defined as a person not born in the United States, then it is not clear what difficulty there can have been in the classification of individuals, or what inter-marriages are referred to. Arrangements by country of origin are only given for the men, and the groups correspond with Fishberg's, except that the series from Lithuania and White Russia are kept separate and there is no group of Hungarian Jews, presumably because their number was too small. The constants in our Table V were obtained from distributions given in the 1912 publication (Table VII, pp. 247—259 and Table XVII, pp. 370—373), the groups of Lithuanian and White Russian Jews having been combined to facilitate comparison with Fishberg's data. The means for the Galician, Polish, Little Russian, Rumanian, and United States series are those given by Boas, and all the other constants were computed by us.

Comparisons may be made first between the constants given for the different groups in our Table V, omitting those for the pooled series. The characters were considered singly and the differences between all possible pairs of means were

TABLE V.

Constants for Groups of adult Jews measured for the Immigrant Commission.

	Galicia	Poland	Lithuania and White Russia	Little Russia	Rumania	United States	Pooled
	Means						
Stature ...	1636±3·2 (144)	1635±3·6 (100)	1639±2·4 (245)	1656±3·9 (117)	1648±3·4 (126)	1672±4·0 (38)	1644±1·4 (770)
Head length ...	185·3± ·33 (144)	188·2± ·40 (101)	188·9± ·27 (244)	187·6± ·41 (115)	185·3± ·38 (126)	188·8± ·59 (39)	187·4± ·16 (769)
Head breadth	156·1± ·31 (144)	156·6± ·37 (101)	154·5± ·23 (244)	156·4± ·32 (115)	154·4± ·35 (126)	153·7± ·49 (39)	155·3± ·13 (769)
Cephalic index	84·3± ·18 (144)	83·3± ·19 (101)	81·8± ·13 (244)	83·5± ·18 (115)	83·4± ·18 (126)	81·4± ·27 (39)	83·0± ·076 (769)
Breadth of face	140·3± ·31 (142)	140·0± ·35 (101)	139·4± ·25 (241)	140·0± ·34 (112)	139·4± ·34 (125)	136·9± ·61 (38)	139·6± ·14 (759)
	Standard deviations						
Stature ...	56·5 ±2·2	53·7 ±2·6	54·8 ±1·7	62·7 ±2·8	56·3 ±2·4	66·0 ±5·1	57·8 ±·99
Head length ...	5·94± ·24	5·93± ·29	6·22± ·19	6·54± ·29	6·33± ·27	5·48± ·42	6·38± ·11
Head breadth	5·60± ·22	5·58± ·26	5·32± ·16	5·10± ·23	5·89± ·25	4·50± ·34	5·53± ·095
Cephalic index	3·19± ·13	2·86± ·14	3·05± ·09	2·87± ·13	3·05± ·13	2·47± ·19	3·14± ·054
Breadth of face	5·53± ·22	5·27± ·25	5·70± ·18	5·41± ·24	5·70± ·24	5·62± ·43	5·63± ·097

divided by their probable errors. These ratios are summarised below, as in the comparison of Fishberg's means (cf. table on p. 7).

	0—3·5	3·5—5	5—10	10—12
Stature	8	4	3	—
Head length ...	7	3	5	—
Head breadth ...	6	9	—	—
Cephalic index ...	5	2	7	1
Breadth of face ...	10	5	—	—

The distribution of the ratios in the case of every character shows that it is impossible to consider that all the groups were drawn from a single homogeneous population. The different types are clearly differentiated from one another, as was found in the case of those derived from Dr Fishberg's measurements, the principal difference between the two sets compared in this way being that the head breadth does not distinguish the latter, while it shows a large proportion of significant differences between the series from different countries measured for the Immigrant Commission. No differences whatever greater than 3·5 times their

probable errors are found between the standard deviations for the same character in Table V, and the suggestion, noted in dealing with Fishberg's data, that the immigrants from Galicia were more variable than the other groups is not confirmed.

Comparisons may be made next between the constants for the groups of Jews measured by Dr Fishberg, given in Tables I and II above, and the corresponding values derived from the data presented by Professor Boas. The latter writes: "The differences between various types found by our observers agree well with those described by Dr Fishberg" (1912, p. 99). Considering the means first and omitting those for the pooled series, no differences greater than 3·5 times their probable errors are found in the case of stature and head length; for the head breadth four of the six differences may be supposed significant, the same is found for the cephalic index, and it is probable that five of the six differences are significant in the case of the bizygomatic breadth, though exact comparisons cannot be made for it as no probable errors can be found for Fishberg's means. These relations are at variance with Boas's statement, and it becomes necessary to look into them more closely. For the head length all Boas's means are slightly *less* than the corresponding values found from Fishberg's data, though not one of the differences is significant. For the head breadth Boas's means are *greater* than Fishberg's in four cases out of six, and for these four the differences divided by their probable errors range from 4·4 to 5·8. For the cephalic index, as would now be expected, all Boas's means are *greater* than Fishberg's, while four of the six differences are significant, and the ratios for these four range from 5·7 to 8·1. Finally, for the facial breadth Boas's means are *greater* than Fishberg's in five cases, and the differences for these five must be supposed markedly significant. Before attempting to find an explanation of this unexpected and disturbing situation, we may compare the standard deviations in Table V with the corresponding values in Table II. Out of the 24 comparisons possible (omitting the values for the pooled series), three significant differences are found, viz. those for the statures of the Galician (Δ/p.e. of $\Delta = 5{\cdot}1$), and Rumanian (3·7) series, and for the head breadth of the Rumanian series (3·6). The standard deviations for Fishberg's material are the greater in the first two cases and the smaller in the third, but judging from all the data it cannot be said that his groups were appreciably more or less variable than those measured for the Immigrant Commission.

It appears that three explanations might possibly account for the fact that several differences which are much greater than any which could be attributed to the chance selection of the samples are found between the corresponding groups of immigrants from different European countries measured in New York on the two occasions. These are concerned with:

(*a*) The possibility that the two sets of measurements were taken in different ways. This appears to us to be quite the most plausible explanation, though other factors may have influenced the result to a lesser extent.

(*b*) The possibility that there were differences in the social constitution of the two samples sufficiently marked to differentiate the mean measurements. Judging from the description of the material, however, the Jews measured on each occasion represented the lower social classes, though there is a possibility that those born in the United States were socially superior to the immigrants. Differences in social standing would be expected to influence stature more significantly than the other characters, but actually the statures for the corresponding groups are practically identical, while some of the head measurements differ significantly.

(*c*) The possibility that the effect observed is due to differences in the types of immigrants from the same country landing in New York at different periods. There is known to be a distinction between the two investigations: Fishberg measured immigrants shortly after they landed, and all his subjects had probably left Europe in the period 1900—1904; the immigrant Jews measured for the Immigrant Commission settled in New York at different times between 1870 and 1909.

Evidence bearing on this last point, which concerns a possible secular change in the type of immigrant, could have been deduced from the material described by Professor Boas. He considered it, but only in a way which appears to be inadequate. After comparing his own means for immigrant Jews from different European countries, irrespective of their years of arrival in New York, he writes: "Furthermore, the composition of the immigrant population subjected to measurement, when distributed according to provinces and to year of arrival in America, did not show material changes in the proportionate number of individuals from each province for different years. For this reason it seemed best to treat the material as a unit, and rather to determine the physical characteristics of the immigrants arriving here from year to year" (1912, p. 99).

In the later discussion of the data all the immigrants are combined for all purposes and, in our opinion, this greatly lessens the value of the analysis. From the published material it is possible to examine the supposition that there was no secular change in the average types of immigrants from each European country (or province), though in a very indirect way. In the case of three head measurements data are given for immigrants of different ages when measured (1912, Tables VII and VIII). Combining some of the distributions for different age groups in order to obtain large enough samples, we obtain the means given in our Table VI.

In the case of the Jews the differences of the constants for the three groups are quite insignificant for head length, head breadth and cephalic index. The facial breadth shows a significant increase with advancing age*. It is possible that this change would be observed in the case of a racially homogeneous population, as an increase in the thickness of facial integuments is a feature of middle age, though not of old age. We have not been able to find a growth curve showing the

* The differences divided by their probable errors give the ratios 6·1, 7·3 and 3·4.

change of the bizygomatic breadth in adult years for any population*. The data for the immigrant Jewesses in Table VI also show an increase for this character with age (Δ/p.e. $\Delta = 8{\cdot}4$). The head length shows a markedly significant increase (6·3) and, since the head breadths for the two groups are identical, the cephalic index shows a decrease with age which is probably quite significant. The conclusions suggested by the male and female means are thus not in agreement. The former are so similar for different age groups that the supposition that the older and younger series of adult Jews represented approximately the same proportions of the Jewish populations in the European countries of origin seems to be justified. For the Jewesses, however, the series under and over 26 years of age seem to have been constituted in different ways. To understand the situation more completely we should need means for different ages referring to immigrants from each of the European countries considered separately.

TABLE VI.

Means for adult Immigrant Jews of different Ages measured for the Immigrant Commission.

Ages	Head length	Head breadth	Cephalic index†	Breadth of face
	Jews			
20—25	187·0 ± ·32 (187)	155·4 ± ·27 (187)	{83·1 (187)}	138·1 ± ·26 (187)
26—49	187·5 ± ·19 (476)	155·5 ± ·17 (475)	{82·9 (475)}	140·0 ± ·17 (467)
50—80	187·5 ± ·45 (97)	155·5 ± ·43 (97)	{82·9 (97)}	141·4 ± ·37 (96)
	Jewesses			
20—25	177·7 ± ·30 (168)	149·8 ± ·25 (169)	{84·3 (168)}	131·1 ± ·23 (168)
26 and over	179·8 ± ·15 (628)	149·8 ± ·13 (629)	{83·3 (628)}	133·4 ± ·15 (605)

† The mean cephalic indices were found from the means of the head length and breadth, as distributions for the indices were not provided.

* More recently we have found data relating to the point in question in a paper by Dr H. Gray ("Body-Build in Illinois Convicts with special reference to Age," *Journal of Criminal Law and Criminology*, Vol. XXV. (1934), pp. 554—575). His means for the bizygomatic breadth for the convicts (whites) are for different ages group:

Age	20—30	30—40	40—50	50—60	60—80
Mean	139·19 ± ·20	140·42 ± ·31	140·73 ± ·41	141·80 ± ·51	143·34 ± ·99
Number ...	281	155	93	46	19

The average rate of increase of the breadth with increasing age is almost exactly the same for this as for Professor Boas's material.

The adult immigrants are grouped by Boas in another way, viz. by different periods within which they arrived in the United States. The total period covered ranges from 1870 to 1909, but few of the immigrants measured reached New York before 1885. Again, combining some of the distributions given by Boas (1912, Table XVIII) in order to have large enough series, we find the means given in our Table VII. The earlier groups here must have been made up by individuals who were older, on the average, than those in the later groups, and this fact may account for parts of some of the differences observed between the means for the groups immigrating at different periods. The male means in Table VII show no significant differences in the case of stature and head and face breadths, and the

TABLE VII.

Means for Groups of Immigrant Jews reaching the United States at different Periods (Immigrant Commission Series).

Period of arrival	Stature	Head length	Head breadth	Cephalic index	Breadth of face
	Jews				
1870—1884	1635±5·0 (51)	188·0±·53 (53)	155·2±·49 (52)	82·7±·23 (52)	140·3±·56 (52)
1885—1899	1644±2·3 (286)	188·5±·26 (286)	155·6±·22 (286)	82·5±·13 (286)	140·0±·23 (285)
1900—1904	1643±2·6 (215)	186·2±·29 (213)	155·1±·26 (213)	83·3±·14 (214)	139·9±·25 (211)
1905—1909	1642±2·8 (210)	186·8±·29 (211)	155·7±·27 (211)	83·4±·15 (211)	139·1±·24 (206)
	Jewesses*				
1870—1884	1544±5·7 (47)	180·9±·54 (47)	151·9±·39 (47)	84·0±·20 (47)	137·7±·56 (46)
1885—1899	1548±2·1 (329)	179·8±·21 (329)	150·0±·18 (332)	83·5±·10 (330)	133·5±·20 (327)
1900—1904	1554±2·4 (250)	178·5±·24 (251)	149·4±·21 (251)	83·7±·12 (251)	132·0±·21 (244)
1905—1909	1550±2·7 (246)	179·2±·25 (246)	149·7±·20 (248)	83·6±·12 (246)	131·9±·22 (233)

* All the Jews whose measurements are included in this table were 20 years old or older when measured. The Jewesses whose measurements are included appear to have been 18 years old or older, though this fact is not stated.

female means show no significant differences in the case of stature and cephalic index. Marked differences are only found between the bizygomatic breadths for the groups of Jewesses. Full interpretation of these results is again impossible owing to the fact that the material is presented in an unsatisfactory way. It seems, however, that some significant deviations between groups of immigrants arriving in the United States at different periods are larger than any which can be supposed solely due to the divergent age constitutions of these groups. It appears to be at least probable that they differ partly because of deviations in the proportions of individuals drawn from various European countries.

We may now ask whether these reductions offer any explanation of the observed differences between the corresponding means given by Fishberg and

Boas for immigrants from various regions in Europe. The Jews measured by the former had probably left Europe in the period 1900—1904 and they give male mean cephalic indices consistently, and in most cases significantly, less than those obtained from measurements taken for the Immigrant Commission and relating to Jews who left Europe between 1870 and 1909 (cf. Tables I and V). An examination of Table VII shows that the group immigrating between 1900—1904 had a higher male mean cephalic index than that for all the remainder, and hence if it were possible to give cephalic indices for the territorial groups in Table V based only on the 1900—1904 immigrants, we should expect them to have indices slightly higher on the average than the values given there. But in being higher they would diverge further from Fishberg's corresponding means, and hence the slight adjustment which seems to be required would make matters worse.

Since it does not appear to be possible to offer any other reasonable explanation, we are forced to conclude that the source of the differences between Fishberg's measurements and those taken for the Immigrant Commission on the corresponding group of immigrants being in the case of some characters unduly large is due to the fact that the readings were obtained in different ways. Personal equation seems, in particular, to be responsible for a difference of more than one unit, on the average, in the cephalic index. If this conclusion be accepted, then one of Fishberg's is rendered doubtful if not entirely invalid. He maintained the theory that in different European countries the Jews approximate to the type of the non-Jewish population among which they live. The evidence offered in support of this view relates almost entirely to the cephalic index. He gives mean values for this character* provided by a number of different observers and relating to series of Jews and non-Jews measured in different European countries. Omitting series measured in the Caucasus, the range for the former is from 80·9 to 83·6, and for the latter from 80·6 to 84·9. The Jewish and non-Jewish means for a particular country clearly tend to be closer to one another than do the Jewish and non-Jewish means for different countries picked at random. But it may be observed that for each of six of the seven countries, the Jewish and non-Jewish measurements were obtained by the same observer. If personal equation may make substantial differences between the readings obtained by two different observers, as is feared, then comparisons of the kind made by Fishberg, which make no allowance for this fact, are likely to be quite fallacious†.

* *Loc. cit.*, Table II, p. 14. The same theory is maintained in Fishberg's book *The Jews* and in several of his other papers. Boas (1912, p. 75) accepts Fishberg's evidence to show that the Hebrew type in Europe varies "generally in accordance with the type of the surrounding population", but he considers that it may be due to the effect of environment rather than to intermixture.

† A considerable amount of evidence of different kinds may be adduced in support of Fishberg's hypothesis regarding the substantial modification of Jewish types in different countries due to intermixture with the surrounding non-Jewish populations. We do not mean to imply that in our opinion this hypothesis is false: our sole purpose is to point out that due regard should be paid to the possible effects of personal equation in measurement when interpreting anthropometric evidence. The average cephalic indices for all the Jewish and all the non-Jewish populations of Central and Eastern Europe are of much the same order, so that a comparison restricted to them is not likely to lead to any very clear conclusions.

Boas's main contention is that owing to the direct effect of environment the children of Jewish immigrants to the United States tended to diverge from the type of their parents, especially in becoming taller, and in acquiring a greater head length, a lesser head breadth and hence a lower cephalic index, on the average. We will now examine the evidence relating to adults only which may have suggested these conclusions, and Fishberg's material can also be considered with the same object in view. In both cases we have measurements for series of immigrants and series of Jewish adults born in the United States, and it is necessary, first of all, to enquire how the two groups were constituted, since it may be possible to explain any differences observed in ways which do not relate to the possible effect of environmental conditions. The factors to be considered are:

(*a*) *Age Constitution.* Of the 39 adult American-born Jews measured for the Immigrant Commission, 27 were aged 20—25 and the remaining 12 were 26 or over (Boas, 1912, Table VII). These individuals were younger, on the average, than the adult immigrant Jews (see our Table VI). Fishberg merely says that all the Jews he measured were over 20 years of age. It is not possible to allow for differences in the age constitutions of the samples compared, but it is unlikely that this omission is of any consequence.

(*b*) *Social Constitution.* More than half of the immigrants measured by Fishberg were applicants for charitable relief. His subjects born in the United States were undoubtedly of a higher social standing, on the average, than the immigrants, and they were further selected owing to the fact that a large proportion of the "native" Jews who were asked refused to submit (see p. 2 above). Boas gives no information regarding the social condition of his groups beyond that referred to above (p. 13). He remarks (1912, p. 82) that "a considerable amount of opposition was offered to observers from time to time," and it is likely that the immigrants were more willing to be measured than the "native" born, as in Fishberg's experience. No exact allowance can be made for these factors, but it is probable that parts of the differences observed can be attributed to them. We should expect that the social selection, and the selection due to willingness to be measured, would be reflected in considerable differences between mean statures, but not between mean cephalic indices.

(*c*) *"Racial" Constitution.* Both Fishberg's and Boas's data reveal significant differences between the types of Jewish emigrants from different European countries. In comparing the immigrants with Jews born in the United States, it is thus essential to have information regarding the European or other countries from which the parents or earlier ancestors of the "native-born" Jews emigrated. No information whatever of this kind is given by either Fishberg or Boas. The latter combines all his immigrants, and compares pooled means for them with means for all the subjects born in the United States (1912, Tables 4, 8 and 12). This procedure might be of some value if it could be assumed that the European countries of origin were represented in approximately equal proportions in the two samples. But it would be highly unreasonable to make such an assumption.

In the history of New York Jewry the period from 1812 to about 1881 is known as that of German immigration*. The new-comers then came predominantly from Germany, though some were from Poland. The period from 1881 to some time later than 1909, which is the last date we need consider, is known as that of Russian immigration, the vast majority of the Jewish immigrants then having left the Russian Empire (including Poland), Austria-Hungary and Rumania. All Fishberg's and Boas's groups of immigrants for which constants are given represent these last countries, and German Jews are not represented. But nearly all the *parents* of the adult Jews and Jewesses born in the United States and measured by Fishberg about 1904 must have left Europe before 1881, unless they were born in the United States. Hence it is reasonable to conclude that these subjects were principally of German Jewish origin. Similarly, the majority of the *adult* Jews and Jewesses born in the United States and measured for the Immigrant Commission in 1909 were most probably of German Jewish origin†. All the immigrants measured by Fishberg for whom data are given are known to be of "Russian" (including Austrian and Rumanian) origin. Of the total 799 adult Jews measured for the Immigrant Commission, 733 are known to have been born in eastern Europe, 39 were born in America, and some of the remaining 27 may have been born in Germany. It appears then that in comparing the immigrant and American-born series, in the case of either investigation, we are probably making a comparison primarily between groups of Jews of "Russian" and German origin. It will be idle to attribute any differences noted to the effect of environment if the two populations represented—one of which is known to be heterogeneous—were different in type in their European homelands. The data considered so far are, in fact, of such a nature that no conclusive answer to the question now considered can be derived from them. A comparison of the means for the American-born and immigrant series is of some interest, however. It can be seen from our Table I that the Jews born in the United States who were measured by Fishberg have a decidedly greater mean stature than those given by all the series of immigrants; their mean head circumference and head length are also the largest observed, while for head breadth and cephalic index the American-born Jews have rather small but not extreme values. A similar comparison of the means derived from the measurements taken for the Immigrant Commission leads to similar conclusions: the stature for the American-born Jews is decidedly the greatest, the head length is large but not extreme this time, while the head breadth and cephalic index are the smallest observed, although they are small but not extreme in the case of Fishberg's material. The only discordance between the two investigations com-

* The facts stated relating to the Jewish population of New York are taken from the article "New York" in *The Jewish Encyclopaedia*, Vol. IX. (1916), p. 259. Estimates of the size of the population at different periods are given there.

† Of the 39 Jews, 12 were 26 years old or older when they were measured and 15 of the 42 American-born Jewesses over 20 were 26 years old or older (Boas, 1912, Table 4). The immigrant ancestors of these people probably reached America before 1870 as Professor Boas says: "The attempt was also made to collect material from American families settled in this country for several generations and living under more favorable conditions" (1912, p. 3).

pared in this way is noted in the case of the facial breadth: for Boas's material the American-born Jews have by far the smallest mean, but for Fishberg's their mean for this character is as large as that for any other group. Ignoring the facial breadth, we can say that there is no clear divergence of the type of the American-born Jews from those of *all* the groups of immigrants except in stature. It appears most reasonable to conclude that the greater stature of the former is probably due partly to the fact that they represented a higher social class (on the average) than the immigrants, and partly to the fact that they were more stringently selected, since a smaller proportion of the American-born Jews who were asked were willing to be measured. An excess in stature of German over "Russian" Jews may also account partly for the difference observed.

(4) *The Investigation of the United States Immigrant Commission reported on by Professor Franz Boas:* (*b*) *The Measurements of Parents and Children.* As the data for unrelated adults provide no evidence of the effect of environment, it may now be asked whether such can be obtained from the measurements of the adult and non-adult related Jewish individuals. Professor Boas evidently considered that the clearest proof of his theory was to be derived from the comparison of parents and children. He writes (1912, p. 69): "In order to overcome all possible objections based on the assumption of a different composition of the immigrant series and of the American-born series, I have also compared the measurements of parents and their own foreign-born and American-born children." We only propose to examine these family data relating to a single character—the cephalic index—since this is the one to which he attaches greatest importance. Means are given for each year of life from 4 to 20 and over, but the numbers for some years are small, and in order to obtain sufficiently large samples we have computed the means for age groups covering five years each in the case of the non-adult individuals. The cephalic index only changes slightly with age, and growth curves for it may be misleading unless they are based on large numbers of individuals for each year of life.

Comparisons are to be made between the cephalic indices of parents and children, and in order to interpret these it is necessary to make corrections for the differences which would be expected owing to difference in age. Data bearing on this question are given in our Table VIII. These are derived from:

(*a*) The series of Jewish individuals measured for the Immigrant Commission (1912, Table VII, p. 255 and Table VIII, p. 269). Distributions are given, and from these we calculated standard deviations and thus obtained probable errors.

(*b*) Series of Germans measured in Dresden, Frankfurt a. M., the Thuringian States (Gotha, Sondershausen and Nordhausen) and Bavaria (Kötzting). Data for these are given by Dr C. Röse in "Beiträge zur europäischen Rassenkunde und die Beziehungen zwischen Rasse und Zahnverderbnis" (*Archiv für Rassen- und Gesellschafts-Biologie*, 2. Jahrgang (1905), S. 689—798). Boas (1912, Tables 63 and 64) quotes Röse's means* (Tabelle 12—24) for ages 6—20 from this paper,

* One is misquoted: the mean index for Nordhausen boys 6 years of age should be 82·9 in place of 82·4.

and he also gives values for the age group 26 years and over. We have been unable to find the source of these last; they do not appear to be derived from the first (*loc. cit.*) or second (*ibid.* 3. Jahrgang (1906), S. 42—110) parts of Röse's memoir.

(*c*) Series measured in Worcester (Massachusetts), for which mean indices are given by Boas (1912, Table 63). These are said (1912, p. 78) to have been obtained from a report on "Statistics of Growth" by Franz Boas and Clark Wissler (*United States Bureau of Education. Report of the Commissioner of Education for 1904*, Chapter II). This report gives mean head lengths and breadths, but no indices and no measurements of adults.

(*d*) Dutch series consisting of visitors calling on the inmates of an asylum at Rotterdam and their families. Mean indices for different ages are given by Dr G. P. Frets in *The Cephalic Index and its Heredity* (1925), p. 18.

(*e*) Series of Australians (whites) measured in Melbourne and reported on by Professor R. A. Berry and Dr S. D. Porteus. (*Intelligence and Social Value. Publications of the Training School at Vineland, New Jersey.* No. 20 (1920).)

(*f*) Series of school children almost entirely "of the English professional classes from Kindergarten to the great public girls' and boys' schools." The ages range from 4 to 19, and the material was collected for Professor Karl Pearson between 1895 and 1900. Mean cephalic indices are given by him and Mr L. H. C. Tippett in a paper "On Stability of the Cephalic Indices within the Race" (*Biometrika*, Vol. XVI (1924), pp. 118—138). The adult (18 years and over) female mean in our Table VIII was obtained from the same "school children" series (Pearson and Tippett, Table V), but there is an insufficient number of male adults in the same series, so we have given the mean provided by them (p. 129) in the same paper for 12,255 Englishmen. These male adults, considered as a single sample, were less stringently selected socially than the boys. A warning relating to this point is given: "When the social class is changed, the cephalic index is changed. We do not look upon this as due to environment and nutrition, but to a slight amount of racial difference in the various social strata."

Data from these sources are given in Table VIII. The age groups are actually not exactly the same for all the series, several of the means for the group 5—9, for example, being based on measurements for children 6—9 as none for the sixth year of age are available. In the case of the Jewesses Boas groups all of 18 years and over as adults, so the same has been done for the other female series, and the previous group is taken from 15—17 instead of from 15—19. The change of the cephalic index with age is so slight that these adjustments are not likely to make any appreciable difference. We have to obtain from this material corrections which can be applied in comparing each of a number of Jewish parents with his own child or children. It may appear, at first, as though the best corrections to use will be those derived from the Jewish series in Table VIII, but this view may be questioned. It has been shown that the adults form a heterogeneous sample (our Table V), that the older adult Jewesses have a lower mean cephalic index than those with ages 20—25 (Table VI), and that the immigrant Jews who arrived

TABLE VIII. *Mean Cephalic Indices of Jewish and other Series for different Age Groups*.*

Ages	Male series							
	Foreign-born New York Jews (Boas)	American-born New York Jews (Boas)	Germans: Dresden I—IV (Röse)	Germans: Gotha & Sondershausen (Röse)	Germans: Nordhausen (Röse)	Germans: Kötzting (Röse)	Germans: Frankfurt a. M. (Röse)	Americans: Worcester (Boas & Wissler)
5—9	84·6±·20 (130)	82·9±·11 (365)	87·1 (1096)	—	82·7 (856)	85·2 (141)	81·1 (345)	79·3 (534)
10—14	84·4±·081 (587)	82·3±·048 (1710)	86·7 (3179)	83·4 (162)	81·6 (1020)	85·8 (107)	81·2 (189)	79·1 (807)
15—19	83·6±·16 (207)	81·6±·12 (301)	85·3 (1560)	81·9 (168)	—	—	—	78·4 (297)
20 and over	83·0±·078 (764)	81·4±·27 (39)	85·2 (2861)	81·8 (299)	80·7 (1234)	83·7 (226)	79·5 (417)	77·8 (1222)
	Means for young age groups—adult mean							
5—9	+1·6±·21	+1·5±·29	+1·9	—	+2·0	+1·5	+1·6	+1·5
10—14	+1·4±·11	+0·9±·27	+1·5	+1·6	+0·9	+2·1	+1·7	+1·3
15—19	+0·6±·18	+0·2±·30	+0·1	+0·1	—	—	—	+0·6

Ages†	Male series			Female series†				
	Dutch: Rotterdam (Frets)	Australians (Berry & Porteus)	English (Pearson & Tippett)	Foreign-born New York Jewesses (Boas)	American-born New York Jewesses (Boas)	Dutch: Rotterdam (Frets)	Australians (Berry & Porteus)	English (Pearson & Tippett)
5—9	80·8 (159)	78·7 (148)	78·9 (497)	84·9±·22 (93)	83·0±·14 (245)	81·4 (125)	79·3 (376)	79·1 (416)
10—14	81·3 (158)	79·2 (858)	78·9 (1367)	84·6±·16 (152)	82·3±·14 (243)	80·6 (153)	78·7 (1151)	78·2 (1225)
15—19	80·6 (131)	79·3 (949)	78·9 (440)	83·9±·16 (115)	81·9±·24 (76)	81·2 (98)	78·8 (1027)	78·2 (449)
20 and over	79·7 (137)	78·7 (149)	78·55 (12,255)	83·6±·063 (895)	82·3±·26 (65)	80·9 (281)	78·8 (163)	78·3 (91)
	Means for young age groups—adult mean							
5—9	+1·1	0·0	+0·35	+1·3±·23	+0·7±·30	+0·5	+0·5	+0·8
10—14	+1·6	+0·5	+0·35	+1·0±·17	0·0±·30	−0·3	−0·1	−0·1
15—19	+0·9	+0·6	+0·35	+0·3±·17	−0·4±·35	+0·3	0·0	−0·1

* See pp. 23—24 of text for a description of the data in this table.

† The two oldest age groups for all the female series are 15—17 years and 18 years and over.

in New York after 1900 had a higher mean index than those arriving before that year (Table VII). But the children of the later immigrants must have been younger when measured, on the average, than the children of the earlier immigrants. Hence when all the material is pooled the younger groups will be expected to have the higher cephalic indices merely on account of the fact that the nature of the population sampled was changing with time. Furthermore, the number of adult American-born Jews and Jewesses are too few to give reliable differences between their means and those of the groups of children belonging to the same class, as can be seen from the probable errors of the differences given in the table. The best corrections to use in comparing parents and children would appear, rather, to be ones derived from a stable and homogeneous population, or populations. For this purpose the remaining data in Table VIII were collected. Unfortunately the different series do not all suggest the same conclusion. It is possible in some cases that the adults and children do not represent exactly the same population (differences in social class possibly being of importance), and in some cases the corresponding adult and non-adult samples were probably measured by different observers so that "errors" of personal equation may have influenced the results to an appreciable extent. There may also be racial differences in the way that the cephalic index changes with age. Data obtained from a stable and homogeneous Jewish population would hence be the most appropriate to use, but there are no reliable data for such a population available in the case of the cephalic index. We are dealing here with small differences, but it will be seen below that any final conclusions it may be possible to reach depend on equally small quantities. We eventually decided to attach greatest importance to average unweighted differences for all the non-Jewish series in Table VIII. In accordance with this decision, the mean cephalic index for fathers will be expected to be 1·24 less than the mean for their sons of 5—9 years, 1·28 less than for their sons 10—14, and 0·44 less than for their sons 15—19: the average index for mothers will be expected to be 0·60 less than for their daughters 5—9, 0·17 *greater* than for their daughters 10—14, and 0·07 less than for their daughters 15—17. The application of these differences to the Jewish material is only a *pis aller*, and the fact that they do not show regular sequences suggests that they are far from ideal, but it may be hoped that will not mislead greatly.

The means of the differences between different groups of Jewish parents in New York and their own children are given in Boas's Table XXIX*. For the cephalic index we have computed from his values for yearly intervals in the ages of the children the "actual mean differences" for different age groups of the

* The way in which the data in this table were compiled is not explained clearly, but it is safe to assume that individual differences were found between each parent and his own child or children. A particular parent may thus have been counted several times, a child may have been counted once or twice according as measurements were available for one or both of his parents, and brothers and sisters will have been included. Further it appears (see 1912, pp. 69—70) that the tabulation was confined to parents having at least one American-born and at least one foreign-born child measured. Thus every parent occurs at least once in a column headed "American-born" and at least once in a column headed "Foreign-born" in our Table IX.

children in the upper part of our Table IX. Probable errors cannot be given as distributions are not provided. "American-born" and "Foreign-born," in this table refer to the children: all the parents of the American-born children must have been foreign-born themselves, if we are right in supposing that every one had a foreign-born child or children. The crude (or actual) mean differences clearly need to be corrected for two factors before it can be seen whether the children are reproducing the types of the parental population or not. The first factor is age, and the average divergences expected between adults and children falling in different age groups given above were applied. The second factor is sex in the comparisons of parents and children of opposite sexes.

The data given in Tables VI and VII above relate to the total samples of which the mothers and fathers entered in Table IX form sub-samples. They suggest that on the average the cephalic index for the New York Jewesses was almost exactly 0·6 greater than that for the New York Jews. Accepting this value we proceeded in the following way. In the comparison of American-born daughters and fathers, say, 0·6 was first subtracted from the four actual differences in Table IX to give the differences expected between daughters and mothers, the mothers' mean indices being inferred from those of the fathers. The female age allowances were then applied to these adjusted values to give the differences corrected for age and sex in the table. In a similar way, in comparing sons and mothers the mean indices for fathers were first inferred from the values for mothers (using the same sex difference of 0·6), and then the male allowances for age were applied.

The divergences from zero of the corrected differences between the mean cephalic indices for parents and children in Table IX should give an estimate of the extent, if at all, to which the American-born and foreign-born Jewish populations had changed in type, for the character in question, in one generation. The differences are all small, and it must be admitted that several of them may differ appreciably from the best possible values which could be deduced from the data owing to the fact that the corrections made for age and sex are not ideal. A comparison may be made first between parents and their adult children. This is the most direct one possible since no corrections for age are required, and only two of the four classes of differences have to be corrected for sex. Combining all the classes, we find from the bottom row of Table IX that the weighted mean difference (adult child − parent) is $-0{\cdot}58\,(100)$ for the American-born and $+0{\cdot}45\,(392)$ for the foreign-born Jews. These values are decidedly smaller than we should have expected after reading Professor Boas's conclusions, and it must be asked whether they can be considered to differ significantly from zero or not. Their true probable errors cannot be given, but approximations to them may be obtained by supposing that the standard deviations of all four distributions involved are 3·0 (cf. Table V above). The mean differences are then $-0{\cdot}58 \pm {\cdot}28$ for the American-born and $+0{\cdot}45 \pm {\cdot}14$ for the foreign-born (the ratio of the difference to its probable error being 2·1 for the first and 3·2 for the second), if we ignore the fact that the indices for parents and their own children will be correlated variates. If this correlation

TABLE IX.

Mean Differences for the Cephalic Index between Jewish Parents in New York and their own Children.*

Ages of Children†	Actual mean Differences							
	Son—Father		Daughter—Mother†		Daughter—Father†		Son—Mother	
	American-born	Foreign-born	American-born	Foreign-born	American-born	Foreign-born	American-born	Foreign-born
5—9	+0·7 (137)	+2·2 (56)	+0·4 (172)	+2·7 (69)	+0·4 (116)	+1·9 (50)	0·0 (181)	+1·5 (74)
10—14	−0·5 (79)	+0·9 (80)	−0·8 (136)	+1·4 (107)	+0·3 (93)	+1·2 (75)	−1·0 (116)	+1·3 (100)
15—19	−1·25 (32)	0·0 (48)	−0·4 (45)	+0·1 (63)	+0·65 (32)	+1·1 (49)	−2·7 (41)	−1·2 (60)
20 and over	−0·3 (18)	+0·5 (69)	−0·6 (31)	+0·5 (127)	+0·7 (36)	+1·0 (120)	−3·1 (15)	−0·2 (76)
	Differences corrected for age and sex							
5—9	−0·5	+1·0	−0·2	+2·1	−0·8	+0·7	−0·6	+0·9
10—14	−1·8	−0·4	−0·6	+1·6	−0·1	+0·8	−1·7	+0·6
15—19	−1·7	−0·4	−0·5	0·0	0·0	+0·4	−2·5	−1·0
20 and over	−0·3	+0·5	−0·6	+0·5	+0·1	+0·4	−2·5	+0·4

* See pp. 26—27 of text for a description of the data in this table.

† For the comparisons Daughter—mother and Daughter—father the two oldest age groups are 15—17 years and 18 years and over.

is supposed 0·5 the mean differences are $-0{\cdot}58 \pm {\cdot}20$ for the American-born and $+0{\cdot}45 \pm {\cdot}10$ for the foreign-born. We thus reach the following unexpected conclusions: The American-born adult Jewish children measured for the Immigrant Commission had a lower average cephalic index than their parents, but the difference need not be considered significant, while the foreign-born Jewish adult children tended to have higher indices than their parents, and the difference in this case appears to be statistically significant.

Combining the four different kinds of comparison, the differences (child − parent) corrected for age and sex in the lower half of Table IX may be summarised in the following way:

Ages of children	American-born	Foreign-born
5—9	−0·50 (606)	+1·21 (249)
10—14	−1·01 (424)	+0·72 (362)
15—19	−1·20 (150)	−0·27 (220)
20 and over	−0·58 (100)	+0·45 (392)
All ages	−0·76 (1280)	+0·55 (1223)

There can be no doubt that both of the mean differences for all ages, and several of the differences for the age groups, differ from zero with marked significance, and the fact that all the differences for the American-born are negative is obviously suggestive. But the age corrections on which the figures are based may deviate appreciably from the most appropriate, and unknown, corrections. If the average age differences for all the foreign-born Jews given in Table VIII are applied to the differences between foreign-born children and parents in Table IX, and the differences for the American-born Jews are applied to the family statistics for the American-born, while also making the sex allowance (female adult mean − male adult mean = 0·6) where necessary, then the mean corrected difference is found to be − 0·79 (1280) for the American-born and + 0·20 (1223) for the foreign-born. If it is supposed that no corrections for age are needed, as the Australian and English material in Table VIII may suggest, while making the sex allowance as before, then the mean corrected difference becomes −0·14 (1280) for the American-born and +0·95 (1223) for the foreign-born. The first of these values probably differs insignificantly from zero, so, after making assumptions of this last kind, we should be obliged to conclude that the foreign-born children were diverging from the type of their parents in having larger cephalic indices, while the American-born children remained true to type. In our opinion the data collected for the Immigrant Commission are not capable of leading to definite proofs of these or any alternative hypotheses of the same kind. The average divergences of the children from their parents are very small in the case of the cephalic index, and it is conceivable that if the ideal corrections for age and sex could be applied—these corrections not being necessarily the same for the American-born and the foreign-born series—then both average divergences might

be found to be no greater than those which would be expected owing to "errors" of random sampling. As far as the Jewish material is concerned, there seems to be no justification whatever for the statement, said to be "amply proved", that there is "a far-reaching change in the type [of immigrants]—a change which cannot be ascribed to selection or mixture, but which can only be explained as due directly to the influence of environment" (1912, p. 2). If no conclusive evidence of the existence of such a change in the cephalic index is given, as we contend, then no explanation of its presumed existence need be accepted*.

(5) *Conclusions.* (I) A comparison between Dr Fishberg's records for groups of immigrant Jews and Jewesses in New York coming from different parts of Europe suggests that the head breadth and eye colours were practically identical for all the populations represented. The horizontal circumference of the head, nasal index and hair colours show a few significant differences, while the stature, head length and cephalic index show differences which are small absolutely, but several of which are large enough to indicate clear differentiation of the populations sampled.

(II) The means of the adult series of Jews from different parts of Europe measured for the Immigrant Commission also show some clearly significant differences, though the incidence of characters of greater or lesser significance in differentiating the types is rather different for the two investigations. The series show few significant differences in variability, and it cannot be said that Dr Fishberg's groups were appreciably more or less variable than those measured for the Immigrant Commission.

(III) A comparison of Dr Fishberg's means for Jews from different parts of Europe with values for corresponding groups given by Professor Boas reveals several markedly significant differences—notably in the case of the cephalic index—which can only be attributed to differences in the ways in which the measurements were taken.

(IV) The adult immigrant Jews and Jewesses measured for the Immigrant Commission show some significant differences between means for different age

* Even if a markedly significant, but small absolute difference had been found between the cephalic indices of the parents and children, it does not follow necessarily that Professor Boas's explanation need be accepted. A comparison of the mean cephalic indices of English-born and foreign-born Jewish boys in London schools has been made by Professor Karl Pearson and Miss Margaret Moul ("The Problem of Alien Immigration into Great Britain, illustrated by an Examination of Russian and Polish Jewish Children, Part I." *Annals of Eugenics*, Vol. I. (1925), pp. 5—127). They find (footnote, pp. 27—28) a significant difference between the breadth-length indices but not between the height-length indices of the head. The difference in the former case is not attributed to the immediate influence of environment as it is pointed out that any crossing would tend to reduce the first cephalic index more than the second. The writers ask: "Is it possible for the group of children born in this country, that a few cases in which the alien has married a Gentile wife or an English Jewess with some Gentile blood may suffice to explain the slight fall in the first cephalic index?" One may go further than this in suggesting that an indubitable proof that environment was solely responsible for such an observed difference would require a guarantee that all the fathers and mothers of the children were of pure Jewish descent. [It may be remarked that all the children had been admitted as Jewish to the *Jews Free School* and the birth-places of Father and Mother were included in the data. Ed.]

groups, and between groups arriving in New York at different periods. By pooling together all the immigrants, Professor Boas lessens the chance of reaching any exact conclusions regarding modification of types from measurements of adults.

(V) A comparison of the adults born in the United States with all the immigrants is primarily one between Jews of German and of "Russian" origin, in all probability. The former series only diverges appreciably from all the groups of immigrants in having a greater mean stature, and social and other forms of selection probably account for the greater part of this difference. Nothing can be inferred from comparisons of the adult series of unrelated individuals with regard to possible modification of type due to changing environmental conditions.

(VI) The direct comparison made by Professor Boas between parents and their own children cannot be objected to on the ground that the material is to some extent racially heterogeneous. In treating his Jewish family data, the cephalic index is the only character considered in the present paper. The mean differences given between parents and their own children are all small, and of the same order as differences expected owing to diversity of age and sex: correct interpretation hence depends on the accuracy with which allowance can be made for these factors. Age corrections ought not to be obtained from the combined Jewish immigrant material, as there is evidence of significant differences between groups of adults of different ages or different periods of arrival in New York.

The age corrections applied are average values derived from a number of other populations believed to have been more stable. This is not a satisfactory procedure, since the age differences for the different non-Jewish series used differ very appreciably, and hence any results based on them are of uncertain value. Sex allowances required in comparing parents and children of opposite sexes can be obtained from the combined Jewish material, and these are less likely to mislead. Comparisons between parents and their adult children require corrections for sex in two of the four possible combinations, but none for age, so greatest importance should be attached to them. They show that the American-born adult Jewish children had a slightly lower mean cephalic index than their parents ($\Delta = -0{\cdot}58$), but this difference is probably not significant, while the foreign-born adult children had a higher cephalic index than their parents ($\Delta = +0{\cdot}45$), and this difference appears to be of greater statistical significance. Comparisons between parents and their children of all ages, after applying what appear to be the best available age and sex corrections, lead to slightly larger differences of the same signs in the two cases. But other reasonable age corrections lead to very different results, and it is conceivable that if the best possible, but unknown, allowances could be made, that the divergences in type of parents and children would be found to be no greater than those which would be expected owing to "errors" of random sampling. Our general conclusion is that considerably larger divergences would have to be found in order to establish the theory that head-form, as estimated by the cephalic index, is modified directly by environment.

ON JEWISH-GENTILE RELATIONSHIPS.

By Karl Pearson, F.R.S.

The following table is provided by Maurice Fishberg in his "Materials for the Physical Anthropology of the Eastern European Jews*." Owing to the fact that we are here dealing with a high degree of brachycephaly, we fear the real significance of this table has been overlooked.

Mean Cephalic Index of Jews and of Gentiles of the same Districts.

District	Jews		Gentiles	
	Index	Observer	Index	Observer
Caucasus	87·5	Pantyukhof	87·9 (Aïssers)	Aruturoff, Pantyukhof
			87·0 (Armenians)	Ivanowski
Galicia	83·6	Majer and Kopernicki	84·4 (Poles)	Majer and Kopernicki
			84·3 (Ruthenians)	
Baden	83·5	Ammon	84·1 (Germans: Baden)	Ammon
Little Russia ...	82·9	Talko-Hryncewicz	83·2	Talko-Hryncewicz
Turin	82·4	Lombroso	84·9 (Italians: Turin)	Lombroso
Lithuania	81·7	Talko-Hryncewicz	80·6 (Letto-Lithuanians)	Talko-Hryncewicz
Russian Poland ...	81·9	Elkind	80·85 (Poles in Russia)	Elkind
White Russia ...	80·9	Yakowenko	83·2 (White Russians)	Talko-Hryncewicz
			81·9 („ „)	Ivanowski
Mean	$m_J = 83{\cdot}05$	—	$m_G = 83{\cdot}50$	—
Standard Deviation	$\sigma_J = 1{\cdot}888{,}121$	—	$\sigma_G = 2{\cdot}093{,}741$	—

Correlation of Jewish and Gentile Cephalic Indices = ·8365.

This is a most notable value, and although the districts are few, the means are based on reasonably long series. I have not attempted to enlarge the data, but I feel fairly confident that if Western European Jews were included the correlation would be even strengthened. The correlation is too high to be treated as due merely to random sampling. It is certain that there is a high relationship between the Jewish cephalic index and that of the environmental Gentiles. This can only be due to one of two causes:

(i) The environment influences the cephalic index. This is Professor Boas' conclusion,

or (ii) There exists a very considerable amount of Jewish-Gentile intermarriage.

The first explanation would be widely destructive of modern anthropological conceptions, and the second I have found hotly contested by many Jews themselves, although they have never to my knowledge appealed to definite statistics.

Luckily we have definite marriage statistics for one nation—the German—which are most valuable at the present time. They have been compiled by Mr Udny Yule†, in a most interesting summary. From this I deduce the following conclusions:

For the 25 years 1906—1930, 15·6 °/₀ of the persons of Jewish descent who married, married Gentiles.

* *Memoirs of the American Anthropological Association*, Vol. 1 (1905—1907), Table II, p. 14.

† *Journal of the R. Statistical Society*, Vol. 96 (1933), pp. 478—480.

For the seven post-war years 1924—1930*, before the National Socialists came into power, 20·8 °/₀ of the persons of Jewish descent who married, married Gentiles.

From this it is clear that the Jewish community was up to 1930 being not slowly but rather rapidly assimilated in important characters to those of its German environment. I am fairly certain that a like assimilation is going on in France and Great Britain. Of course the German-speaking peoples are far from forming a single race. They cover a wide range of racial groups still very imperfectly mixed, and no one pure German type really exists; they, the Germans, are far from a pure Nordic community. Under the circumstances, they have taken the term "Aryan" to describe their commixture—a word stolen from linguistics—and covering a vast variety of racial groups other than German. Not only owing to the assimilation was the Jew in Germany feeling himself by culture and family ties more and more a member of the German community, but actually in the 16 years 1910—1925 the Jews by confession had decreased some 8 per cent.† The present German Government by checking a natural amalgamation, which was advancing quite rapidly, and by throwing the Jews back on themselves, has perpetuated a "racial" distinction, instead of allowing it to disappear. If tyranny must interfere to hasten a natural process, it would be more successful in its purpose if, instead of forcing Jew to marry Jewess, it had forced every Jew and Jewess to marry a Gentile‡!

However, I am moving far from my point, which has been to emphasise the fact that intermarriage in itself is adequate to explain the observed fact, that the Jewish population approaches in its physical characters the population in the midst of which it lives; there is no necessity to call on environment to account for changing cephalic index.

It may be objected that I am applying German statistics of Jewish-Gentile intermarriages to the condition of affairs in America. But failing available data for Jewish-Gentile marriages in the United States, I am strongly of opinion that they will ultimately be found to correspond closely with the condition of affairs among the western nations of Europe.

* Excluding the post-war period of postponed marriages.

† I do *not* put this forward as an advantage because, stripped of its formalism and tribalism, the Jewish Unitarianism seems to me personally a higher form of religious faith than the Gentile Trinitarianism.

‡ Some idea of this sort seems at last to have struck the National Socialist Government, for I read in *The Times* of Nov. 16th, 1935—sometime after the above was written—that *Mischlinge*, or persons of 25 °/₀ or 50 °/₀ Jewish blood, may marry *with consent of the authorities* an "Aryan" or a 25 °/₀ *Mischling*, if the said authorities approve of the moral and physical characters of the applicant and his family history. This is far from a complete adoption of a natural assimilation policy, for it stops assimilation for full Jews and Jewesses, wherein the chief field for assimilation must lie.

METHOD OF MOMENTS AND METHOD OF MAXIMUM LIKELIHOOD.

By KARL PEARSON, F.R.S.

Wasting your time fitting curves by moments, eh?

"Perhaps the most extended use of the criterion of consistency has been developed by Pearson in the 'Method of Moments.' In this method, which is without question of great practical utility, different forms of frequency curves are fitted by calculating as many moments of the sample as there are parameters to be evaluated. The parameters chosen are those of an infinite population of the specified type having the same moments as those calculated from the sample.... Moreover for that class of distribution to which the method can be applied, it has not been shown except in the case of the normal curve, that the best [sic!] values will be obtained by the method of moments. The method will in these cases certainly be serviceable in yielding an approximation, but to discover whether this approximation is a good or a bad one, and to improve it, if necessary, a more adequate criterion is required." **R. A. Fisher,** *Phil. Trans.* Vol. 222 A, p. 321 (1922).

"...the omission of such simple tests of significance as "Student's" test of the arithmetic mean is for the working biologist, poorly compensated by the inclusion of an account of Professor Pearson's frequency curves, and the traditional, but inefficient method of fitting them by moments." Review by R. A. Fisher of Feldman's *Biomathematics. Annals of Eugenics*, Vol. VI. p. 252.

In view of the "inefficiency" of the Method of Moments asserted in the last quotation and in the need for a "more adequate criterion" suggested in the first quotation, I wrote to Professor Fisher, as I am always ready to adopt new methods if they show better results and are practical in application, asking if he would point out to me the more efficient way of fitting frequency curves than by moments, and hoping for an illustration of its application. He most kindly referred me to his paper in the *Phil. Trans.* cited above, to his *Proc. Camb. Phil. Soc.* paper in Vol. XXII. pp. 700—725, and to a paper by Koshal in the *Journal of the Royal Statistical Society*, Vol. 96, pp. 303—313 (1933).

I re-read the first two papers, and read the third, which as far as I am aware is the only case at present published of the application of Professor Fisher's method. To my astonishment that method depends on first working out the constants of the frequency curve by the Method of Moments, and then superposing on it, by what Fisher terms the "Method of Maximum Likelihood," a further approximation to obtain, what he holds he will thus get, "more efficient values" of the curve constants. The additional computing work after the determination by moments is

as long as, I think longer than, the first fitting by moments. The practical value of the method therefore turns on the gain in accuracy obtained by the increased labour. Koshal in the paper cited writes:

"I am highly indebted to Dr R. A. Fisher, Sc.D., F.R.S. for suggesting to me this problem, and for the very valuable advice which he has given at every important step" (p. 312).

This statement—i.e. that the paper was worked out under the supervision of Professor Fisher—and the latter having referred me to the paper as an illustration of his method, justify me in speaking of the paper as the Koshal-Fisher paper in order to distinguish it from another memoir, the Koshal-Turner memoir, to which I shall have to refer directly.

Now the results of this paper, which was clearly planned to show how the "Method of Moments" is much inferior to the "Method of Maximum Likelihood," are given at the bottom of Table II (p. 311), where we find it stated that

	Method of Moments	*Method of Maximum Likelihood*
χ^2 } $n=7$	7·6552	5·5375
P }	(0·371)	(0·596)
L (Likelihood)	−947·134,306	−946·494,469

Now the first thing to note here is that my P, χ^2 method is far more sensitive than the Likelihood L, where the negative value has only been reduced from −947·1 to −946·5, or about ·061 % in the value, by the additional labour. The χ^2 has however been reduced 27·7 %. It may be doubted whether the change in the Likelihood is not within the number of decimal places retained in certain portions of the calculations. We need not trouble about this, for there are serious blunders in the calculations by both methods, which entirely vitiated the results just given! Let us consider what are the conditions under which a true comparison between the two methods could be made:

(i) The raw statistical data must be tabled in a manner suitable to the problem to be investigated.

(ii) The moments must be calculated and properly corrected.

(iii) There must be some agreed common standard of what is meant by comparative efficiency, i.e. some accepted definition of "best."

(iv) There must be accuracy in arithmetic so that no preference is given to either method by blunders in computing, or by inexplicable change of constants.

All these points seem to be entirely overlooked in the Koshal-Fisher paper. Lastly, how can it be a waste of time fitting curves by moments, if such a process enables one to see the dangers in classifying material, the need for properly computing moments, and trains one in accurate numerical work? Had Koshal and Turner had such experience, they might have saved me from writing this paper. Had Professor Fisher, besides giving very valuable advice at every important step, investigated also Koshal's arithmetic, he would probably have been less dogmatic about the importance of the Koshal-Fisher results, which he has doubtless

emphasised to others as well as to myself as an illustration of the inefficiency of the Method of Moments.

The reader will be anxious now to learn what Turner has to do with the Koshal-Fisher paper. The fact is that in the latter paper there is *no* statement whatever as to how the moments have been calculated on which the theoretical frequencies are based in order to reach the final comparison of the two methods. The values of these moments are not even given. If the curve be put into the form

$$y = y_0 (x - \alpha)^{m_1} (\beta - x)^{m_2},$$

we are simply told that

$$\alpha = 0{\cdot}32980, \quad \beta = 16{\cdot}67375, \quad m_1 = 0{\cdot}702{,}432, \quad m_2 = 4{\cdot}948{,}333,$$

and no value is given for y_0.

In order to throw light on these somewhat mysterious results, we are bound to go back to a memoir by Ram Sarum Koshal, M.Sc., and A. James Turner, M.A., D.Sc., published in the *Journal of the Textile Institute Transactions*, Vol. XXI, pp. 325—370 (*Technological Laboratory, Bombay*, April 16, 1930). This paper deals with many other characters than the strength of cotton fibres, but it provides in groups of 1000 the strengths of 3000 Surat cotton fibres (1027, A.L.F. (1925—1926)). It is the frequency distribution of the last 1000 which we are called upon to deal with. This runs*:

Weight in grammes

0·0—0·9	*1·0—1·9*	*2·0—2·9*	*3·0—3·9*	*4·0—4·9*	*5·0—5·9*	*6·0—6·9*	*7·0—7·9*	*8·0—8·9*	*9·0—9·9*	*10·0—10·9*	*11·0—11·9*	*12·0—12·9*	*13·0—13·9*
38	165	188	159	137	114	81	48	29	19	15	6	—	1

No statement is given as to what decimal of a gramme the strengths were read to and there must, since the strength of cotton fibres is continuous, have been strengths between 1·9 and 2·0 say. The only possible conclusion to be drawn from this is that strengths were read to $\frac{1}{10}$ of a gramme. If this be so, then the ranges should not be as above but 0·95—1·95, 1·95—2·95 etc. This is confirmed by the authors' Table XVI on p. 360, where they take moments about the point 4·45, and treat it as the centre of the group 3·95—4·95. From the group they go back by units to 0·45, which if all the groups are to be of *equal* size must run from −0·05 to +0·95, and which they centre at +0·45. Hence as they have tabled their data, they include negative strengths in their first subrange! Now we know that 38 cases fall into this first range, but is it reasonable to centre them at 0·45? In the first place there are the negative strengths; in the second place can we have a fibre of zero strength? It could not support even its own weight and could hardly be brought to the testing-machine. But thirdly and most important: all the curves fitted to the above classification rise vertically and at some point well within the first

* Table IV, last column, p. 327.

range—some even beyond 0·45 itself, the point at which the whole group has been centred. The Koshal-Turner curve starts at 0·32980; the Koshal-Fisher curve at 0·40756; and the curve with corrected moments at 0·628,908 grs. It is clear that the first curve would exclude fibre-strengths at 0·0, 0·1, 0·2 and 0·3. The Koshal-Fisher curve would also exclude 0·4, and the corrected moment curve 0·5 and 0·6 in addition. It is thus quite clear that centering the first group at 0·45 is fallacious, and since this is one of our extreme groups the influence of the erroneous centering is bound to make itself felt on the moments. Neither the Koshal-Turner nor the Koshal-Fisher paper draws any attention to this important point of the start of the curve being well above zero, but accept moments based on the hypothesis that the curve starts at $-0·05$*. It is now clear that our first condition has hardly been satisfied; the data have not been thought out before classification. For example, if the lowest observed strength had been 0·5 gramme, then the first subrange might have been taken from 0·45—1·45, which would have contained the ten strengths 0·5, 0·6, 0·7, 0·8, 0·9, 1·0, 1·1, 1·2, 1·3, 1·4, and so on for the second group 1·45—2·45. Had this been done we should have been able to test not only whether our curve was starting properly, but to obtain possibly better values of the moments. Koshal and Turner working in India had the raw materials in their hands, but grouped them without discussing the start of their materials; they do not even tell us what was the lowest strength observed, and further they leave us to make out from their results what was the decimal of a gramme to which the readings were taken, a knowledge of which is essential to proper grouping. As we are not in possession of the original raw data, all we can do is to follow Koshal and Turner's defective method of attack.

Passing on, Koshal and Turner give the crude moments about 4·45 grammes as

$$\nu_1' = -0·410, \quad \nu_2' = 5·7280, \quad \nu_3' = 3·7660, \quad \nu_4' = 91·2640,$$

denoting a mean of 4·04.

These are correct if we accept their centering. Starting from them they proceed to calculate the crude moments about the mean, and give (p. 361)

$$\nu_2 = 5·5599, \quad \nu_3 = 10·6619, \quad \nu_4 = 103·11237.$$

The first is correct, the last two values are in error. On the basis of these erroneous values they obtain on p. 361 the values

$$\beta_1 = 0·661,406, \quad \beta_2 = 3·33562,$$

and from these they calculate the curve as "given by the method of moments†."

The only reason they give for using uncorrected moments, eventually to be used to test the Method of Moments against the Method of Likelihood, is stated on p. 361:

"In this case Sheppard's corrections are not applied because the frequency curve does *not* have high contact at both ends."

* On p. 311 of the Koshal-Fisher paper the ranges are properly given as 0·95—1·95 etc. except in the first, which should be from $-0·05$—0·95 instead of 0·0 to 0·95, since the group was centred at 0·45.

† The true crude moments are $\nu_3 = 10·673,598$ and $\nu_4 = 103·132,728$, leading to $\beta_1 = 0·662,858$ and $\beta_2 = 3·336,2804$.

This is no excuse in 1930 for not correcting the moments for abruptness! Using the crude moments, two of which be it remembered are in error, Koshal and Turner provide the final frequency curve referred to the mode on p. 362 as*

$$y = 181{\cdot}275\left(1 - \frac{x}{2{\cdot}07266}\right)^{0{\cdot}702,4318}\left(1 - \frac{x}{14{\cdot}60109}\right)^{4{\cdot}948,333} \quad \text{.....(i).}$$

The mean is at 4·04, and the distance from mode to mean is 1·63754; consequently the mode is at 2·40246, which gives the start of the curve at

$$\alpha = 2{\cdot}40246 - \ 2{\cdot}07266 = \ 0{\cdot}32980,$$

and its finish at

$$\beta = 2{\cdot}40246 + 14{\cdot}60109 = 17{\cdot}00355,$$

while the range will be 16·67375.

On p. 362 we have a section of the Koshal-Turner paper entitled *The Determination of the Theoretical Frequencies.* Here we meet with the astonishing statement that

"the value of x is calculated as from the mid-point of each class interval, and the values of y as calculated from the equation give us the corresponding theoretical frequencies."

This statement invalidates the entire contents of their comparisons of observed and theoretical frequencies in their Tables XVII, XIX, XX and XXI. It indicates that the authors had no training in quadrature, previously to writing their paper. They infringe our conditions (i), (ii) and (iv) above; namely, the data are doubtfully tabled, the moments are not corrected and there are blunders in the arithmetic.

The mid-ordinates in the Koshal-Turner paper, there stated to be frequencies, are given as calculated from the corrected form of equation (i):

Subrange	Mid-ordinate	Observed Frequency	Subrange	Mid-ordinate	Observed Frequency
−0·5 —0·95	45·0028	38	7·95— 8·95	34·0993	29
0·95—1·95	160·4758	165	8·95— 9·95	19·6295	19
1·95—2·95	180·8133	188	9·95—10·95	10·4590	15
2·95—3·95	166·7703	159	10·95—11·95	4·9137	6
3·95—4·95	138·7433	137	11·95—12·95	1·9587	0
4·95—5·95	107·0217	114	12·95—13·95	0·6072	1
5·95—6·95	76·7411	81	13·95—14·95	0·1241	0
6·95—7·95	52·8714	48	—	—	—

They form the basis of the frequencies said to be obtained by the "Method of Moments" in the Koshal-Fisher paper. For this reason we have had to go back to the Koshal-Turner paper to explain what has been done in the Koshal-Fisher paper.

* The sign in the first bracket is clearly in error, but the ordinates are calculated correctly with a positive sign.

The question must now be answered as to whether the faults in calculating the raw moments about the mean, as apart from using raw moments, seriously alter the frequency curve (i) and the resulting frequencies obtained from it. To ascertain this we must start with the β's which flow from the exact raw moments, i.e.

$$\beta_1 = 0{\cdot}662{,}858, \qquad \beta_2 = 3{\cdot}336{,}2804.$$

Working with these we obtain the equation for the frequency curve deduced from crude moments:

$$y = 178{\cdot}7989\left(1 + \frac{x}{2{\cdot}140{,}4171}\right)^{0{\cdot}722{,}5248}\left(1 - \frac{x}{14{\cdot}513{,}4658}\right)^{4{\cdot}905{,}9841} \quad \ldots\ldots\text{(ii)},$$

$$\text{Mode (as origin)} = 2{\cdot}4182{,}6425, \qquad \text{Range} = 16{\cdot}653{,}8829,$$

while the Koshal-Turner curve calculated from the crude moments, but with errors in the third and fourth, is with corrected sign in first bracket (p. 38)

$$y = 181{\cdot}275\left(1 + \frac{x}{2{\cdot}07266}\right)^{0{\cdot}702{,}4318}\left(1 - \frac{x}{14{\cdot}60109}\right)^{4{\cdot}948{,}333} \quad \ldots\ldots\text{(i)},$$

$$\text{Mode (as origin)} = 2{\cdot}40246, \qquad \text{Range} = 16{\cdot}67375.$$

Now I have no intention whatever of calculating the ordinates and resulting frequencies of curve (ii). We can clearly see that the ordinates will not be the same, for the first curve rises to a maximum ordinate of only 178·799 while the second curve rises to 181·275. I leave it to Professor Fisher or Mr Koshal to deal with curve (ii) and superpose upon it the labour of improving the constants by the Method of Likelihood. They are quite at liberty to admit that the frequency curve, which they say has arisen from the Method of Moments, is in error and then to assert that the Method of Likelihood will correct these errors. But to raise this plea is to discard the assertion that the Koshal-Fisher memoir as it stands shows the superiority of the Method of Likelihood over that of Moments, because the curve peculiar to the latter has never been ascertained. In other words, columns (3), (5) and (6) of their Table II, p. 311, must be put aside as not representing in any way the Method of Moments. They are based on improper moments, i.e. crude moments, two of which are wrongly computed.

But it is time to turn to columns (4), (7) and (8) of Table II, p. 311, of the Koshal-Fisher memoir. I shall now show that the results obtained in these columns are hopelessly in error, for a bad blunder has been made at the very start! In other words the results stated to be provided by the Method of Maximum Likelihood are also *wholly erroneous*.

Professor Fisher prefers to write my curve Type I in the form

$$y = y_0\,(x - \alpha)^{m_1}\,(\beta - x)^{m_2},$$

where α and β are measured not from the mode, but from the zero of the x variable. He has every right to do this, if he finds the constants thus expressed are best suited to his method. We will give the numerical value of α, β, m_1 and m_2 as found from the frequency curves under consideration.

Constants of Frequency Curves.

Character of Distribution	Curve from exactly calculated crude moments	Curve from erroneously calculated crude moments	Curve provided by the Method of Maximum Likelihood
α	0·277,847	0·32980	0·40756
β	16·931,730	17·00355	16·79948
m_1	0·723,5248	0·702,4318	0·691,329
m_2	4·905,9841	4·948,333	4·888,138
Mode	2·4182,6425	2·40246	—
Range	16·653,8829	16·67375	—

To my astonishment, however, the β given in the third column above, in the Koshal-Fisher memoir appears on p. 307 as 16·67375 instead of 17·00355, that is to say the range had been substituted for the true value of Fisher's constant β! This error is repeated again in the Table at the top of p. 311. This error has no influence at all on the values of the frequencies deduced from the erroneously calculated crude moments, because these were obtained not from the Fisher form of equation with α, β, but from the ordinates computed in the Koshal-Turner paper from equation (i) (see our p. 38).

But now comes the all-important point. In the Koshal-Fisher paper the correction to β is given on p. 310 as $y' = +0{\cdot}12573$. Added to the true value of β, i.e. 17·00355, this gives $\beta = 17{\cdot}12928$ for the Method of Maximum Likelihood. On p. 311 the value given for β is not this, but is 16·79948, i.e. 16·67375 + 0·12573, or the *range* of the curve calculated from erroneous crude moments + the correction of its β due to the Method of Maximum Likelihood. As the curve from which the theoretical frequencies of that method (Table II, p. 311, column 4) were computed is nowhere provided in the Koshal-Fisher memoir, it was needful to ascertain whether the entirely erroneous value of β given at the top of p. 311* had been used or the more correct value 17·12928. Accordingly it was needful to form the two curves for the two β's. They are:

For the erroneous β (16·79948),

$$\log y = \bar{4}{\cdot}393,4876 + 0{\cdot}691,329 \log (x - 0{\cdot}40756) + 4{\cdot}888,138 \log (16{\cdot}79948 - x) \quad \text{.....(iii).}$$

For the β as provided by Koshal-Turner's curve (17·12928),

$$\log y = \bar{4}{\cdot}336,5667 + 0{\cdot}691,329 \log (x - 0{\cdot}40756) + 4{\cdot}888,138 \log (17{\cdot}12928 - x) \quad \text{.....(iv).}$$

From which of these curves were the ordinates and the frequencies given as those due to the Method of Maximum Likelihood computed? This could only be

* This might be solely due to an oversight in copying after the frequencies had been found from the true curve.

answered—owing to suppression of such constants as $\log y_0$—by actually working out sample cases of the frequencies arising from curves (iii) and (iv). Here are specimens:

Subrange	Curve (iii)	Table II, Column (4)	Curve (iv)
0·95—1·95	155·66	154·94	151·06
5·95—6·95	78·55	78·65	80·40

There can be no doubt that the frequencies attributed to the Method of Maximum Likelihood go entirely astray, and were obtained from using a false value of β, that is to say by using the range of the Koshal-Turner curve instead of the abscissa of its terminal. I am unable to explain why curve (iii) does not agree absolutely with Table II, column (4), but suspect that the Koshal-Fisher y_0 does not agree with mine, which has been twice and independently worked out. This may be due to inadequate determination of the complete Γ-functions.

We can now sum up our criticisms of Table II in the Koshal-Fisher memoir to which Professor Fisher has referred me as indicating the superior efficiency of the Method of Maximum Likelihood to that of Moments:

(i) The results given as representing the Method of Moments are unfair, because (*a*) they do not give the best the Method of Moments can achieve, being based merely on *crude* moments and (*b*) these crude moments are not even correct owing to faulty arithmetic*.

(ii) By an oversight or blunder of some sort the results given by the Method of Maximum Likelihood are obtained by supposing the abscissa of the terminal of the frequency curve to be the same as the range, and this error wholly invalidates the comparison made between the two methods.

The Koshal-Fisher paper therefore must be put on one side as having no bearing on the relative merits of the two processes. Can we, however, deduce any results from the data provided? I think we can. In the first place we can deduce the curve which arises when moments are properly corrected, and secondly we can use the value of β which really flows from the β of the crude moments.

Now before we start on this new investigation we must remark on two points involved. The Method of Maximum Likelihood has been spoken of as a powerful method, and it is supposed to deduce even from poorly presented material a good or much improved result for the parameters. If this were not so, no one would have proposed to apply it to the Koshal-Turner crude moments. We therefore feel less hesitation in following up the Koshal-Fisher paper with the true value of β as given by Koshal-Turner. Secondly we shall adopt without in the least expressing approval the quadrature method of the Koshal-Fisher paper. This consists in deducing the area on the sth subrange by aid of the formula, where h is in our case unity:

$$h\,(y_s + \tfrac{1}{24}\delta^2 y_s).$$

* It should be noted that in the Koshal-Fisher paper, the solution obtained by using crude moments, in themselves erroneously computed, is invariably spoken of as the solution obtained by the "Method of Moments." No student who had been through a laboratory training in curve fitting would accept such an assertion.

The Koshal-Fisher paper gives it on p. 309 as

$$y_s - \tfrac{1}{24}\delta^2 y_s\text{*},$$

and states that it

"was found to be adequate for all the frequency classes, except the first, for which the area was calculated by subtraction from the total frequency unity." [†1000.]

By direct independent quadrature of the areas on the first and second subranges (and even on the third subrange) we assured ourselves that this process is inadequate. However, as it has been used throughout in the Koshal-Fisher memoir we will adopt the same process, so that any controversy arising from that paper may not be increased by differences as to the suitable methods of quadrature†.

Starting with the corrected curve to replace that of the Koshal-Fisher paper, namely (iv) of our p. 40, we determined its midordinates and the resulting frequencies. These are given in columns (3), (7) and (8) of the Table on p. 44 below.

Next we took the correct crude moments (not those of Koshal-Turner) and corrected them for abruptness by the formulae provided by Miss Pairman and K. Pearson‡. We thus obtained the following results:

Crude moments about $-\cdot05$, the start of the first subrange are

$$\nu_1' = 4{\cdot}090, \quad \nu_2' = 22{\cdot}288, \quad \nu_3' = 147{\cdot}3115, \quad \nu_4' = 1115{\cdot}6215.$$

After adding the abruptness corrections we have about the same point $-\cdot05$,

$$\mu_1' = 4{\cdot}0957{,}4456, \qquad \mu_2' = 22{\cdot}2069{,}2771,$$
$$\mu_3' = 146{\cdot}2875{,}4337, \qquad \mu_4' = 1104{\cdot}5045{,}2771.$$

Transferring to the mean we have:

$$\mu_2 = 5{\cdot}4318{,}0421, \text{ giving } \sigma = 2{\cdot}330{,}6231,$$
$$\mu_3 = 10{\cdot}8390{,}7492, \text{ giving } \beta_1 = 0{\cdot}7330{,}8125^+ \text{ and } \sqrt{\beta_1} = +\,0{\cdot}8562{,}0164,$$
$$\mu_4 = 98{\cdot}8083{,}1764, \text{ giving } \beta_2 = 3{\cdot}3489{,}2398,$$

whence with the customary symbol, $r = 6{\cdot}4573{,}6213$.

Then in the usual way we reach:

$$b = \text{range} = 15{\cdot}2719{,}3483,$$
$$m_1 = 0{\cdot}4447{,}2515, \qquad m_2 = 4{\cdot}0126{,}3698,$$
$$a_1 = 1{\cdot}5237{,}2935, \qquad a_2 = 13{\cdot}7482{,}0548.$$

Again: Mode from $-\cdot05$ = Mean $- 1{\cdot}8931{,}0679 = 2{\cdot}2026{,}3777$.

Start of curve from $-\cdot05 = \alpha = 0{\cdot}6789{,}0842$.

Terminal of curve from $-\cdot05 = \beta = 15{\cdot}9508{,}4325$.

* It is presumed that Koshal took $\delta^2 y_s = 2y_s - y_{s-1} - y_{s+1}$ instead of the more usual $y_{s-1} + y_{s+1} - 2y_s$, but it would have saved trouble had he said so.

† The method employed does not provide an adequate check on the system of quadratures. Again, the first three ordinates of Koshal-Turner curve are 45·0028, 160·4758 and 180·8133; $\delta^2 y_2 = -3{\cdot}96$ and $y_2 + \frac{1}{24}\delta^2 y_2 = 156{\cdot}51$. The Koshal-Fisher paper has for the second frequency 156·52.

‡ *Biometrika*, Vol. XII (1918), and *Tables for Statisticians*, Part II. cxciii—cxciv (on the latter page for $H_{1,\alpha}$ in the numerator $-4{,}586$ should of course be read for $+4{,}586$).

Equation to curve in Professor Fisher's form:

$$y = y_0 (x - 0{\cdot}6789{,}0842)^{0{\cdot}4447{,}2515} (15{\cdot}9508{,}4325 - x)^{4{\cdot}0126{,}3068},$$

where $\log y_0 = \bar{3}{\cdot}629{,}3626$.....(v).

This is the best result that the Method of Moments can give, so long as the data are grouped in the doubtful manner we have already referred to, and this curve is what should be compared against curve (iv), which is the one that should have been reached in the Koshal-Fisher paper. The ordinates and then the frequencies were calculated for curves (iv) and (v) and the following results (Table, p. 44) reached to replace Table II of the Koshal-Fisher paper.

It will be seen from our Table on p. 44 that

(1) Higher Likelihoods have been obtained for both the Method of Moments and that of Likelihood.

(2) The superiority of the latter has been reversed, when (*a*) the blunder in the Koshal-Fisher paper has been corrected, and when (*b*) the curve deduced by the Method of Moments is based on properly computed moments.

Thus both in respect to the P, χ^2 test, and to Likelihood, the Method of Moments is superior.

But the attentive reader will doubtless remark one point. The χ^2 worked out by the Method of Maximum Likelihood from a frequency curve based on a bad blunder (i.e. using range for a terminal abscissa) gives less likelihood than the correct curve, it gives a lower χ^2. This may be due to further arithmetical blunders, or to a more important point. The χ^2 is a measure of the difference between the observed and theoretical groups. The test of χ^2 is *not* the same thing as making the constants of the curve cause

$$L = S(n_s \log p_s)$$

to be a maximum, where the p_s's are the theoretical group probabilities and the n_s's the observed group frequencies. To make χ^2 a minimum will certainly give the best fit as judged by the χ^2 test, and to make L a maximum will do the same if judged by the test of Maximum Likelihood. Sometimes the two tests may point in the same sense and sometimes in opposite directions. We have no proof that χ^2 will always be least when the Likelihood is a maximum. This is indeed all the more to be regarded, when we are told:

"For the solution of problems of estimation we require a method which for each particular problem will lead us automatically to the statistic [sic!] by which the criterion of sufficiency is satisfied. Such a method is, I believe, provided by the Method of Maximum Likelihood although I am not satisfied as to the mathematical rigour of any proof which I can put forward to that effect*."

The reader will see how this applies to our third condition on p. 35: "There must be some agreed common standard of what is meant by comparative efficiency."

* R. A. Fisher, *Phil. Trans.* Vol. 222 A (1922), p. 323.

Comparison of the Method of Moments and the Method of Maximum Likelihood as illustrated in Table II of the Koshal-Fisher Memoir.

Category in grammes	Observed Frequency	Theoretical Frequency		Calculation of χ^2			
		Method of Moments Curve (V)	Method of Maximum Likelihood Curve (IV)	Method of Moments Curve (V)		Method of Maximum Likelihood Curve (IV)	
	m	m'	m''	$m-m'$	$\frac{(m-m')^2}{m'}$	$m-m''$	$\frac{(m-m'')^2}{m''}$
-0·05— 0·95	38	32·87	41·99	5·13	·8006	− 3·99	·3791
0·95— 1·95	165	169·17	151·06	−4·17	·1026	+13·94	1·2864
1·95— 2·95	188	186·72	176·48	1·26	·0085	+11·52	·7520
2·95— 3·95	159	167·20	166·83	−8·20	·4022	− 7·83	·3675
3·95— 4·95	137	137·04	140·44	0·04	·0000	− 3·44	·0843
4·95— 5·95	114	105·48	109·71	8·52	·6882	+ 4·29	·1678
5·95— 6·95	81	76·72	80·40	4·28	·2385	+ 0·60	·0045
6·95— 7·95	48	52·60	55·38	4·60	·4023	− 7·38	·9835
7·95— 8·95	29	33·82	35·70	−4·82	·6869	− 6·70	1·2574
8·95— 9·95	19	20·07	21·33	−1·07	·0570	− 2·33	·2545
9·95—10·95	15	10·76	11·65	4·24	1·6708	+ 3·35	·9633
10·95—11·95	6	5·02	5·67				
11·95—12·95	—	1·91	2·36				
12·95—13·95	1	·52	·79	−0·53	·0373	− 2·03	·4564
13·95—14·95	—	·08	·18				
14·95—15·95	—	·00	·03				

	$\chi^2=5·1151$,	$\chi^2=6·9567$,
P, if we follow Koshal-Fisher and take 7 degrees of freedom*	0·658,	0·429,
$L=$ Likelihood	$=-944·8175$,	$-945·2786$,
Previous results from the Koshal-Fisher paper:	$\chi^2=7·6552$,	$\chi^2=5·5375$†,
	$P=0·371$,	$P=0·596$,
	$L=-947·1343$,	$L=-946·4945$.

* In a footnote, p. 366, Koshal and Turner cite Fisher's paper in the *Journal of R. S. Society*, Vol. LXXXV (1922), p. 93, as stating that the number of moments used must be subtracted from the number of classes before entering the P, χ^2 Table. Now each moment involves a linear relation of the class frequencies and *if the moment be kept constant for all samples*, there exists a constraint, or a degree of freedom is lost. But those who recognise this should be very careful in applying it, to see that the moment *is* constant for all the samples taken. This is usually *not* the case. I hope to return to this matter shortly, as it appears to me that the blind use of so-called degrees of freedom is doing a grave amount of damage to statistical practice.

† Columns (7) and (8) of Table II, p. 311 of the Koshal-Fisher memoir are not in agreement. The values ·3880 and ·5371 are erroneous and errors in the last figure of the numbers in column (8) are frequent owing to the computer having failed to raise the last figure when he ought to have done. The total change in χ^2 is not actually serious. The errors, however, indicate careless computing which I fear taints all the numerical work of this author.

The very choice of words *Maximum Likelihood** seems to beg this question, although no rigorous proof can be given of the method which bears that name, nor is there any proof that we shall reach the minimum χ^2 when we reach the Maximum Likelihood.

In fact we do *not*. This is well illustrated by the example Professor Fisher discusses† drawn from Dr Kirstine Smith's paper in *Biometrika*, Vol. XI (1915), pp. 265—266. He seems to have quite misunderstood her purpose and not explained his own. They both seek to find a standard deviation for a series of 300 observations, which are supposed to follow a normal distribution of which the mean is known. Dr Smith sets as her problem the finding of the standard deviation of a normal curve which will most closely reproduce the observed group frequencies. She does this by making χ^2 a minimum, and finds $\sigma = 2{\cdot}355{,}860$ and $\chi^2 = 9{\cdot}720$. Her problem is perfectly definite and her answer definite. This value of σ will most closely reproduce the series of group frequencies, which is all we know of the observations. How Professor Fisher can object to this I fail to grasp, but he indicates that the standard deviation ought to have been found by the Method of Maximum Likelihood. Applying this method he finds for his standard deviation $\sigma = 2{\cdot}26437$, while with Sheppard's correction $\sigma = 2{\cdot}26421$. This accordance‡ seems to give him much satisfaction, but he has failed to notice two points, (i) that with the σ he has reached he ought to have thrown out the last observation, that between 8″ and 9″, for the odds against it are about 2500 to 1! and (ii) that his standard deviation fails to reproduce the observed group-frequencies with the same accuracy as Dr Smith's σ. It raises the χ^2 from 9·720 to 11·828, and this because it has disregarded that a low standard deviation is not compatible with an individual between 8″ and 9″. The values of the Likelihood are: for the Minimum χ^2 test $-202{\cdot}1660$; for the Maximum Likelihood test $-202{\cdot}0535$. Thus the example discussed by Professor Fisher has taught us two things, (i) that the

* The Method of Maximum Likelihood has come in various forms widely into use without much regard being paid to its validity. This has gone even so far that after finding the Likelihood its frequency curve has been determined by the Method of Moments; whereas if the Method of Likelihood is to be applied, the frequency curve of Likelihood should surely be found by the Method of Likelihood itself! Otherwise we have the tacit assertion that for *practical* purposes the Method of Moments is as good as that of Likelihood.

† *Phil. Trans.* Vol. 222 A (1922), pp. 360—361. The actual frequencies in seconds of angle are

0″—1″	1″—2″	2″—3″	3″—4″	4″—5″	5″—6″	6″—7″	7″—8″	8″—9″	Total
114	84	53	24	14	6	3	1	1	300

No attention was paid by the observer to sign.

‡ This "accordance" seems to suggest a difference in functioning of the $S(n_s^2/p)$ and $S(n_s \log p_s)$ criteria. The latter leads to corrections of the parameters which would be unimpeachable, if the parent population were actually of the assumed type. The former does not assume the parent-population to be of the assumed type, but modifies the parameters so as to obtain a curve of the assumed type, which has a minimum of improbability. This suggestion is only made on the basis of a single illustration, and so of small value, but might still be worth following up.

methods of Minimum χ^2 and of Maximum Likelihood do not give us the same values of the statistical constants, and (ii) that whereas with the former we know what we shall achieve—the curve which gives us as nearly as possible by the χ^2 test the grouped frequencies observed—we only know of the latter that it makes a certain quantity which Prof. Fisher has termed the Likelihood a maximum, but what advantages the resulting curve gives to the practical statistician remain so far unrevealed. We do know that it does not necessarily provide the curve of which the "goodness of fit" is a maximum, or the curve which provides the closest accordance with the limited knowledge the statistician possesses of the observations. That both the Minimum χ^2 test and the Maximum Likelihood test are hard to apply may be admitted, but one is not necessarily harder than the other. At any rate we know what we shall achieve by the first, but Professor Fisher has not yet explained what advantages will be achieved by the second.

Actually the two processes lead to determining the constants of a curve by:

(*a*) Minimising $S\left(\frac{n_s^2}{p_s}\right)$ which will lead to a curve in the closest possible accordance with the observed data judged by the χ^2 test; and

(*b*) Maximising $S(n_s \log p_s)$, which leads to an "efficient estimate" of any parameter of a proposed curve according to Professor Fisher. But in what respect is that curve found by (*b*) for the practical statistician preferable to that found by (*a*)*? It has reduced L from $-202{\cdot}1660$ to $-202{\cdot}0535$, again indicating the insensitive character of the Method of Maximum Likelihood. But what additional information has it brought to the statistician? Is there any proof that the curve thus obtained, although a worse fit to the data, will enable the statistician to interpolate better, or tell him something about the original data of which he only knows the grouped frequencies? Professor Fisher will no doubt reply: But my method provides "efficient parameters" for the required curve. Why more "efficient" than those provided by minimising χ^2 we ask? Surely Professor Fisher is arguing in a circle. He provides no rigorous or even non-rigorous proof that the minimising of $L = S(n_s \log p_s)$ will provide "better" values of the parameters than other functions of n_s and p_s, and then he begins under the name of "Maximum Likelihood" to impress on the statistical world that it is the "best" method of finding the constants of frequency curves! In reality no definition of "best" is provided. There is no answer to our condition (iii). The Method of Least Squares used to be spoken of as the "best" method of curve fitting. But who can say that the method is better than the Method of Least Fourth Powers, if both are judged by their own minimum? The Method of Least Squares is now known as a *utile* method and in my opinion that is all that can be said of the Method of Maximum Likelihood, but it fails to give any advantage of a tangible kind like that which attaches to the minimum χ^2. Like the Method of Moments it gives values for frequency curve parameters, but which of the sets of the parameters obtained is the "better"? Lack of a criterion of "better" forbids an answer.

* Of course either (*a*) or (*b*) may form a suitable jumping-off point for determining (*b*) or (*a*) respectively.

In the one published example to which Professor Fisher refers me, that worked out by Koshal (under Fisher's supervision at every important step), the final tests made are by both the value of χ^2 and the value of the Likelihood. We know what the former aims at—a practical measure of the degree of closeness of fit, but we do not know what practical value arises from making the Likelihood a minimum. We do learn, however, from the Koshal-Fisher paper that we are first to apply the "inefficient Method of Moments" before we can apply the Method of Likelihood, thus doubling our labours, and rendering at least a training in fitting curves by moments desirable as a needful preliminary to applying the Likelihood method. The two steps are taken in the Koshal-Fisher memoir, and a series of conclusions provided at the end of the memoir intended to indicate how superior the Method of Likelihood is to the Method of Moments.

Unfortunately the basis of comparison is made by using raw moments for the Method of Moments—and two of these are erroneously computed. Thus the curve obtained does not represent the best obtainable from that method.

In the next place, the curve which is supposed to represent the result of the Method of Likelihood is quite erroneous, for at the important step of writing down its parameters the range of the Method of Moments curve has been taken to represent the abscissa of the curve terminal, and the correction for the latter has been added to the range to represent the abscissa of the terminal!

Finally, if we take the properly computed raw moments and correct them for abruptness, and then correct the erroneous application of the Method of Likelihood as applied to the Koshal-Turner curve, we find that by both the χ^2 and the Likelihood tests, the Method of Moments is superior, not inferior to that of Likelihood!

We might go further and improve the Method of Moments results by making χ^2 a minimum in the manner proposed in the able, but not sufficiently regarded paper by Dr Kirstine Smith. But that seems at present unnecessary (i) until Mr Koshal and Professor Fisher have started afresh, and corrected the errors in the Koshal-Fisher paper, and again indicated that they obtain a "better" result; and (ii) because a $P = 0{\cdot}658$ goodness of fit is adequate for most statistical purposes, and because the other two samples each of 1000 cotton fibre strengths given in the Koshal-Turner paper indicate how wide the divergence in samples of the same cotton can be.

But the main point in this paper is to suggest to Professor Fisher that he should state, now that the Method of Likelihood has taken hold of so many of the younger generation of mathematical statisticians, wherein he conceives it to give "better" results than the method of minimum χ^2. The latter has a firmer basis and gives the practical statistician what he requires, the "best" fitting curve to his data according to χ^2. But what does the Maximum Likelihood give him? If it gives the "best" parameters to the frequency curve, how does he define "best"? It cannot be because it gives the Minimum Likelihood, for that would be arguing in a circle. He must define what he means by "best," before he can prove that the principle of Likelihood provides it.

NOTE TO PAGES 43—47 OF THE ABOVE MEMOIR.

It may be as well to place on record here the assumptions made with regard to the two criteria, i.e. that of making $S\left(\frac{n_s^2}{p_s}\right)$ a minimum, and that of making $L = S(n_s \log p_s)$ a maximum (lowest negative value of L).

If we obtain a frequency curve graduating a series of observations in any way either by guess work, or the method of moments for example, and treat this frequency curve as representing the parent population, then P_{χ^2} will under certain conditions to be mentioned immediately measure the percentage of samples, taken from this theoretical parent population, which have a combined system of deviations $n_s + \delta n_s$* greater than that of the observed series of observations. On account of this property I termed P_{χ^2} a measure of the "goodness of fit" of a graduation curve to a sample. If we make χ^2 a minimum, then P_{χ^2} would take a maximum value, or we should obtain the parameters of a theoretical frequency curve of a chosen type, which gave the highest possible percentage of samples having a combined system of deviations greater than that of the observed series.

In order to reach these results I had to make a fundamental assumption, namely that the true distribution of n_s being given by a binomial, that binomial might for practical purposes be replaced by a normal curve. I thus reached a multivariate normal surface, by aid of which it was relatively easy to deal with combined systems of sample deviations, and to give them by aid of the contour lines of this surface a scale of magnitude.

Now the reader has to remember that any unimodal curve, of which the ordinate rises to a maximum and then falls again, can be replaced in the neighbourhood of the mode by a normal curve. But something more than this happens when we replace a binomial by a normal curve. If we take the binomial $(\frac{1}{2} + \frac{1}{2})^n$, the corresponding normal curve, even when n is as low as 10 or even 5, will give with remarkable accuracy the terms of the binomial. If we take the binomial $(p+q)^n$ and p be not too divergent from q, even with n of the order 15 to 20, there is a striking accordance between the terms of the binomial and the corresponding areas of the proper normal curve†. We may here call to mind: (i) that the frequency in the cell of a sample will with much greater probability come from the neighbourhood of the

* Here δn_s is what I have termed a statistical differential, not a mathematical (or indefinitely small) differential.

† I do not believe that any grave practical error will arise even if the ratio p_1/q be as small as $\frac{1}{15}$ or $\frac{1}{18}$, because such binomials only occur where the cell content n_s is small compared with the size of the sample N.

mode than from the tails of the binomial, i.e. from the part where the normal curve supplies a really good fit, and (ii) that if the raw material is at the command of the statistician, he can to some extent modify its grouping to avoid very low values of n_s/N.

In consideration of these matters I think we may conclude that the P, χ^2 criterion gives if not an absolutely accurate, still for practical purposes an excellent measure of what most statisticians need to know, namely the relative superiority of one graduating curve over another.

Now let us turn to Professor Fisher's likelihood, which may be defined for the case we are considering as

$$\mathrm{L} = p_1^{n_1} p_2^{n_2} \dots p_s^{n_s} \dots p_t^{n_t},$$

where $S(p_s) = 1$.

To make this a maximum, p_1, p_2, ... p_s, ... p_t being functions of the fitted curve's parameters, is the same thing as making

$$L = S(n_s \log p_s)$$

a maximum, or since each p_s is < 1, giving L its lowest *negative* value. Now at first sight it might seem not unreasonable to choose the p_s's of the fitted curve such that they give the highest probability to the observed system n_1, n_2, ... n_s, ... n_t, but in what sense will the curve parameters thus obtained be the "best"? How is "best" to be defined? Let us note what Professor Fisher has himself to say about this Principle of Maximum Likelihood. The italicising is mine:

> The form in which the criterion of sufficiency has been presented is not of direct assistance in the solution of problems of estimation. For it is necessary first to know the statistic* concerned and its surface of distribution with an *infinite* number of other statistics*, before its sufficiency can be tested. For the solution of problems of estimation we require a method which for each particular problem will lead us automatically to the statistic* by which the criterion of sufficiency is satisfied. Such a method is *I believe* provided by the Method of Maximum Likelihood, *although I am not satisfied as to the mathematical rigour of any proof, which I can put forward to that effect*†.

It will be seen that in 1922 Fisher spoke with much doubt and hesitancy of the Method of Maximum Likelihood. But since that date it has been raised by him to the rank of the most efficient method of curve fitting!

Now let us look at two further points submitted in the *Phil. Trans.* paper and again in the *Camb. Phil. Proc.* paper‡. Professor Fisher points out that if the size of a sample be indefinitely increased the distribution of any statistical parameter will ultimately be given by a normal curve, and of any series of statistical parameters by a multivariate normal surface. This had already been shown by L. N. G. Filon and Karl Pearson in 1898§, some twenty-four years earlier. Further Professor Fisher has shown that as the size N of the sample approaches

* I object very strongly to the use of the word "statistic" for a statistical parameter. Are we also to introduce the words, a mathematic, a physic, an electric, etc., for parameters or constants of other branches of science?

† *Phil. Trans.* Vol. 222 A, p. 323.

‡ Vol. 22, pp. 700 et seq.

§ *Phil. Trans.* Vol. 191 A, p. 229, etc.

infinity the standard deviation (and accordingly probable error) of the statistical parameter approaches a minimum. But can any justification be drawn for the method of Maximum Likelihood from what happens in the case of $N \to \infty$? The senior author of the paper of 1898 very soon learnt as more and more curves and surfaces of the distribution of statistical parameters came to be worked out, that very little practical knowledge could be really ascertained from $N \to \infty$ values! They amounted to very little more than asserting that any curve in the neighbourhood of its mode could be represented by a normal curve! What is the practical value of showing that the distribution of a statistical parameter may be *approximately* represented by a normal curve when $N \geqslant 400{,}000$ when you are seeking what happens when $N = 400$? Or again if a parameter λ has a standard deviation σ_λ given by

$$\sigma_\lambda = \frac{A}{n^{\frac{1}{2}}} + \frac{B}{n^{\frac{2}{2}}} + \frac{C}{n^{\frac{3}{2}}} + \text{etc.},$$

what is the practical value of A, if B and C, etc. are very large numbers, and for practical purposes B and C have to be included? It does not appear to me that anything of value as to the nature of the Method of Maximum Likelihood can be deduced from $N \to \infty$ cases.

I again ask what that method provides the statistician with in the case of samples of finite size?

APPENDIX I.

SINCE the preceding memoir was written, my attention has been drawn to the following three papers, which have been written since the Koshal-Fisher paper was published. I assume they were unknown to Professor Fisher, or he would have referred to them as seriously modifying the results of the Koshal-Fisher paper.

(*a*) W. Palin Elderton, F.I.A. and G. H. Hansemann, Ph.D.: "Improvement of Curves fitted by the Method of Moments." *Journal of the R.S. Society.* Vol. XCVII, 1934, pp. 330—333.

(*b*) R. J. Myers: "Note on Koshal's Method of Improving the Parameters of Curves by the Use of the Method of Maximum Likelihood." *The Annals of Mathematical Statistics.* Vol. V, 1934, pp. 320—323.

(*c*) R. S. Koshal, M.Sc.: "Application of the Method of Maximum Likelihood to the Derivation of Efficient Statistics for Fitting Frequency Curves." *Journal of the R.S. Society.* Vol. XCVIII, 1935, p. 128.

The authors of (*a*) and (*c*) papers do not realise that the discrepancies they are bringing to light are in great part due to the arithmetical errors of the Koshal-Turner paper or to the blunders in the Koshal-Fisher application of the Method of Maximum Likelihood. Both of them appear to assume that the latter method has in itself a justifiable existence, i.e. will "better" the method of moments.

Elderton and Hansemann, not having apparently seen the Koshal-Turner paper, calculated the crude or unadjusted moments from all that is obtainable from the

Koshal-Fisher paper treating the 7 "*above* 10·95" as grouped between 10·95—11·95; whereas 6 are actually in that group and 1 in the group 12·95—13·95. Hence the Elderton-Hansemann raw moments agree neither with the true raw moments, nor with those provided by Koshal. They suppose Koshal may have adjusted in some way his moments. Koshal in (*c*) suggests* that the source of the difference is the treatment of the "*above* 10·95" group. That difference does in part arise from this fact, but it also springs from the occurrence of blunders in Koshal's arithmetic! See our pp. 37, 40, above.

Elderton and Hansemann then proceed by a "practical" method to compute their curve from two raw moments. They do *not* adjust their moments†. The main assumptions they make are that the curve should start at 0·45 gr. and the range be 16 grs. This seems somewhat arbitrary. In fact I know it to be erroneous since one of the 38 individuals of the first group occurs actually at 0·3 gr.‡ The unsuitable grouping of Turner and Koshal did not suggest anything of this kind (see our pp. 36—37), and the start of the Elderton-Hansemann curve is slightly better than that of those curves fitted by Koshal and others in the memoir above. Given that we know nothing but the distribution of frequency given on our p. 36, one may use any process one chooses to get a curve to fit it and Elderton and Hansemann show they can get a better χ^2 (4·65 as against 5·54) than Koshal does with his process of Maximum Likelihood. They do this by comparing their result with that of the Koshal-Fisher paper. They do not realise, however, that the latter result is hopelessly in error! Elderton and Hansemann did not calculate the Likelihood, but I have done so:

Elderton-Hansemann	Koshal-Fisher
$L = -944{\cdot}7218$	$-946{\cdot}4945$

Thus even with their simple practical methods both their χ^2 and their Maximum Likelihood are better than Koshal's! Now what answer does Koshal make to this? He first refers to the "above 10·95" difference, but this is of no importance because both the Koshal-Fisher paper and the Elderton-Hansemann paper are occupied

* He has of course only to thank himself for this interpretation for nowhere in the Koshal-Fisher paper does he state the value of his moment-coefficients or how they were obtained!

† W. P. Elderton: *Frequency Curves and Correlation*, 2nd Edition, 1927, pp. 121—2. The present writer pointed out in *Phil. Trans.* Vol. 186 A (1895), p. 371 that if the start of a curve Type I were known, then the constants of the curve could be obtained from *three* moments about the start. Elderton reduces this number to *two* by *guessing* the start and the range. If the start and range be *known*, it was shown by the present writer in the same paper (p. 370) that two moments about the start would suffice. The "practical" method adopted by Elderton consists in *guessing* the two ends of the range. He uses *raw* moments about his guessed start and he gets a curve arising abruptly at his guessed start of 0·45 gr. Actually this is a good guess, it only excludes one individual at 0·3 gr. But this "guess" involves a previous working out of the curve by raw moments (*loc. cit.* p. 331), when it is found to start abruptly at 0·412 gr., and have a range of 15·314 grs. Without this previous investigation, one may question whether the guesses of 0·45 gr. and 16 grs. would have been obtainable. Elderton and Hansemann do not ascertain whether their *raw* moment curve would give better or worse results than their guessed curve. Their method provides *no* adjustments of moments, although their curve shows that both μ_1' and μ_2' require them.

‡ I owe the actual values of the 38 strengths in the first group to the kindness of the Director of the Technological Laboratory, Bombay.

with the problem of fitting a curve which shall "best" describe a certain set of grouped frequencies, and whether χ^2 or L is taken as the test the Elderton-Hansemann curve gives the better result. This may, perhaps, be considered not unnatural as Koshal's results are in error whether he is dealing with what *he* calls* the Method of Moments or with the Method of Likelihood.

Elderton-Hansemann's $L(-944{\cdot}72)$ is almost the same as that found by aid of abruptness coefficients in the preceding memoir (p. 44: $L=-944{\cdot}82$).

Koshal further states that:

> The Method of Maximum Likelihood aims at getting an efficient estimate of parameters, and the efficient statistics [sic!] thus derived will not always lead to a better fit as judged by the χ^2, P method than that obtained by the method of moments (*c*, p. 128).

He does not state that the method of maximum likelihood lacks proof, and that applied even to well adjusted moments, it is not only very laborious but very liable to inaccuracies.

These points are brought out very well in our second paper (*b*) by R. J. Myers. He points out not only that Koshal has made serious numerical errors†, but that by modifications of Koshal's method other and greater values of the Likelihood could be obtained.

> This would seem to show that better results could be gotten by judicious guessing than by using this method of approximation (*b*, p. 322).

And again:

> Another disadvantage of this method is the fact that a great deal of time is required to apply it. Approximately sixty hours were required to carry [out] the calculations for the Type I curve.
>
> Another interesting fact was brought out when the method of Pearson and Pairman for correcting moments for grouping was applied to the original data. The frequency distribution obtained was far better than any previously obtained as shown by the fact that the L for this distribution was highest of all‡. χ's [?χ^2] for this distribution was 4·64. The time required to apply this method was considerably less than that needed for Koshal's method (*b*, p. 323).

Unfortunately Myers does not give the frequencies nor the equation to the curve thus found. Nor does he state its L. But it is only fair to quote a paper which has covered some of my ground, but was unknown to me, when I wrote the preceding memoir. Unfortunately I do not agree with his value of χ^2 for the abruptness coefficient solution§, but this can be because he may have made a single group of the 7 "above 10·95," and so have obtained different raw moments before correction. He apparently accepts the Method of Maximum Likelihood for he writes: "It has been shown by R. A. Fisher that the most efficient parameters for Pearsonian curves may be found by the Method of Maximum Likelihood" (*b*, p. 320). How has it been shown? Fisher himself has so far failed to produce a proof!

* Mr Koshal's paper was written in 1933 and that in association with Turner in 1930. The corrections for moments were given in 1919. Mr Koshal writes to me personally that these corrections were unknown in 1929 when the Koshal-Turner paper was written.

† I cannot say whether these are identical with those found by me.

‡ That is, L had the lowest negative numerical value.

§ He gets $\chi^2=4{\cdot}64$, while I find it 5·12, twice working independently.

On the whole, I venture to think, these papers tend to confirm the views of my preceding memoir, that there is nothing whatever in the Koshal-Fisher memoir to induce a really practical statistician to impose the method of Likelihood on well-adjusted moment work.

APPENDIX II.

On p. 36 I have drawn attention to the facts (i) that Koshal and Turner in their paper give three samples, each of 1000 frequency of Surat cotton fibres. We are only directly concerned with the third sample, as its frequency is that used as a test case by Koshal and Fisher. But on p. 47 it was pointed out how widely samples of the same cotton could in three cases differ from one another. My suspicions were aroused as to whether the material might not be "lumpy," i.e. more fibres taken from one plant or part of the field than was admissible on the assumption of random sampling; (ii) that in the Koshal-Turner paper we were nowhere told what was the unit to which the strength of the cotton fibres' strengths were measured; but we could only conclude from the method of tabulating that they were read to $\frac{1}{10}$ of a gramme.

Being interested in the nature of the third sample, quite apart from its use by Koshal and Fisher as a test case between the Method of Maximum Likelihood and the Method of Moments, I asked the Director of the Technological Laboratory (Indian Central Cotton Committee), Matunga, Bombay, if he would be willing to supply me with the Surat cotton fibre strengths grouped in a somewhat different manner, namely in subranges 0·25—1·25, 1·25—2·25, ... 14·25—15·25. This he most kindly undertook to do, and the following table, actually prepared by Mr Koshal, was sent by the Director to me.

Frequency Distributions of Strength Values of 3000 *fibres of Surat cotton* 1027 *A.L.F.* (1925—26). *Second Classification.*

Strength class grammes	3000 Tests	Sets of 1000 Tests		
		First 1000	Second 1000	Third 1000
0·25— 1·25	315*	103*	102	110
1·25— 2·25	535	192	204	139
2·25— 3·25	502·5	172	164	166·5
3·25— 4·25	556·5	172	189·5	195
4·25— 5·25	404	148	121	135
5·25— 6·25	237	72	75·5	89·5
6·25— 7·25	178	67	48	63
7·25— 8·25	117	28	43	46
8·25— 9·25	60	23	17	20
9·25—10·25	37	7	12	18
10·25—11·25	27	8	7	12
11·25—12·25	16	4	7	5
12·25—13·25	7	2	4	1
13·25—14·25	4	1	3	—
14·25—15·25	4	1	3	—

* N.B. These include one fibre of 0·1 gr. strength.

This table was a revelation in more than one way. Clearly the strengths of at least some of the fibres were read to 0·05 of a gramme and not to 0·1 of a gramme; this is evidenced by the appearance of the ·5's in the frequency columns. But if the measurer read to 0·05 and not to 0·1 of a gramme, it is singular that no ·5's occur in the frequency distribution on our p. 36 for the third sample, nor indeed in the frequencies for the First 1000 or Second 1000 of the samples with the first classification*. One seems driven to conclude that the measurer did *not* read to 0·05 of a gramme, but during the course of his work read at times to 0·1 of a gramme and at times to 0·25, i.e. one quarter of a gramme. It may be that the strengths were not all determined by the same tester, but by two testers differently graduated. Anyhow there is clearly a possible source of heterogeneity in this difference.

Again, if the reader will examine the modal frequency and its class, in the six samples, he may find further evidence for lumpiness and probably heterogeneity.

	First Classification			Second Classification		
	First 1000	Second 1000	Third 1000	First 1000	Second 1000	Third 1000
Class of Modal Frequency	2·0—2·9	1·0—1·9	2·0—2·9	1·25—2·25	1·25—2·25	3·25—4·25
Value of Modal Frequency	184	203	188	192	204	195
Theoretical Mode ...	2·357 for total 3000 tests[a]		2·2526[b] 2·4025[c]	—	—	2·6123[d] 2·3996[e]

a Calculated by Koshal and Turner from raw moments for 3000 fibres.

b Calculated by Pearson for Third 1000 from moments corrected for abruptness.

c Calculated by Koshal and Turner from raw moments partly in error.

d Calculated by Pearson from moments corrected for abruptness, supposing start of curve and range free.

e Calculated by Pearson from moments corrected for abruptness, supposing curve to start at 0·25 gr., but range to be free.

It will be seen that while in the First Classification the theoretical modes of the First and Third 1000's lie in the maximum frequency group, this is not the case in the Second 1000. In the Second Classification we only know the theoretical mode for the Third 1000, that with which we are directly concerned, but it is widely removed from the group of maximum frequency, indeed the frequency takes a minimum value (139) in the subrange in which the theoretical mode appears. The discrepancy is so great that it tends to confirm the view that the material dealt with in the third sample was really "lumpy." Anyhow the chief interest of these classifications of the third sample is the fact that they demonstrate how small is the reliance to be placed on subrange of maximum frequency—i.e. the

* *Technological Bulletin*, Section B, No. 8 (April 1930), p. 12, or *Journal of Textile Institute Transactions*, Vol. xxi. p. 327.

highest column of a histogram—as indicating the whereabouts of the mode. And further how idle is the practice of some writers to break up a frequency distribution into two or more groups by the existence of two or more subranges of maximum frequency in the histogram*.

When we start fitting a Type I curve to the third sample of the Second Classification, we are met by alternative methods of procedure. We know that the lowest strength observed in the third sample was 0·3 of a gramme. Hence:

A. We may suppose the curve to start at 0·25 gr., and thus our Second Classification will be appropriate. We leave the range free to be determined from the data. Thus our *A* curve may be described as "start-fixed range-free" curve.

B. We may leave the start to be determined as well as the range, or *B* will be described as the "start-free range-free" curve.

A requires a knowledge of three, *B* of four moments. Turning our attention first to *A*, we find first for the raw moments about 0·25 of the frequencies of the Third 1000 given in the table on p. 53,

$$\nu_1' = 3{\cdot}7535, \quad \nu_2' = 19{\cdot}6700, \quad \nu_3' = 126{\cdot}509{,}375, \quad \nu_4' = 940{\cdot}4765.$$

Assuming that the abruptness corrections would be required, their values were calculated†:

$$H_{1,a} = 0{\cdot}0065{,}5911, \; H_{2,a} = -0{\cdot}0008{,}5048, \; H_{3,a} = 0{\cdot}0022{,}1965, \; H_{4,a} = -0{\cdot}0007{,}3920.$$

We now proceed to the corrected moments about 0·25,

$$\mu_1' = \nu_1' - H_{1,a} = 3{\cdot}7535 - 0{\cdot}0065{,}5911 = 3{\cdot}7469{,}4089.$$

But this is not the mean strength, it is measured from 0·25. Hence

$$\text{Mean strength} = 3{\cdot}9969{,}4089,$$

$$\mu_2' = \nu_2' - \tfrac{1}{2} - H_{2,a} = 19{\cdot}5866{,}6667 + 0{\cdot}0008{,}5048 = 19{\cdot}5875{,}1715,$$

$$\mu_3' = \nu_3' - \tfrac{1}{4}\nu_1' + H_{3,a} = 126{\cdot}509{,}375 - 0{\cdot}938{,}375 + 0{\cdot}0022{,}1965$$
$$= 125{\cdot}571 + 0{\cdot}0022{,}1965 = 125{\cdot}5732{,}1965,$$

$$\mu_4' = \nu_4' - \tfrac{1}{2}\nu_2' + \tfrac{7}{240} + H_{4,a} = 940{\cdot}4765 - 9{\cdot}8350 + 0{\cdot}0291{,}6667 - 0{\cdot}0007{,}3920$$
$$= 930{\cdot}6706{,}6667 - 0{\cdot}0007{,}3920 = 930{\cdot}6699{,}2747.$$

Proceeding from μ_1', μ_2' and μ_3' the constants of the curve were calculated;

$$m_1 = 0{\cdot}7528{,}9901, \quad m_2 = 5{\cdot}2050{,}1003, \quad b = \text{range} = 17{\cdot}0105{,}7198,$$

$$\text{Mode} = 2{\cdot}1496{,}1679 \text{ from } 0{\cdot}25 = 2{\cdot}3996{,}1679 \text{ from 0 grammes},$$

leading to $$y = y_0 x^{0{\cdot}7528{,}9901} (17{\cdot}0105{,}7198 - x)^{5{\cdot}2050{,}1003},$$

whence $\log y_0 = \bar{5}{\cdot}9060{,}2834$, and the ordinates y are to be calculated from

$$\log y = \bar{5}{\cdot}9060{,}2834 + 0{\cdot}7528{,}9901 \log x + 5{\cdot}2050{,}1003 \log (17{\cdot}0105{,}7198 - x).$$

* Theoretical modes are determined from all available observations. Pseudomodes are judged from maxima of the histogram columns. The latter may result from the action of really random selection or from "lumpiness" in the material. The material may be perfectly homogeneous, but the gatherer has not collected purely randomly.

† Cf. p. 42 above.

The origin of x is at 0·25, and the value of m_1 being less than unity, the use of abruptness corrections is justified.

The mid-ordinates of the subranges were calculated and the subrange frequencies deduced, as exhibited in the table below:

Range	Observed Frequency	Theoretical Frequency (Curve *A*)	Contributions to χ^2
0·25— 1·25	110	99·27	1·1598
1·25— 2·25	139	167·01	4·6977
2·25— 3·25	166·5	176·52	0·5688
3·25— 4·25	195	157·94	8·6960
4·25— 5·25	135	128·46	0·3330
5·25— 6·25	89·5	97·20	0·6100
6·25— 7·25	63	68·93	0·5103
7·25— 8·25	46	45·82	0·0071
8·25— 9·25	20	28·38	2·4744
9·25—10·25	18	16·21	0·1977
10·25—11·25	12	8·40	1·5429
11·25 and over	6	5·86	0·0033
		Total $\chi^2 = 20·8010$	
		P for 12 groups = 0·036	

It is clear that the suggestion made on p. 37 will not work in this case, and the source of the failure is clear, namely, the paucity of the frequency where the mode should be (subrange 1·25—2·25) and the excess in the subrange 3·25—4·25.

B. We will now turn to the curve in which both start and range are free and are to be determined from the data. It is needful in the first place to transfer the moments found under Curve *A* to the mean. The moments about 0·25 are

$\mu_1' = 3·7469,4089$, $\mu_2' = 19·5875,1715$, $\mu_3' = 125·5732,1965$, $\mu_4' = 930·6699,2747$.

These give us

$\mu_2 = 5·5479,5112\,(\sigma = 2·3554,0891)$, $\mu_3 = 10·6042,6089$, $\mu_4 = 107·2814,0131$.

Whence $\beta_1 = 0·6585,1084$, $\beta_2 = 3·4834,5264$,

$$r = \frac{6\,(\beta_2 - \beta_1 - 1)}{6 + 3\beta_1 - 2\beta_2} = 10·9111,6223,$$

$$b = \text{range} = \tfrac{1}{2}\sigma\sqrt{\beta_1\,(r+2)^2 + 16\,(r+1)} = 20·4103,7280,$$

$$\text{Mode} = \text{mean} - \tfrac{1}{2}\sigma\sqrt{\beta_1}\,\frac{r+2}{r-2} = 3·9969,4089 - 1·3846,7793$$

$$= 2·6122,6296, \text{ measured from 0 gr.}$$

Proceeding we find

$m_1 = 1·1574,0988$, $m_2 = 7·7537,5235$,

$a_1 = 2·6509,6364$, $a_2 = 17·7594,0916$,

Mode $- a_1 = -0·0387,0068$, Mode $+ a_2 = 20·3716,7212$.

Equation to curve

$$y = y_0 (x + 0{\cdot}0387{,}0068)^{1{\cdot}1574{,}0988} (20{\cdot}3716{,}7212 - x)^{7{\cdot}7537{,}5235},$$

with $\log y_0 = \bar{8}{\cdot}0766{,}2882$, or the ordinates must be found from

$$\log y = \bar{8}{\cdot}0766{,}2882 + 1{\cdot}1574{,}0988 \log (x + 0{\cdot}0387{,}0068) + 7{\cdot}7537{,}5235 \log (20{\cdot}3716{,}7212 - x),$$

x being measured from 0 gr.

Mid-ordinates were calculated and frequencies determined with the results provided in the table below:

Range	Observed Frequency	Theoretical Frequency (Curve *B*)	Contributions to χ^2
0·25— 1·25	110	100·91	0·8188
1·25— 2·25	139	160·05	2·7685
2·25— 3·25	166·5	176·71	0·5899
3·25— 4·25	195	161·30	7·0409
4·25— 5·25	135	131·53	0·0092
5·25— 6·25	89·5	98·54	0·8293
6·25— 7·25	63	68·72	0·4761
7·25— 8·25	46	44·82	0·0311
8·25— 9·25	20	27·33	1·9659
9·25—10·25	18	15·51	0·3997
10·25—11·25	12	8·13	1·8422
11·25 and over	6	6·45	0·0314
		Total $\chi^2 = 16{\cdot}8030$	
		P for 12 classes = 0·114	

This is a distinct improvement on Curve *A*, but the peculiarities on the sub-ranges 1·25—2·25 and 3·25—4·25 are still conspicuous. The free start of the curve is now about $-0{\cdot}038$, approaching $-0{\cdot}05$, which would be the start as Koshal and Turner originally classified the material.

But although $P = 0{\cdot}115$ can hardly be called a "good fit," there can be little doubt the reader will have recognised a serious objection to this curve.

Its m_1 exceeds unity, and therefore abruptness corrections ought not to be used.

It is true that the contact at the low strength end of the range is only of the lowest order, and that the abruptness corrections are also of a low order. But the latter should be removed, and the calculations started afresh with Sheppard's adjustments only. The moments are easily obtained from the previous work, namely

Curve *C*. Mean from zero = 4·0035, and from 0·25 = 3·7535,

$$\mu_2' = 19{\cdot}5866{,}6667, \quad \mu_3' = 125{\cdot}571, \quad \mu_4' = 930{\cdot}6706{,}6667,$$

these three moments being measured from 0·25.

Transferring to the mean we have

$\mu_2 = 5{\cdot}4979{,}0442$ ($\sigma = 2{\cdot}3447{,}6106$), $\mu_3 = 10{\cdot}7799{,}7818$, $\mu_4 = 105{\cdot}5793{,}4731$,

whence we obtain $\beta_1 = 0{\cdot}6992{,}6889$, $\beta_2 = 3{\cdot}4928{,}8753$.

Accordingly $$r = \frac{6(\beta_2 - \beta_1 - 1)}{6 + 3\beta_1 - 2\beta_2} = 9{\cdot}6775{,}2332.$$

Further b = range = 19·1279,8032, Mode = 2·5123,5356 from zero.

Next we find $m_1 = 0{\cdot}9427{,}0825^5$, $m_2 = 6{\cdot}7348{,}1506^5$,

$a_1 = 2{\cdot}3486{,}8775$, $a_2 = 16{\cdot}7792{,}9257$,

and the curve starts at Mode $- a_1 = 0{\cdot}1636{,}6581$, or at about 0·16 gr. The start, the mode, the range are all suitable, but we note that m_1 is less than unity, or the abruptness corrections ought to have been used! We have thus reached an important point in fitting frequency curves: If we use abruptness corrections m_1 may turn out more than unity, whereas if we leave them out m_1 may be found less than unity, and we ought to have used them. Fuller treatment is really required for curves verging on abruptness, i.e. with low order of contact. The difficulty is unlikely to occur, if we are fitting to a mathematical curve of frequencies, but the random sampling effects (especially in the case of the third sample under the Second Classification they are so great (139 in second, 195 in fourth subrange)) make it extremely difficult to ascertain the nature of the frequency curve at the start. Notwithstanding the m_1 is < 1, we will proceed to calculate the subrange frequencies of this curve for comparison with Curves *A* and *B*. The equation to Curve *C* may be written as

$$y = y_0 (x - 0{\cdot}1636{,}6581)^{0{\cdot}9427{,}0825} (19{\cdot}2916{,}4613 - x)^{6{\cdot}7348{,}1507}$$

with the origin of x at 0 gr.

This leads us to $\log y_0 = \bar{7}{\cdot}6626{,}0140$, and our ordinates are to be found from

$$\log y = \bar{7}{\cdot}6626{,}0140 + 0{\cdot}9427{,}0825 \log (x - 0{\cdot}1636{,}6581) + 6{\cdot}7348{,}1507 \log (19{\cdot}2916{,}4613 - x).$$

Whence computing mid-ordinates and thence the frequencies, we find:

Range	Observed Frequency	Theoretical Frequency (Curve *C*)	Contributions to χ^2
Below 1·25	110	95·59	2·1723
1·25— 2·25	139	164·71	4·0131
2·25— 3·25	166·5	178·80	0·8461
3·25— 4·25	195	161·15	7·1103
4·25— 5·25	135	130·52	0·1538
5·25— 6·25	89·5	97·70	0·6882
6·25— 7·25	63	68·36	0·4203
7·25— 8·25	46	45·24	0·0128
8·25— 9·25	20	27·50	2·0401
9·25—10·25	18	15·69	0·3401
10·25—11·25	12	8·25	1·7045
11·25 and over	6	6·49	0·0370
		Total χ^2 =	19·5440
		P for 12 groups =	0·053

If we adopt Professor Fisher's limitation as to degrees of freedom we have:

Curve	Number of moments used	Number of classes	Value of P
A	Three ($\chi^2=20{\cdot}8010$)	Nine	0·008
B	Four ($\chi^2=16{\cdot}8030$)	Eight	0·002
C	Four ($\chi^2=19{\cdot}5440$)	Eight	0·007

Thus while judged by my interpretation of the test, A, B and C give unfavourable fits ($P=0{\cdot}036$, $0{\cdot}114$ and $0{\cdot}053$), judged by Professor Fisher's they are all most unsatisfactory. Taking L as a measure of Likelihood we have:

Curve	Likelihood
A	$L=-949{\cdot}4469$
B	$L=-959{\cdot}8587$
C	$L=-946{\cdot}9641$

It is clear again that the order of Likelihood is not the order of χ^2, and while the Likelihood may indicate which curve has the "better" parameters, it is accompanied by no interpreter like P_{χ^2}, the meaning of which can be at once stated. As I have said previously this Appendix has little to do with the Koshal-Fisher claims for the superiority of the Method of Maximum Likelihood, but it does lead us to conclusions of some importance for the theory of curve-fitting, namely:

(i) The position, shape and "goodness of fit" of the fitted curve depend much more on the nature of the grouping than has been so far recognised. The subranges need careful consideration before we start our calculations.

(ii) Any assumptions as to the start of the distribution may further modify the position, shape and "goodness of fit" of the fitted curve.

(iii) A further study of the corrections to the moments needful when the start of the curve has *low* contact, but is not necessarily abrupt, is very desirable. The data selected by Koshal and Fisher for a test case suffer from the double defect that we do not know what is the minimum strength of a cotton fibre that can be placed on the tester, nor do we know whether the curve rises abruptly from this point or not. The Surat cotton strengths probably belong to the class of frequencies with low contact at the start, but not abruptness. I seriously doubt whether the Koshal approximation method if applied respectively to any two of the Method of Moments' curves dealt with in this Appendix II and in the previous paper would give a unique result.

CLEFT PALATE, HARE LIP, DACRYOCYSTITIS, AND CLEFT HAND AND FEET.

By E. A. COCKAYNE, D.M., F.R.C.P.

Cyril C. has been an in-patient in the Hospital for Sick Children, Great Ormond Street, on several occasions. He is a thin active boy, aged 5 years in 1933, inquisitive, but disinclined to talk or play with other children. His projecting premaxilla, the scars of his operations for hare lip, and the peculiar shape of his skull give him an odd appearance. At birth he had a double hare lip and a cleft affecting both hard and soft palates, for which several operations have been performed by Mr H. Tyrrell-Gray. He also had a squint, for which an operation on both eyes was performed with success, but the notes have been mislaid. His hare lip has been repaired, and after his last operation for cleft palate in 1934 only a small hole remained. At an early age he developed bilateral dacryocystitis, secondary to a congenital stenosis of the lachrymal ducts, and in October 1933 the right lachrymal sac was excised by Mr Penman and the cavity was curetted, and the left sac was washed out and curetted in November.

His left hand is normal and a radiogram shows that the bones are well formed. His right hand is cleft; the distal part of the index finger with the two distal phalanges is bent at a right angle towards the ulnar side, the middle finger does not show externally and its sole phalanx, the proximal one, is bent at a right angle towards the ulnar side, the other two fingers are nearly normal, though the middle phalanx of the ring finger is very small. In both feet there is a deep cleft between the big toe and the rest of the foot. The metatarsal of the second toe on each side is very slender and has no phalanges, the metatarsal of the third toe is equally short, but it is broader and has one phalanx; the other toes are normal.

The mother, who is a native of Hampshire, gave the family history and a description of the other affected members of her family on three occasions at considerable intervals, and, since there was no discrepancy, her account is probably reliable. She says that the defect first appeared in her own sibship and that her parents and ascendants were normal. I have only seen the mother and son and the other information given below was obtained from the mother.

II, 1 and 2. Males, both normal and both dead.

II, 3. Female, normal with a normal boy and girl, III, 1 and 2.

II, 5. The mother of Cyril C. has a hare lip and cleft palate, but her lachrymal ducts are not occluded. The right hand has three digits, the middle and ring fingers are represented by a fibrous nodule, but the metacarpals are present. In

Photograph of III, 7's Feet

Skiagram of III, 7's Feet

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Skiagram of III, 7's Left Hand

Skiagram of III, 7's Right Hand

Photograph of III, 7's Hands

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the left hand the middle and ring fingers are very large and there is a small hard nodule between the index and middle fingers. The right foot is cleft and has two large and one small toe; the left foot is normal in appearance. She refused to have either an X-ray or photograph taken.

III, 3. Female, aged 17 years, has no hare lip, cleft palate, or abnormality of the lachrymal ducts, but has all four extremities cleft. The index and middle fingers of each hand are missing or concealed, and there are only two large toes visible on each foot.

III, 4. A female, who died. She had a hare lip and cleft palate, and all four extremities were cleft.

III, 5 and 6. Males, aged 14 and 12 years, both normal.

III, 7. Cyril C., the patient.

Reverting to the second generation, II, 7, a female, with a normal husband, has two children, a boy and a girl, both normal.

II, 9. A male, normal.

II, 10. Female, married with one normal daughter, III, 10.

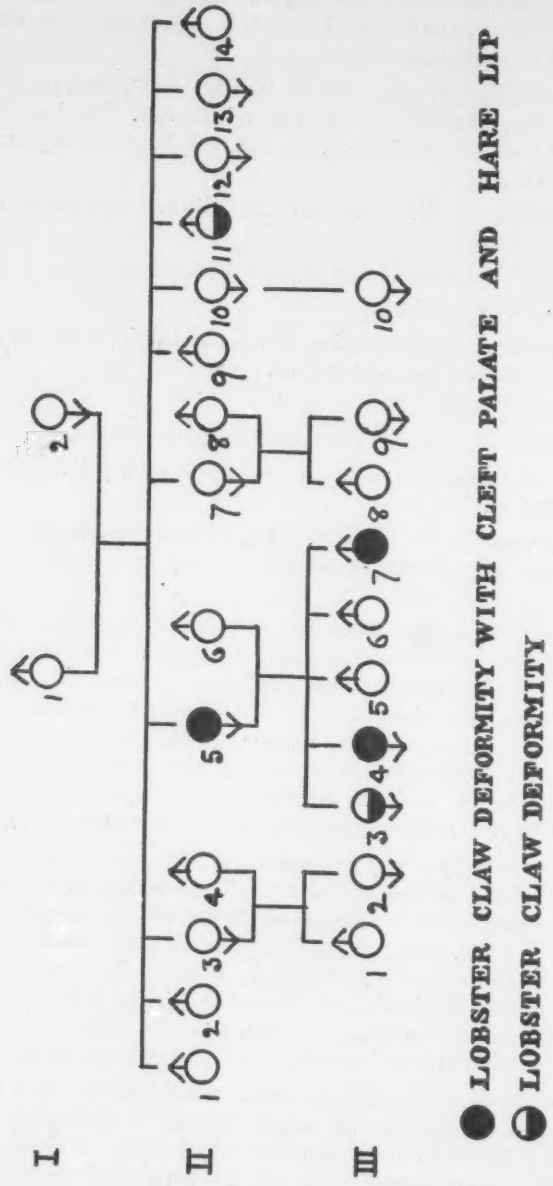
II, 11. A male, unmarried, with no hare lip or cleft palate, but all the four extremities cleft.

I can find no account of a family in which all these abnormalities were present, though there are many records of families in which each of them was inherited alone.

Cleft palate and hare lip is much the commonest of them, and probably has an incidence of one in one to two thousand of the general population, if such minor manifestations of the defect as bifid uvula are included. In some families it appears to behave as a simple dominant and pedigrees showing this type of descent have been published in the *Treasury of Human Inheritance*. As a rule however only one sibship, and often only one member of that sibship, is affected in a given family. Hogben, analysing Rischbeith's data, says that the condition may be determined by two dominant genes, one of which is comparatively common.

The lobster deformity is very uncommon, but is due to a single dominant gene. Tubby has published an account of a family in which fifteen members were affected in four generations, the parents of the first affected member being normal and unrelated by blood. Other similar pedigrees are to be found in the literature, but isolated cases also have been described. It occurs in various races in widely separated countries. Nobécourt and Babonneix give a figure of an Annamite family, a father and his daughter and son, with the 'Déformation en pince de homard,' and it is said to occur frequently in a tribe of South American Indians.

Congenital atresia of the lachrymal ducts usually affects only a single sibship in a family, but more than one sib may be affected as the reports of Groenouw, A. Peters, and Schmeichler show. Leboucq indeed reports a sibship, in which six successive children were affected, a girl and five boys. Their parents were normal.


I
II
III
1
2
1 2 3 4
5 6 7 8 9 10 11 12 13 14
1 2 3 4 5 6 7 8 9 10
LOBSTER CLAW DEFORMITY WITH CLEFT PALATE AND HARE LIP
LOBSTER CLAW DEFORMITY

Stenosis of the ducts with resultant dacryocystitis, starting in early childhood or even in adult life, usually affects only one member of a family, but Schnyder gives a pedigree of a family, of which five members in three generations developed dacryocystitis, and in all but one case descent was direct. Nieden finds that in 9 per cent. of all cases of developmental errors of the ductus naso-lachrimalis, epiphora, stenosis, and dacryocystitis, there is a family history of the defect, and in many cases a parent is the sufferer, in approximately 37 per cent. the father, and 63 per cent. the mother of the affected child is also affected.

It must surely be more than a coincidence that in this family only the members with the lobster deformity had cleft palate and hare lip, three out of five being affected in this way.

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INFLUENCE OF POSITION ON STRUCTURE OF INFLORESCENCES OF *CICUTA MACULATA*.

By WILLIAM DOWELL BATEN, University of Michigan.

The study of the relations of floral or inflorescence parts to position on the plants has been made by others. Investigations of the blossoms of *Caltha palustris* by Burkill* indicate that the number of stamens and the number of carpels on terminal flowers are significantly larger than the corresponding numbers on laterals. Counts of Vogler† of the heads of *Arnica montana* show that terminal heads have more ray flowers than lateral heads. His work with *Eupatorium album* suggests that the flower number is different for terminals and non-terminals on branches. Haacke‡ found that the number of ligulate flowers of *Tanactum corybosum* differed for stem and branch terminals and that there was a considerable difference between high and low branches. Samples of *Centaurea atropurpurea* gathered by MacLeod§ show the number of flowers per head (both ray and disk) to differ significantly for stem and branch terminals. Schüepp‖ in his report of the investigations of *Aconitum Napellus* suggests that the number of floral parts is a function of the position of the flower on the plant. The study made by Klebs¶ of *Sempervivium* reveals little difference between terminal and axial inflorescences. Pearson** studied the relation of number of veins of leaves to position on plants of *Fagus sylvatica*. Harris†† found correlation between first and second flowers of *Adoxa*. Stout and Boas‡‡ made a very thorough investigation of *Cichorium intybus* and showed, among other interesting things, that number of flowers per head

* Burkill, I. H. On some variations in the number of stamens and carpels. *Jour. Linn. Soc. Bot.* xxxi. pp. 216—245.

† Vogler, P. Variation der Anzahl der Strahlenblüten bei einigen Kompositen. *Beihefte Bot. Centralbl.* xxv[1]. pp. 387—396. Neue variationsstatistische Untersuchungen. *Jahrb. St. Gal. Naturwiss. Ges.* 1—32.

‡ Haacke, W. Entwicklungsmechanische Untersuchungen. *Biol. Centralbl.* xvi. pp. 481—497, 529—547.

§ MacLeod, J. Over de varanderlijkheid van het aantal randbloemen en het aantal schijfbloemen bij de Korenbloem (*Centaurea Cyanus*) en over Correlatieverschijnselen. *Handel. vlaamsch nat. en geneeskundig Congres*, 61—72.

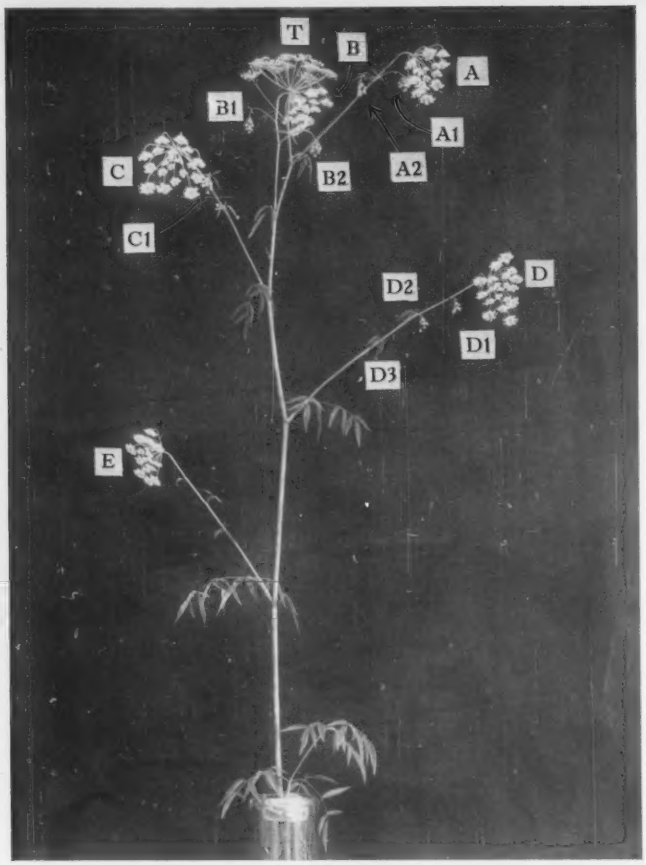
‖ Schüepp, O. Variationsstatistische Untersuchungen an *Aconitum Napellus*. *Zeitschr. Induk. Abstam. u. Vererb.* x. pp. 242—268.

¶ Klebs, G. Ueber Variationen der Blüten. *Jahrb. Wiss. Bot.* xlii. pp. 155—320.

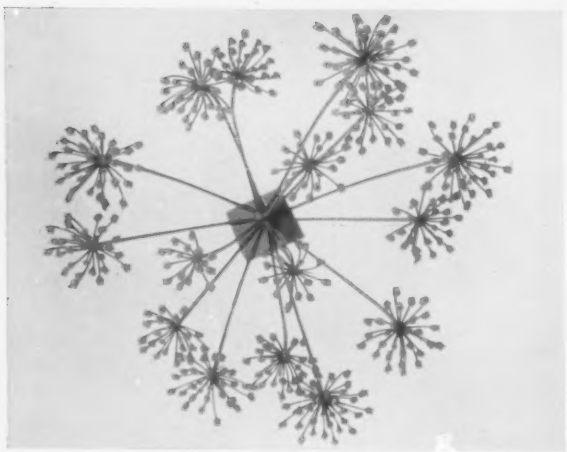
** Pearson, K. On differentiation and homotyposis in the leaves of *Fagus sylvatica*. *Biometrika*, iii. 1904, pp. 104—107.

†† Harris, J. A. Note on variation in *Adoxa*. *Biometrika*, vii. 1909—10, pp. 218—222.

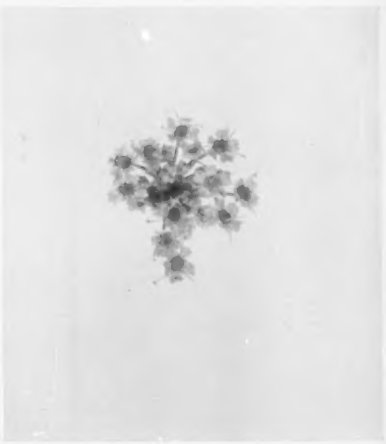
‡‡ Stout, A. B. and Boas, Helene M. Statistical studies of flower number per head in *Cichorium intybus*: Kinds of variability, heredity, and effect of selection. *Torrey Bot. Club Memoirs*, No. 17, 1917, pp. 334—459.



Cicuta maculata.



Cluster of *Cicuta maculata.*



Umbellet of *Cicuta maculata.*

is a function of position. This article contains a comprehensive review of the literature on this subject. Cool and Koopmans* investigated variation and correlation of the number of umbel rays of *Pastinaca sativa*, *Anthriscus sylvestris* and *Sium latifolium* and found correlation between stem and branch terminal inflorescences. The author's† counts of the number of pedicels per umbel of *Zizia aurea* indicate that stem and branch terminals differ significantly, that terminal and non-terminal branch umbels differ greatly, and that branch terminals which bloom later in the flowering season have on the average more rays than those blooming earlier.

The object of this article is to present results of an investigation concerning the influence of position on structure of inflorescences of *Cicuta maculata*.

During the summer of 1934 counts were made of pedicels per umbel of plants from three plots located in different environments. Counts were obtained for umbels at different positions on the plants to ascertain the relation between position and number of pedicels per umbel.

Plot I is located on the banks of a rivulet flowing into the north bank of the Huron River about five miles west of Ann Arbor, Michigan. Plots II and III are situated near the Ann Arbor Air Port on the banks of a ditch which contained water most of the summer. Plot III, which is across the road from Plot II, contained much more vegetation than Plot II. Only terminal clusters were taken from plants growing on the first two plots; all clusters were used for most of the plants on Plot III.

Stem inflorescences will be designated by T, first branch terminal inflorescences by A, second branch terminals by B, etc., and branch non-terminals by $A1$, $A2$, $B1$, $B2$, $C1$, $C2$, $C3$, etc. According to these symbols, D represents the terminal inflorescence on the fourth branch, $E3$ the third non-terminal on the fifth branch. These inflorescences with their respective names are shown in Plate I. Branches are numbered in descending order from the stem terminal.

I. ANALYSIS OF TERMINAL UMBELS.

Table I contains distributions of stem and axillary terminal umbels for the three plots.

Plants on Plots II and III produced on the average more terminals than those on Plot I; this may have been due to the fact that sampling was done on the former two at the end of the flowering season, after the lower branches had had ample time in which to develop. Only 13·1 per cent. of the plants from the first plot produced seven or more terminals; over 22 per cent. of those from the other two produced seven and more. Plants on Plot I were much taller than those on

* Cool, C. and Koopmans, A. N. Variation and correlation of the number of umbel rays of some Umbelliferae. *Biometrika*, XI. 1915—17, pp. 38—49.

† Baten, W. D. Influence of position on structure of inflorescences of *Zizia aurea*. *Papers of the Mich. Acad. of Science, Arts and Letters*, 1935.

TABLE I.

Distributions of Terminal Umbels.

No. of terminal umbels per plant	Frequency for Plots			Frequency in per cent. for Plots		
	I	II	III	I	II	III
3	18	7	8	8·4	8·1	2·5
4	37	17	28	17·4	19·8	8·7
5	75	19	95	35·2	22·1	29·5
6	55	24	118	25·8	27·9	36·6
7	16	10	45	7·5	11·6	14·0
8	12	9	25	5·6	10·5	7·8
9	0	0	3	0·0	0·0	1·0
Totals	213·00	86·00	322·00			
Mean	5·23	5·47	5·79			
S.D.	1·23	1·41	1·18			

Plots II and III; the crown spreads on the latter two were much larger than those on the former. Every plant taken from the plots produced a stem terminal cluster. The average number of terminals is between five and six.

Comparison of Stem and Axially Terminal Inflorescences.

Table II contains the average number of pedicels per inflorescence for stem and branch terminals.

TABLE II.

Average number of Pedicels for Terminal Inflorescences.*

Plot		*T*	*A*	*B*	*C*	*D*	*E*	*F*	*G*
I	Mean	13·39	17·46	17·91	18·18	18·35+	18·93	18·85	18·31
	S.D.	1·97	2·26	2·54	2·53	2·52	2·50	2·46	3·29
	No.	213·00	213·00	212·00	195·00	157·00	86·00	27·00	13·00
II	Mean	12·81	19·52	20·41	20·61	19·91	18·87	18·61	17·20
	S.D.	2·55	3·41	3·61	4·26	3·74	3·23	3·32	2·44
	No.	86·00	85·00	81·00	77·00	66·00	46·00	18·00	10·00
III	Mean	12·09	17·20	18·03	18·33	18·22	18·07	17·96	17·33
	S.D.	2·10	2·38	2·62	2·78	2·74	2·66	2·35+	0·74
	No.	322·00	323·00	319·00	317·00	287·00	192·00	75·00	30·00

* Compare Tables A, B and C below.

TABLE A.

Frequency of Pedicels for Terminal Umbels, Plot I.

Number of pedicels per umbel	Umbels							
	T	*A*	*B*	*C*	*D*	*E*	*F*	*G*
6	—	—	—	—	—	—	—	—
7	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—
9	2	—	—	—	—	—	—	—
10	10	—	—	—	—	—	—	—
11	29	—	—	—	—	—	—	—
12	23	1	—	—	—	—	—	—
13	45	5	7	2	2	—	1	—
14	57	11	13	11	11	3	1	—
15	23	27	13	18	7	3	1	1
16	10	32	40	21	18	9	1	1
17	5	38	20	30	17	9	3	3
18	5	28	29	27	28	14	4	2
19	3	33	34	26	27	16	5	2
20	1	19	25	26	14	12	3	2
21	—	12	19	15	18	7	4	2
22	—	2	3	10	9	6	3	—
23	—	2	4	5	2	2	1	—
24	—	3	3	2	2	3	—	—
25	—	—	—	—	—	1	—	—
26	—	—	2	2	2	1	—	—
27	—	—	—	—	—	—	—	—
28	—	—	—	—	—	—	—	—
29	—	—	—	—	—	—	—	—
30	—	—	—	—	—	—	—	—
Totals	213·00	213·00	212·00	195·00	157·00	86·00	27·00	13·00
Mean	13·39	17·46	17·91	18·18	18·35+	18·93	18·85+	18·31
S.D.	1·97	2·26	2·54	2·53	2·52	2·50	2·46	1·81

TABLE B.

Frequency of Pedicels for Terminal Umbels, Plot II.

Number of pedicels per umbel	Umbels							
	T	*A*	*B*	*C*	*D*	*E*	*F*	*G*
6	1	—	—	—	—	—	—	—
7	—	—	—	—	—	—	—	—
8	2	—	—	—	—	—	—	—
9	3	—	—	—	—	—	—	—
10	9	—	—	—	—	—	—	—
11	13	—	1	—	1	1	1	—
12	12	1	—	—	—	—	—	—
13	17	—	—	1	—	—	—	—
14	8	2	2	1	2	—	1	2
15	5	7	2	4	2	3	1	—
16	9	8	6	3	8	4	3	2
17	3	13	6	12	4	14	—	1
18	3	4	9	3	10	4	2	4
19	1	7	10	14	6	2	2	—
20	—	11	8	7	6	5	2	—
21	—	6	7	4	8	3	2	—
22	—	8	8	5	4	4	1	—
23	—	9	5	6	4	2	3	1
24	—	3	6	3	4	—	—	—
25	—	2	3	3	1	3	—	—
26	—	1	3	5	1	—	—	—
27	—	2	3	3	2	—	—	—
28	—	—	—	1	2	1	—	—
29	—	1	2	—	—	—	—	—
30	—	—	—	—	1	—	—	—
31	—	—	—	—	—	—	—	—
32	—	—	—	1	—	—	—	—
33	—	—	—	—	—	—	—	—
34	—	—	—	—	—	—	—	—
35	—	—	—	—	—	—	—	—
36	—	—	—	—	—	—	—	—
37	—	—	—	—	—	—	—	—
38	—	—	—	1	—	—	—	—
Totals	86·00	85·00	81·00	77·00	66·00	46·00	18·00	10·00
Mean	12·81	19·52	20·41	20·61	19·91	18·87	18·61	17·20
S.D.	2·55	3·41	3·61	4·26	3·74	3·23	3·32	2·44

TABLE C.

Frequency of Pedicels for Terminal Umbels, Plot III.

Number of pedicels per umbel	Umbels							
	T	*A*	*B*	*C*	*D*	*E*	*F*	*G*
8	9	—	—	—	—	—	—	—
9	15	—	—	—	—	—	—	—
10	52	—	—	—	—	—	—	—
11	64	—	—	—	—	—	—	—
12	57	3	1	1	—	2	—	—
13	55	10	4	6	5	5	2	1
14	33	28	17	18	15	11	2	3
15	13	31	33	26	30	11	9	2
16	14	64	36	31	40	30	10	4
17	7	51	50	49	29	24	9	7
18	2	48	58	37	45	29	11	5
19	2	40	35	48	36	24	11	3
20	—	25	31	34	26	18	4	1
21	—	5	29	33	21	15	8	3
22	—	9	9	16	20	15	2	1
23	—	3	4	4	10	3	2	—
24	—	4	6	2	5	4	—	—
25	—	2	2	7	4	1	—	—
26	—	—	3	3	1	—	—	—
27	—	—	—	2	—	—	—	—
28	—	—	—	—	—	—	—	—
29	—	—	1	—	—	—	—	—
30	—	—	—	—	—	—	—	—
Totals	323·00	323·00	319·00	317·00	287·00	192·00	75·00	30·00
Mean	12·09	17·20	18·03	18·33	18·22	18·07	17·96	17·33
S.D.	2·10	2·38	2·62	2·78	2·74	2·66	2·35	0·74

The values in Table II show that the average number of pedicels for stem umbels is significantly smaller than the average number for lateral terminals. This was true for all three plots. Stem terminal inflorescences bloom before any of the others and produce the smallest number of pedicels. This was found to be true for *Zizia aurea* *, but not for *Daucus carota*†. Means for stem terminals differ significantly for different plots, which suggests that environmental conditions have a great deal to do with the number of pedicels of the clusters. Several of the means for corresponding branch terminals are significantly different for different environments.

* Baten, W. D. Influence of position on structure of inflorescences of *Zizia aurea*. *Papers of the Mich. Acad. of Science, Arts and Letters*, 1935.

† Results that have not been published.

Bar diagrams in Fig. 1 exhibit magnitudes of means for various terminal clusters. The following inequalities for means hold for the three plots:

$$M_T < M_A < M_B < M_C,$$

and

$$M_E > M_F > M_G;$$

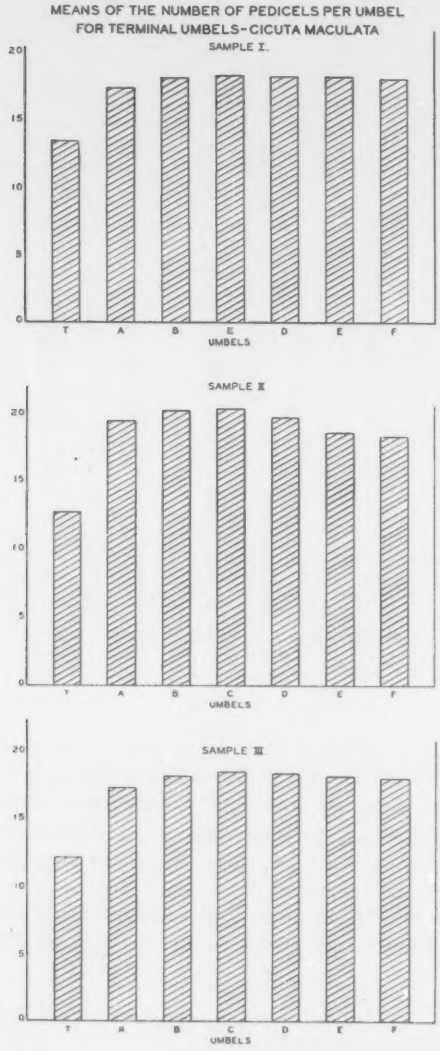


Fig. 1.

for Plots II and III:

$$M_C > M_D > M_E > M_F > M_G;$$

for Plot I:

$$M_T < M_A < M_B < M_C < M_D < M_E.$$

For all the plots the number of pedicels per cluster increases from T to some branch terminal cluster and then decreases from there on to the last terminal to bloom. Cluster C is the optimum cluster for the last two plots; cluster E is that for Plot I.

Table II includes also the numbers of the plants having various umbels. From these numbers one sees that umbels A, B and C are on 90 per cent. of the plants, and that the percentages of plants producing D, E, F or G decrease rapidly. Others have found similar results for other plants*. Terminal branch clusters which bloomed at the end of the season on Plot I had more rays than those terminals which bloomed at the beginning of the season; on the other two plots earlier ones had more rays than later ones.

Fig. 2 contains frequency polygons of the distributions for T, A and B.

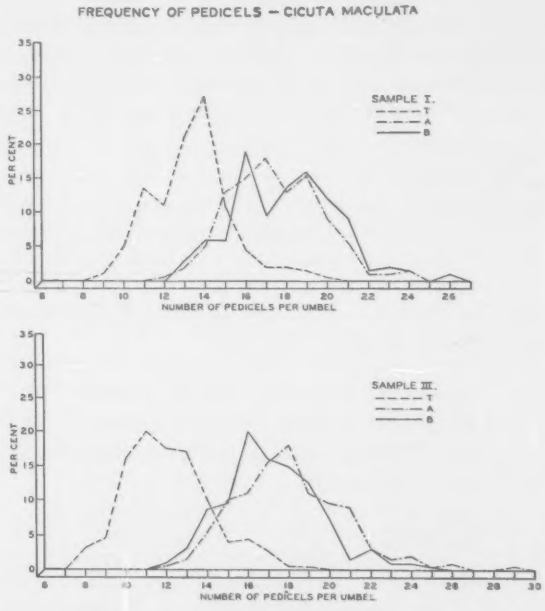


Fig. 2.

* Stout, A. B. and Boas, Helene M. Statistical studies of flower number per head in *Cichorium intybus*: Kinds of variability, heredity, and effect of selection. *Torrey Bot. Club Memoirs*, No. 17, 1917, pp. 334—459.

Correlation between Terminal Inflorescences.

Questions arose as to whether or not there was any correlation between the number of rays on stem and branch terminal inflorescences, or whether the size, in the number of rays, of branch terminals was influenced by the size of the stem terminals which bloomed first. Answers to these questions may be found in Table III, which contains linear correlation coefficients between T and A and between T and D.

TABLE III.

Correlation Coefficients for Stem and Branch Terminal Umbels.

Name of correlation coefficient	Plot I	Plot II	Plot III
r_{TA}	$0{\cdot}615^{-} \pm 0{\cdot}043$	$0{\cdot}622 \pm 0{\cdot}066$	$0{\cdot}648 \pm 0{\cdot}032$
r_{TD}	$0{\cdot}668 \pm 0{\cdot}044$	$0{\cdot}656 \pm 0{\cdot}070$	$0{\cdot}584 \pm 0{\cdot}040$

The coefficients in the last table indicate high correlation between the number of rays on stem and branch terminals. This was not true for *Zizia aurea**. Each of the correlation coefficients is about 0·6 or larger and indicates the influence of the number of rays on T upon the number produced by axillary terminals.

Another question might be asked concerning the size of the plant and the size of the cluster. The answer to this may be found in the coefficients of correlation between the size of the plants and the size of the umbels. Size of plant is designated by the number of terminal umbels it has and the size of the umbel by the number of rays it has. A large plant is one that produced a large number of terminals, and a large umbel is one that had a large number of pedicels. The correlation coefficients were found for the size of plant S and size of terminal umbels for Plots I and III. These coefficients and their standard errors are:

	Plot I	Plot III
r_{TS}	$= 0{\cdot}121 \pm 0{\cdot}067$	$-0{\cdot}149 \pm 0{\cdot}065^{+}$
r_{AS}	$= 0{\cdot}248 \pm 0{\cdot}064$	$0{\cdot}128 \pm 0{\cdot}055^{-}$
r_{BS}	$= 0{\cdot}296 \pm 0{\cdot}062$	$0{\cdot}181 \pm 0{\cdot}054$
r_{CS}	$= 0{\cdot}307 \pm 0{\cdot}065^{-}$	$0{\cdot}200 \pm 0{\cdot}054$
r_{DS}	$= 0{\cdot}313 \pm 0{\cdot}072$	$0{\cdot}285^{+} \pm 0{\cdot}054$
r_{ES}	$= 0{\cdot}225 \pm 0{\cdot}102$	$0{\cdot}223 \pm 0{\cdot}068$
r_{FS}	$= 0{\cdot}012 \pm 0{\cdot}192$	$-0{\cdot}051 \pm 0{\cdot}115^{+}$

The first two values are contradictory in sign and, having such large standard errors, indicate that the size of T is not correlated with the size of the plant. From the standard errors it appears that the sizes of A, B, C and D for Plot I are

* Baten, W. D. Influence of position on structure of inflorescences of *Zizia aurea*. *Papers of the Mich. Acad. of Science, Arts and Letters*, 1935.

correlated to some extent with the size of the plants, and that the sizes of *B*, *C*, *D* and *E* for Plot III are also correlated with the size of the plants, but not to a very great extent. The last coefficients with their standard errors reveal no correlation between the size of the plant and the number of rays on the last clusters to bloom. The above values reveal very little correlation between the size of the terminal umbels and the size of the plants.

Table IV contains the coefficients of correlation between the various lateral terminal inflorescences.

TABLE IV.

Correlation Coefficients between the number of Pedicels of Branch Terminal Inflorescences.

Name of correlation coefficient	Plot I	Plot II	Plot III
r_{AB}	0·896	0·897	0·908
r_{BC}	0·903	0·878	0·932
r_{CD}	0·908	0·852	0·953
r_{DE}	0·925	0·936	0·903
r_{EF}	0·908	0·876	0·856
r_{FG}	0·909	0·897	0·919

Values from Table IV reveal *very* high correlation between the number of pedicels on any branch terminal umbel and the number of pedicels on the terminal directly above it. This indicates that the size of a terminal umbel depends on the umbel above it. This was found to be true for the axillary umbels of *Zizia aurea*. In every instance the correlation coefficient is 0·9 or larger.

An inspection of Correlation Tables 1 and 2 will enable one to appreciate the large amount of correlation between consecutive lateral terminal inflorescences. The eye sees at once the trends and notes how closely the items lie along these trend lines, although the lines are not actually drawn in the tables. Space will not permit other correlation tables which are similar to these.

The multiple correlation coefficient between *B* and *T* and *A*, to determine how much *T* and *A* together affected *B*, is

$$r_{B \cdot TA} = 0{\cdot}904,$$

which shows that the number of pedicels on *B* was influenced very little by the size of *T*, for the total correlation coefficient between *B* and *A* is 0·896. This indicates that the size of *A* is more influential than the size of *T* on the size of *B*.

The multiple correlation coefficient between *C* and *A* and *B* is

$$r_{C \cdot AB} = 0{\cdot}939,$$

which indicates that the combined influence of *A* and *B* did not affect the size of *C* very much. The correlation coefficient was increased by only 0·007.

CORRELATION TABLE 1.

Correlation between Umbels A and B for Plot I.

Number of pedicels per umbel *B*

Number of pedicels per umbel *A*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	Totals
12	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
13	3	2	—	—	—	—	—	—	—	—	—	—	—	—	5
14	4	4	1	1	1	—	—	—	—	—	—	—	—	—	11
15	—	6	8	11	2	—	—	—	—	—	—	—	—	—	27
16	—	1	3	16	4	6	2	—	—	—	—	—	—	—	32
17	—	—	—	11	12	8	7	—	—	—	—	—	—	—	38
18	—	—	—	1	—	13	7	5	2	—	—	—	—	—	28
19	—	—	—	—	1	2	15	8	4	1	1	—	—	—	32
20	—	—	—	—	—	—	3	7	7	2	—	—	—	—	19
21	—	—	—	—	—	—	—	5	5	—	2	—	—	—	12
22	—	—	—	—	—	—	—	—	—	—	1	1	—	—	2
23	—	—	—	—	—	—	—	—	—	—	—	2	—	—	2
24	—	—	—	—	—	—	—	—	1	—	—	—	—	2	3
Totals	7	13	13	40	20	29	34	25	19	3	4	3	—	2	212

$r_{AB} = 0{\cdot}8964.$

CORRELATION TABLE 2.

Correlation between Umbels C and D for Plot III.

Number of pedicels per umbel *D*

Number of pedicels per umbel *C*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	Totals
13	3	1	1	—	—	—	—	—	—	—	—	—	—	—	5
14	2	8	5	—	—	—	—	—	—	—	—	—	—	—	15
15	—	5	10	6	—	—	—	—	—	—	—	—	—	—	21
16	—	1	7	16	5	1	—	—	—	—	—	—	—	—	30
17	—	—	4	15	12	10	1	—	—	—	—	—	—	—	42
18	—	—	1	2	5	18	7	1	—	—	—	—	—	—	34
19	—	—	—	2	5	13	18	5	—	2	—	—	—	—	45
20	—	—	—	—	1	3	6	13	3	4	3	—	—	—	33
21	—	—	—	—	1	—	2	5	16	6	1	—	—	—	31
22	—	—	—	—	—	—	1	2	2	5	3	1	—	—	14
23	—	—	—	—	—	—	—	—	—	2	—	2	—	—	4
24	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
25	—	—	—	—	—	—	—	—	—	1	2	2	1	—	6
26	—	—	—	—	—	—	—	—	—	—	—	—	2	1	3
27	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	5	15	28	41	29	45	35	26	21	20	11	5	3	1	285

$r_{CD} = 0{\cdot}9526^{5}.$

II. ANALYSIS OF NON-TERMINAL INFLORESCENCES.

The writer decided to make counts of the number of rays on all the clusters on the last 224 plants taken from Plot III. This investigation would be more nearly complete had this been done for every plant taken from all the plots.

Table V contains the frequency distribution of terminal and non-terminal clusters for the last 224 plants from the third plot.

TABLE V.

Distribution of Umbels for 224 plants from Plot III.

Number of umbels per plant	Frequency	Number of non-terminal umbels per plant	Frequency
7—10	14	2—5	6
11—14	56	6—9	68
15—18	67	10—13	83
19—22	43	14—17	38
23—26	24	18—21	20
27—30	12	22—25	6
31—34	5	26—29	2
35—38	2	30—33	1
39—42	1		
Total	224	Total	224
Mean	17·911	Mean	12·018*
S.D.	5·853	S.D.	4·627

* The standard deviations were obtained by the formula for grouped discrete variables.

The frequencies in Table V give a general impression of how the non-terminals were distributed. There were about twice as many non-terminals as terminals, for the average number of terminals multiplied by two is nearly equal to the mean of the non-terminals. The next table shows the branches on which these non-terminals were found.

There were only four plants which had three non-terminals on the first branch. The lower branches had more different non-terminals. It seems characteristic of this species to have only two non-terminals on the first branch. There were 67 plants which had three non-terminals on the second branch, and 154 plants that had three on the third branch. The lower branches had more different umbels than the higher branches. On all the first branches there were 631 clusters; on all the second, 726; on all the third, 817; on the fourth, 753; and so on. The third branch appears to contain more clusters than any other branch. The numbers in Table VI show that the first three branches do not have four non-terminal

TABLE VI.

Number of Branch Inflorescences for 224 plants on Plot III.

Number of plants with	Branch									Totals
	1	2	3	4	5	6	7	8	9	
Terminal	224	223	224	205	134	59	25	4	1	1099
First non-terminals	222	219	222	199	125	57	23	4	1	1072
Second non-terminals	181	216	217	188	118	52	21	4	1	998
Third non-terminals	4	67	154	146	89	46	18	4	1	529
Fourth non-terminals	—	1	—	14	34	19	11	2	1	82
Fifth non-terminals	—	—	—	1	1	1	2	1	—	6
Totals	631	726	817	753	501	234	100	19	5	3786

umbels, whereas the others had four or more. More than one-half of the third non-terminals were found on the third and fourth branches. More than one-half of the fourth non-terminals were found on the fourth and fifth branches; one-half of the fifth clusters were found on the seventh and eighth branches. Over 60 per cent. of all the branch clusters are found on the first three branches. The arrangement of the non-terminals is quite different from that of *Zizia aurea*; the first two branches produced most of the clusters. A bar diagram in Fig. 3 shows the percentages of umbels for the various branches.

The correlation coefficient between the number of terminals and non-terminals is 0·904. This was found from Correlation Table 68 (not published).

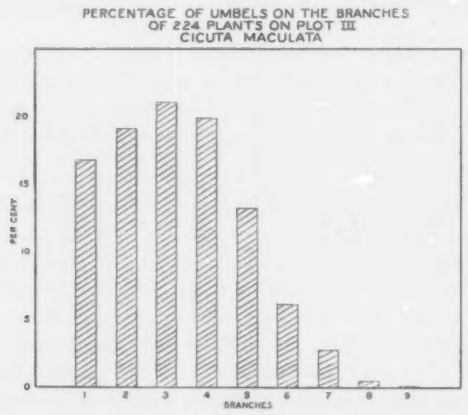


Fig. 3.

Comparison of Branch Terminals and Non-terminals.

Table VII contains the means for the number of pedicels of lateral inflorescences.

TABLE VII.

Average number of Pedicels per Lateral Inflorescence.*

Umbel	Means and S.D.	No.	Umbel	Means and S.D.	No.	Umbel	Means and S.D.	No.
A	17·20 ±2·38	323	*B*	18·03 ±2·62	319	*C*	18·33 ±2·78	317
*A*1	17·65^{+}±2·87	225	*B*1	18·35^{-}±2·82	226	*C*1	18·27 ±2·79	223
*A*2	18·30 ±2·85^{+}	187	*B*2	18·81 ±3·13	217	*C*2	19·04 ±3·18	218
*A*3			*B*3	19·48 ±2·81	67	*C*3	19·56 ±3·13	151

Umbel	Means and S.D.	No.	Umbel	Means and S.D.	No.	Umbel	Means and S.D.	No.
D	18·22 ±2·74	287	*E*	18·09 ±2·66	192	*F*	17·96 ±2·35	75
*D*1	18·19 ±2·95^{-}	201	*E*1	18·43 ±2·61	127	*F*1	19·09 ±3·05^{+}	58
*D*2	19·04 ±3·23	189	*E*2	19·24 ±2·96	119	*F*2	20·08 ±3·27	53
*D*3	19·18 ±3·35^{+}	147	*E*3	19·34 ±2·76	89	*F*3	20·15^{-}±3·44	47
*D*4	20·57 ±3·31	14	*E*4	20·44 ±3·30	34	*F*4	20·37 ±2·62	19

Umbel	Means and S.D.	No.
G	17·33 ±0·74	30
*G*1	19·43 ±2·41	23
*G*2	20·29 ±3·00	21
*G*3	21·06 ±3·03	18
*G*4	21·64 ±2·46	11

The values of the means in Table VII show how the means of rays for the non-terminals increase for lower branches. In every instance except two the means for non-terminals are larger than the means for the terminals. This was not the situation with *Zizia aurea*†. The following inequalities hold for the various branches:

$M_A < M_{A1} < M_{A2}$ (when M_A means the average number of rays for cluster A)

$M_B < M_{B1} < M_{B2} < M_{B3}$

$M_C > M_{C1} < M_{C2} < M_{C3}$

$M_D > M_{D1} < M_{D2} < M_{D3} < M_{D4}$

$M_E < M_{E1} < M_{E2} < M_{E3} < M_{E4}$

$M_F < M_{F1} < M_{F2} < M_{F3} < M_{F4}$

$M_G < M_{G1} < M_{G2} < M_{G3} < M_{G4}$.

* Compare Tables VIII and IX below.

† Baten, W. D. Influence of position on structure of inflorescences of *Zizia aurea*. *Papers of the Mich. Acad. of Science, Arts and Letters*, 1935.

TABLE VIII. *Frequency of Pedicels for Non-terminal Umbels, Plot III.*

Number of pedicels per umbel	Umbels											
	A1	*A2*	*B1*	*B2*	*B3*	*C1*	*C2*	*C3*	*D1*	*D2*	*D3*	*D4*
12	2	1	1	—	—	2	1	—	—	1	—	—
13	6	4	4	1	—	3	3	3	5	1	1	—
14	16	6	8	8	4	4	3	3	9	6	7	—
15	19	17	16	18	—	20	17	7	16	11	12	1
16	47	29	31	30	7	33	26	8	33	30	15	1
17	30	32	42	30	5	38	24	16	28	11	15	1
18	27	19	22	30	10	32	26	23	33	33	22	1
19	25	19	29	17	9	32	32	24	20	21	12	2
20	20	18	29	24	8	15	24	12	12	21	13	—
21	14	14	15	18	6	14	19	16	22	17	12	2
22	9	17	16	10	8	13	14	14	12	11	17	2
23	3	3	5	14	3	5	11	11	3	9	6	2
24	4	4	—	8	6	6	8	7	1	6	6	—
25	—	2	4	2	—	3	2	—	2	4	3	1
26	2	—	2	2	1	—	3	3	2	1	2	—
27	—	1	—	3	—	2	2	1	2	3	1	1
28	—	—	—	—	—	—	—	2	—	1	2	—
29	1	1	1	1	—	1	2	—	—	1	—	—
30	—	—	1	1	—	—	—	—	—	—	—	—
31	—	—	—	—	—	—	—	1	—	—	1	—
32	—	—	—	—	—	—	—	—	—	1	—	—
33	—	—	—	—	—	—	1	—	1	—	—	—
Totals	225·00	187·00	226·00	217·00	67·00	223·00	218·00	151·00	201·00	189·00	147·00	14·00
Mean	17·65+	18·30	18·35−	18·81	19·48	18·27	19·04	19·56	18·19	19·04	19·18	20·57
S.D.	2·87	2·85+	2·82	3·13	2·81	2·79	3·18	3·13	2·95−	3·23	3·35+	3·31

TABLE IX. *Frequency of Pedicels for Non-terminal Umbels, Plot III.*

Number of pedicels per umbel	Umbels											
	E1	*E2*	*E3*	*E4*	*F1*	*F2*	*F3*	*F4*	*G1*	*G2*	*G3*	*G4*
13	—	—	—	—	—	1	—	—	—	—	—	—
14	5	2	1	1	1	—	1	—	1	—	—	—
15	9	6	2	2	6	2	2	—	—	1	1	—
16	17	19	8	2	4	4	3	—	1	1	—	—
17	23	9	14	3	10	5	5	2	5	2	2	1
18	15	20	15	3	7	6	9	4	2	3	2	—
19	19	13	10	2	5	4	2	4	2	2	1	1
20	14	11	10	2	7	9	5	—	2	1	—	2
21	7	12	9	5	8	5	4	2	5	4	2	1
22	8	10	5	4	3	7	4	2	3	2	4	2
23	6	6	8	2	2	4	3	4	2	1	2	1
24	2	4	1	5	3	1	3	—	—	3	3	2
25	1	3	5	2	—	3	4	—	—	—	—	—
26	—	2	—	—	—	—	—	—	—	—	—	1
27	1	2	1	1	1	1	—	1	—	1	1	—
28	—	—	—	—	—	—	1	—	—	—	—	—
29	—	—	—	—	1	—	1	—	—	—	—	—
30	—	—	—	—	—	—	—	—	—	—	—	—
31	—	—	—	—	—	1	—	—	—	—	—	—
Totals	127·00	119·00	89·00	34·00	58·00	53·00	47·00	19·00	23·00	21·00	18·00	11·00
Mean	18·43	19·24	19·34	20·44	19·09	20·08	20·15−	20·37	19·39	20·33	21·06	21·64
S.D.	2·61	2·96	2·76	3·30	3·05+	3·27	3·44	2·62	2·41	3·00	3·03	2·46

Later umbels on the branches produce more rays than earlier ones. The increases of the means may be detected immediately by a glance at Fig. 4. It seems evident that the position of the inflorescence on the plant has much to do with its size.

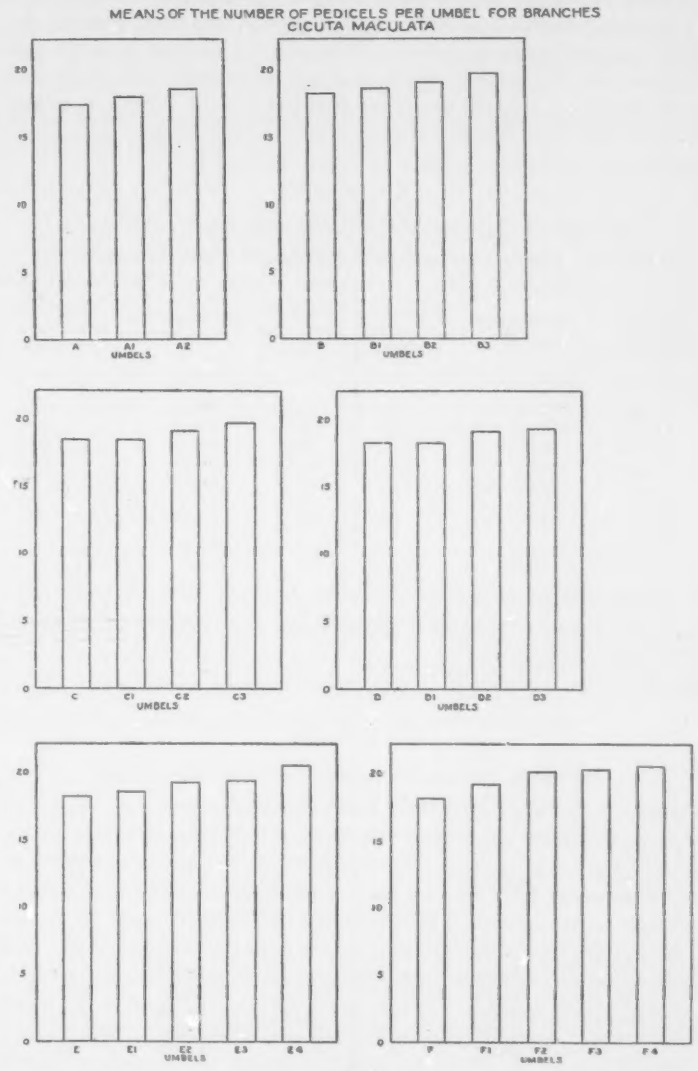


Fig. 4.

Correlation between Axillary Terminal Inflorescences.

Table X contains coefficients of correlation between the number of rays, for branch terminals, and first non-terminals on the same branch, and also between *T* and the first non-terminals.

The values in the first column of coefficients in Table X reveal high correlation between branch terminal and first non-terminal on the same branch. In every case, except one, this value is equal to 0·6 or greater; most of them are about 0·7. This suggests that the second cluster to bloom on a branch has many pedicels if the first cluster has many, and few if the first has few, etc. There is a little more correlation between first and second branch umbels than between stem and branch terminals, as may be seen by comparing Tables III and X.

TABLE X.

Correlation Coefficients between Terminals and First Non-terminals.

Name of correlation coefficient	Correlation coefficient	Number of plants	Name of correlation coefficient	Correlation coefficient	Number of plants
$r_{A\,A1}$	0·679 ± 0·038	203	$r_{T\,A1}$	0·463 ± 0·055+	203
$r_{B\,B1}$	0·711 ± 0·035−	202	$r_{T\,B1}$	0·499 ± 0·053	202
$r_{C\,C1}$	0·696 ± 0·036	205	$r_{T\,C1}$	0·490 ± 0·053	205
$r_{D\,D1}$	0·691 ± 0·039	183	$r_{T\,D1}$	0·416 ± 0·061	183
$r_{E\,E1}$	0·537 ± 0·067	113	$r_{T\,E1}$	0·335+ ± 0·084	113
$r_{F\,F1}$	0·600 ± 0·092	48	$r_{T\,F1}$	0·459 ± 0·106	48
$r_{G\,G1}$	0·186 ± 0·201	23	$r_{T\,G1}$	0·285+ ± 0·192	23

The second column of values in Table X shows that there is correlation between *T* and second branch inflorescences, but not so much as between *T* and first branch inflorescences.

Table XI gives coefficients of correlation between second and third branch clusters.

TABLE XI.

Coefficients of Correlation between Second and Third and Third and Fourth Branch Umbels.

Name of correlation coefficient	Correlation coefficient	Number of plants	Name of correlation coefficient	Correlation coefficient	Number of plants
$r_{A1\,A2}$	0·940	164	—	—	—
$r_{B1\,B2}$	0·936	196	$r_{B2\,B3}$	0·935+	61
$r_{C1\,C2}$	0·912	198	$r_{C2\,C3}$	0·950	142
$r_{D1\,D2}$	0·913	171	$r_{D2\,D3}$	0·946	129
$r_{E1\,E2}$	0·896	108	$r_{E2\,E3}$	0·923	80
$r_{F1\,F2}$	0·932	53	$r_{F2\,F3}$	0·928	39

The correlation coefficients in Table XI indicate *very high* correlation between consecutive branch non-terminal inflorescences. Every value is equal to 0·9 or greater, which shows that the number of rays to the umbel is dependent upon the number in the umbel on the same branch just above it. There is greater correlation between consecutive non-terminals on the same branch than between terminals and the umbel just below it on the same branch. The correlation between third and second branch clusters is about the same as that between fourth and third, as is seen by comparing the second and fifth columns of Table XI.

Table XII contains the coefficients of correlation between corresponding clusters on consecutive branches.

TABLE XII.

Correlation Coefficients between corresponding Umbels on consecutive Branches.

Name of correlation coefficient	Correlation coefficient	Number of plants	Name of correlation coefficient	Correlation coefficient	Number of plants
r_{A1B1}	0·892	200	r_{A2B2}	0·939	161
r_{B1C1}	0·901	202	r_{B2C2}	0·949	190
r_{C1D1}	0·867	183	r_{C2D2}	0·934	167
r_{D1E1}	0·873	112	r_{D2E2}	0·921	105
r_{E1F1}	0·802	49	r_{E2F2}	0·923	45
r_{B3C3}	0·935+	59	r_{C3D3}	0·921	103
r_{D3E3}	0·906	78	r_{E3F3}	0·883	45

The values in Table XII show that there is a *great deal* of correlation between corresponding inflorescences on consecutive branches. All these coefficients, except one, are equal to 0·9 or more. Values for second clusters on consecutive branches are a little greater than those for first non-terminals in every case.

III. Comparison of Stem and Branch Inflorescences.

In *Zizia aurea* stem inflorescences produced staminate flowers altogether while branch inflorescences produced perfect flowers. Microscopic examinations reveal no difference between stem and branch florets for *Cicuta maculata*. Plate I (see p. 65) contains a picture of an umbel and also a picture of an umbellet, showing the flowers. There was no difference between the florets of branch terminals and non-terminals. Every flower on the cluster shown produced fruit. On umbels of *Zizia aurea* there is one circular row of imperfect florets; this does not occur on umbels of *Cicuta maculata*.

Bimodal distributions arise many times because the sampler is not careful to separate data pertaining to flowers at different positions on the plants. The numbers of pedicels per stem terminal should not be mixed with the numbers per

branch umbel. This study should serve as a caution to the reader, who desires to make statistical investigations concerning flowers, to keep a close check on the positions at which the flowers grow. Some investigators who took samples at different times during the flowering season concluded that the later blooms differed significantly from the earlier ones. Their conclusions might have been different had position been taken into consideration. Bimodal, trimodal and polymodal distributions give the statistician trouble, are difficult to interpret and cause the computer many hours of work dissecting them into monomodal distributions. If the positions of flowers, inflorescences, bracts, leaves, etc., were taken into consideration, there would not arise so many multimodal distributions. If an investigator is comparing flower parts for different times in the season, he should compare flowers at the same positions for certain plants. At one time in the season the terminals may be the only flowers that are mature enough to gather, at another time branch non-terminals may be the only ones from which to sample. Perhaps stem terminals might not differ significantly for different periods of the season. Distributions from different plots might not differ significantly if positions or corresponding parts were compared instead of comparing just random samples. Some object to many of the samples because they are not random; this study clearly shows that a sample might be too random, for it might mix things that should not be mixed.

Summary.

This investigation shows that:

1. The average number of stem and branch terminal inflorescences is about five.

2. The average number of pedicels per stem terminal inflorescence is significantly smaller than the average for branch terminal and non-terminals.

3. The average number of rays per stem and branch terminal is significantly different for different environments.

4. The number of rays on branch terminal umbels increases for new branches, reaches the greatest value on the optimum branch, and then decreases for each new branch until the end of the season.

5. There is high correlation between the number of pedicels on stem and branch terminals.

6. There is practically no correlation between the size of the plant and the number of pedicels per stem terminal cluster.

7. There is very little correlation, if any, between the size of the plant and the size of branch terminals.

8. *Very high* correlation exists between the number of rays per umbel for consecutive branch terminal umbels, the correlation coefficient being as great as or greater than 0·9.

9. The multiple correlation between third branch terminals and first and second branch terminals is about the same as that between third and second branch terminals.

10. The multiple correlation between second branch terminals and stem and first branch terminals is not much different from that between second and first branch terminal inflorescences.

11. The average number of axillary non-terminals per plant is about twelve.

12. The first branch produces only two non-terminal clusters.

13. Each of the first three branches produces no more than three non-terminals, whereas the other branches produce more than three.

14. The third branch produces more inflorescences than any other, and is the "optimum branch."

15. The average number of pedicels per cluster increases for each new cluster on the same branch.

16. There is a significant difference between the mean number of rays of branch terminals and of the last non-terminal on the same branch.

17. High correlation exists between branch terminals and first non-terminals.

18. There is some correlation between the number of rays per cluster for stem terminals and first branch non-terminals.

19. There is *very high* correlation between the number of rays on consecutive branch non-terminal clusters.

20. High correlation exists between the number of pedicels per umbel for corresponding umbels on consecutive branches, the coefficient of correlation being greater than 0·9.

21. There is no difference between the florets on stem and branch inflorescences.

22. The number of pedicels per cluster is a function of the position of the cluster on the plant.

23. The correlation between the number of non-terminals and the number of terminals is 0·904.

24. One should be very careful when taking a random sample of flowers, for flowers at different positions on certain plants are different. Distributions pertaining to them should not be mixed.

A BIOMETRIC STUDY OF THE HUMAN MANDIBLE.

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(1) *Introduction.* Although measurements of several thousand mandibles taken for anthropological purposes have been published, they are actually of little value. This is primarily due to the fact that the technique of measurement has never been standardized, and many of the data obtained by following the techniques which have been most widely used cannot be compared accurately in cases where direct comparison should be possible. The value of the material is also lessened considerably since many of the series described are unsexed, and hence the statistical treatment generally used in comparing different racial series cannot be applied. The main object of this paper is to present the measurements of two ancient Egyptian series of mandibles—made up by 141 and 125 specimens respectively—the sexes of which were determined by sexing anatomically the accompanying crania or complete skeletons.

We are indebted to Professor Karl Pearson for permission to work on this material. It is extensive enough to make profitable the calculation of constants of variation and correlations, and the application of those methods of racial comparison which have already been applied extensively to cranial measurements. Our aim was to explore the possible usefulness of the methods rather than to make any positive contribution to the problem of racial classification. Considerably more material than is yet available will be needed to reveal how far mandibular measurements are capable of aiding the discovery of racial relationships.

(2) *The Choice and Accuracy of the Measurements.* The technique followed is that which has been used by several writers in *Biometrika* since it was given by the present writer in this Journal in 1923†. There are now far more data relating to sexed mandibles taken in accordance with the biometric than in accordance with any other technique. The measurements of a bone, or any other part of the body, which will be most useful for anthropological purposes can only be determined empirically in the present state of our knowledge. A number which appear to provide an adequate description of the bone as a whole, and of all its principal

* Miss Collett measured the Kerma series and calculated the means and constants of variation for them, and also some of the coefficients of racial likeness. The measurements of the male Qau series were taken by Mr Adyanthāya. The writer is responsible for the remaining measurements and calculations and for the conclusions.

† "A First Study of the Tibetan Skull," Vol. xiv. pp. 193—260. Measurements of the mandible are dealt with on pp. 253—260.

parts, must first be defined. Laboratory experience may make it possible to reject or modify some of these, and statistical treatment, after they have been found for series representing several different races, should give an estimate of their relative value as racial criteria. In the technique referred to 30 absolute measurements and 7 angles found directly from the bone were defined, while some pairs of these provide measures of the same character taken in slightly different ways. In such cases a choice of the more effective measurements may now be made as a result of experience in the laboratory. In applying definitions of a number of measurements, the observer gains a general impression of their relative degrees of reliability, and he may be able to grade them roughly according to this qualitative appreciation. A few may be felt to be so unreliable that they should be rejected without further enquiry, and this is so in the case of all measurements depending on placing the mandible *in situ* on the cranium. The condition of the teeth, and the difficulty of adjusting the condyles in the glenoid fossæ in positions which can be supposed to correspond with their normal positions in life, render these determinations extremely uncertain and it is suggested that they should be rejected*. It was felt unsafe to reject any of the remaining measurements before obtaining statistical estimates of the personal equations involved in determining them. For this purpose a series (the Kerma) measured by Miss Collett (C) was re-measured by the present writer (M_1) until fifty comparisons were available for each character. The majority of the mandibles included are male, but some female had to be included in order to obtain fifty readings for a few of the characters, and it is reasonable to assume that the personal equations will be approximately the same for series of either sex. After an interval of two years, the present writer obtained new readings (M_2) on the same material. The information required can be obtained from the two sets of differences, $(M_1 - C)$ and $(M_1 - M_2)$, and data for these are given in Table I. It would be of considerable interest to include a third comparison with readings obtained by a third observer who followed the written definitions only, before receiving instruction in the same laboratory, but this has not been possible so far. Before discussing the data available, it may be noted that all estimates of personal equation are likely to be influenced considerably by measurements taken on imperfect specimens. It is often difficult to decide whether an entry should be made, or not, if the region of one terminal of a measurement is defective, and there can be little doubt that the variabilities of the differences in the present case would have been reduced considerably if several readings which are questionable owing to this cause had been omitted. Another factor may be supposed to have influenced the results. In the interval between the times when the senior author obtained his first and second readings he modified some of his definitions to a slight extent as the result of experience gained in measuring other material. These modifications consist almost entirely of additional instructions needed in dealing with abnormal specimens.

* According to the technique referred to, the measurements in question are $L\angle$, $L'\angle$, $F\angle$, $S\angle$ and $p_a p_b$. The "mandibular profile angles," $L\angle$ and $L'\angle$, are of considerable interest, but it appears to be impossible to determine them with sufficient accuracy to indicate small inter-racial differences.

The maximum differences found are given in Columns 2 and 3 of Table I, and it will be seen that they are of much the same order for the two sets of differences. For 20 characters the maximum difference for $(M_1 - C)$ is the greater, for 11 the reverse is true, and there is equality for two characters. The male means given by Miss Collett's measurements of the total male Kerma series are in Column 6, and it is somewhat alarming to find that the maximum differences are in some cases greater than 10 per cent. of the means. This is only so for some of the smaller measurements, viz. the breadth of the condyle ($c_y b$), the height of the incisura (ih') and three chords in the symphyseal plane ($p_a d_t$, $p_a d_f$ and $p_a g_n$). For the last of these the maximum difference found is more than half the mean! The fact that relatively large differences are found occasionally when the same specimen is measured by two observers, or by the same observer at different times, may not be of much consequence when the measurements are only used for some particular purposes. The effect of personal equation on mean values is of more importance. Columns 4 and 5 of the table give the differences of the means (Δ)—which are the same, of course, as the mean differences—and the probable errors of these constants. For the first set (Δ_{M_1-C}) 12 of the 33 differences differ from zero by more than 3·5 times their probable errors, and in five of these cases the ratio of Δ_{M_1-C} to its probable error is greater than 5·0, the largest ratio being 9·4. For the second set ($\Delta_{M_1-M_2}$) there are also 12 of the 33 differences which differ from zero by more than 3·5 times their probable errors, while in nine of these the ratio of $\Delta_{M_1-M_2}$ to its probable error is greater than 5·0, the largest ratio being 10·6. There are 14 characters showing differences of means diverging insignificantly from zero in the case of both sets of differences. We may next ask whether the differences of Δ_{M_1-C} and $\Delta_{M_1-M_2}$ are significant or not. For this purpose a knowledge of the correlations of the differences $(M_1 - C)$ and $(M_1 - M_2)$ for individual mandibles is required, since it is clear that these will be correlated variates owing to the fact that the readings M_1 are used in both cases. The correlation coefficients were calculated for all the characters—though they are not given in Table I as they are not of much interest in themselves—and their range is from + ·161 to + ·864. Using these, the probable error of the difference of Δ_{M_1-C} and $\Delta_{M_1-M_2}$, with regard to the signs of the individual and mean differences throughout, is given by the formula

$$\sqrt{\{(\text{p.e. } \Delta_{M_1-C})^2 + (\text{p.e. } \Delta_{M_1-M_2})^2 - 2r_{(M_1-C)(M_1-M_2)} \times \text{p.e. } \Delta_{M_1-C} \times \text{p.e. } \Delta_{M_1-M_2}\}}.$$

It is found that 11 of the 33 differences of Δ_{M_1-C} and $\Delta_{M_1-M_2}$ exceed 3·5 times their probable errors, in 7 of these 11 cases the differences exceed 5·0 times their probable errors, and the most significant difference ($g_n g_o$, R) is 10·0 times its probable error. The most significant differences are shared almost equally between cases where Δ_{M_1-C} is numerically the greater and those for which the converse is true. It is clear that the two sets of differences of means cannot be supposed to differ merely on account of chance causes, though in general there is a fairly close correspondence between them.

TABLE I.

TABLE I.

Data for estimating the Personal Equation of Mandibular Measurements from Readings by Collett (C) and Morant (M_1 and M_2) on the same fifty Specimens (Kerma Series).

Character	Maximum individual differences		Differences of Means Δ		Kerma male Means (m_C)*	$\frac{\Delta_{M_1-C}}{\text{p.e. } m_C}$	Standard Deviations of differences σ		Standard Deviations of Kerma male series (Σ_C)	$\frac{\sigma_{M_1-C}}{\Sigma_C}$
	M_1-C	M_1-M_2	M_1-C	M_1-M_2			M_1-C	M_1-M_2		
ml	+1·6	+ & −1·4	+·02±·070	+·18±·054	106·0±·46	·04	·73±·049	·57±·038	5·07±·33	·14
rl	−2·1	+1·0	−·02±·058	+·02±·036	61·7±·55	·04	·61±·041	·38±·026	6·04±·38	·10
$d_t h$	−2·6	−1·3	−·01±·079	−·37±·045	36·4±·24	·04	·83±·056	·47±·032	2·45±·17	·34
$g_o g_o$	+1·5	−2·5	−·04±·051	−·44±·066	88·9±·67	·06	·53±·036	·69±·047	7·18±·47	·07
$c_y h$	−1·5	+2·8	−·06±·052	+·16±·057	56·3±·59	·10	·54±·036	·60±·040	6·84±·42	·08
$m_2 p_1$	−1·3	+ & −1·2	+·02±·041	+·07±·048	28·4±·16	·13	·43±·029	·50±·034	1·77±·11	·24
$p_a g_n$	−5·3	−4·5	−·02±·121	+·06±·112	9·1±·15	·13	1·27±·086	1·17±·079	1·80±·11	·71
$c_y l$	−0·7	+0·9	−·02±·019	+·00±·022	20·8±·14	·14	·20±·013	·23±·016	1·54±·10	·13
$m_2 h$	−1·3	−1·3	−·03±·041	−·11±·045	26·5±·20	·15	·43±·029	·47±·032	2·33±·14	·18
w_1	+ & −1·0	+ & −1·4	−·09±·043	+·01±·047	113·8±·50	·18	·45±·030	·49±·033	5·34±·36	·08
$R\angle$	+1°·5	+2°·5	−0°·12±·063	+0°·14±·083	76°·6±·65	·18	0°·66±·045	0°·87±·059	6°·95±·46	·09
$g_n g_o$, R	+4·6	+4·6	−·09±·125	+1·21±·157	85·8±·39	·23	1·31±·088	1·65±·111	4·40±·28	·30
$M\angle$	+3°·6	−1°·3	+0°·14±·090	+0°·03±·056	120°·9±·53	·28	0°·94±·063	0°·59±·040	5°·79±·37	·16
$g_n g_o$, L	+3·1	+2·6	−·12±·098	+·57±·088	84·9±·38	·32	1·03±·069	·92±·062	4·17±·27	·25
$c_y c_r$	+1·8	+1·6	−·09±·056	+·15±·055	35·2±·26	·35	·59±·040	·58±·039	2·88±·19	·20
ih	−2·8	−1·6	+·16±·064	+·03±·050	47·8±·46	·35	·67±·045	·52±·035	5·43±·33	·12
$c_y b$	+ & −2·8	−3·1	+·04±·095	+·26±·083	9·5±·11	·36	1·00±·067	·87±·059	1·33±·08	·75
$g_n d_f$	+1·3	+ & −1·0	+·12±·052	−·07±·036	31·2±·27	·44	·54±·036	·38±·026	2·51±·19	·22
w_2	+3·0	−1·9	+·33±·075	+·05±·063	92·7±·71	·46	·79±·053	·66±·045	8·04±·50	·10
$c_p l$	−1·9	−1·8	−·21±·044	+·13±·052	77·7±·41	·51	·46±·031	·55±·037	4·51±·29	·10
rb'	−1·6	+1·2	+·13±·039	+·17±·028	33·9±·24	·54	·41±·028	·29±·020	2·96±·17	·14
$c_r c_r$	+1·0	+1·4	+·34±·036	+·40±·049	92·3±·52	·65	·38±·026	·51±·034	5·18±·36	·07
$c_r h$	+1·6	−1·3	−·35±·062	−·41±·048	66·7±·51	·69	·65±·044	·50±·034	5·90±·36	·11
zz	+0·7	−1·2	+·16±·024	−·00±·035	44·3±·23	·70	·25±·017	·37±·025	2·70±·16	·09
$p_1 h$	−1·5	−1·8	−·21±·052	−·24±·052	31·3±·27	·78	·54±·036	·54±·036	2·53±·19	·21
$g_o p_a g_o$	+5·0	+ & −4·0	−·72±·179	−·65±·172	195·7±·89	·81	1·88±·127	1·80±·121	9·84±·63	·19
$C\angle$	−6°·5	+4°·5	+0°·48±·194	+0°·60±·128	70°·1±·59	·81	2°·03±·137	1°·34±·090	5°·35±·42	·38
h_1	−1·3	−1·8	−·24±·052	−·12±·057	33·6±·28	·86	·54±·036	·60±·040	2·78±·20	·19
G_2'	+2·7	−2·5	+·27±·067	−·14±·059	43·8±·27	1·00	·70±·047	·62±·042	3·11±·19	·22
ik'	−1·4	−1·1	−·13±·040	−·33±·031	12·9±·11	1·18	·42±·028	·33±·022	1·26±·08	·33
$C'\angle$	−3°·7	+ & −3°·0	+0°·73±·136	+0°·30±·133	72°·2±·60	1·22	1°·43±·096	1°·39±·094	5°·77±·42	·25
$p_a d_t$	−5·9	−3·2	−·50±·108	−·69±·100	27·4±·33	1·52	1·13±·076	1·05±·071	3·25±·23	·35
$p_a d_f$	−2·9	+4·8	−·60±·073	−·39±·122	25·0±·33	1·82	·76±·051	1·28±·086	3·06±·23	·25

* The numbers of mandibles on which these means are based are given in Table IV, except in the case of w_2 for which the number is 58, $c_y b$ (61), $g_n d_f$ (38) and $C\angle$ (37). The numbers for all the characters range from 37 to 67.

The value of the differences of means in aiding the selection of the more reliable measurements may now be considered. We should like to be able to compare them with average inter-racial differences, but these cannot be approximated to at present since insufficient material is available. The influence that personal equation in measuring may have on inter-racial comparisons can be estimated roughly, however, in an indirect way. The means, with their probable errors, derived from Miss Collett's measurements of the total male Kerma series are in Column 6 of Table I, and the numbers of individuals on which these are based range from 37 to 67. In the next column are quantities obtained by dividing the differences of the means found from Miss Collett's and Morant's measurements of the same 50 specimens (Δ_{M_1-C}) by the corresponding probable errors for the total male series. These values will give estimates of the extent to which the means for the different characters are made unreliable owing to the personal equations of the measurers*. We may hope to derive a fairly reliable knowledge of racial differences in type from series of mandibles giving means based on 35—70 individuals, and if for any particular measurement the probable error of the constant is found to be no greater than the average difference between the readings of two observers then the measurement in question is clearly an unsatisfactory one. The four ratios in the table greater than unity indicate this state of affairs. It may be decided, arbitrarily, that any measurement showing the ratio greater than 0·7 is so unreliable that its use should be discontinued. Suppose that the ratio is exactly 0·7, and that the means of two series found by two different observers, and having the same probable errors, are being compared. The difference of the means for a particular character may be supposed significant if it exceeds 3·5 times its probable error. The part contributed to this 3·5 by personal equation will be $0{\cdot}7/\sqrt{2}$, or almost exactly 0·5. The danger of personal equation making a difference appear significant which would be considered insignificant if there were no errors in measurement is thus not very great. If the means could be based on much larger numbers of individuals these errors would be more disturbing, but under present conditions we may assume that they will not affect inter-racial comparisons of means appreciably, after all characters showing the ratio Δ_{M_1-C}/p.e. m_C in Table I

* It may be suggested that a more reliable ratio to use for the purpose in view would be one obtained by dividing each difference of means by the average of two probable errors for the means of the total male series, one set being obtained from Miss Collett's measurements and the other from Morant's—i.e. by using $\Delta_{M_1-C}/\frac{1}{2}$ (p.e. m_{M_1} + p.e. m_C) instead of Δ_{M_1-C}/p.e. m_C. The two probable errors were used in this way in the case of 7 characters—these being the only characters for which Morant had measured all possible male specimens—and they lead to the following comparisons:

	$d_i h$	$c_r c_r$	$p_1 h$	h_1	$C' \angle$	$p_a d_i$	$p_a d_f$
Δ_{M_1-C}/p.e. m_C	·04	·66	·78	·87	1·21	1·52	1·84
$\Delta_{M_1-C}/\frac{1}{2}$ (p.e. m_C + p.e. m_{M_1})	·04	·67	·79	·88	1·11	1·56	1·90

The corresponding ratios found in the two ways are so closely similar that there would be little practical advantage in using the two sets of probable errors instead of one only.

greater than 0·7 have been excluded. The selection of the more reliable measurements should be based primarily on the differences $(M_1 - C)$, since most comparisons are made between series dealt with by different observers. The ratios

$$\Delta_{M_1-M_2}/\text{p.e. } m_C$$

are not given in Table I: 13 of them are greater than 0·7, and six of these 13 are for characters which have the ratio greater than 0·7 for the differences $(M_1 - C)$.

It is not sufficient to show that for a selected group of measurements the differences between means found by two different observers for the same series are so small that they will not affect inter-racial comparisons appreciably. The measurements may be needed for other purposes, as in providing constants of variation and correlations, and hence it is also necessary to consider the distributions of differences of repeated readings. The standard deviations for the two sets of differences are in Columns 8 and 9 of the table. In estimating the significance of the differences between these quantities the correlations between the individual differences of the two sets have to be taken into account again. The probable errors of the quantities in question are given by the formula

$$\sqrt{\{(\text{p.e. } \sigma_{M_1-C})^2 + (\text{p.e. } \sigma_{M_1-M_2})^2 - 2r^2_{(M_1-C)\,(M_1-M_2)} \times \text{p.e. } \sigma_{M_1-C} \times \text{p.e. } \sigma_{M_1-M_2}\}}.$$

The ratios of the differences $\sigma_{M_1-C} - \sigma_{M_1-M_2}$ to their probable errors in cases where these quantities exceed 3·5 are: $g_n d_f$, + 4·0; $C\angle$, + 4·3; $c_r h$, + 4·5; rb', + 4·6; $M\angle_2$ + 4·8; rl, + 4·8; zz, − 5·6; $d_l h$, + 5·7 and $p_a d_f$, − 5·8. The preponderance of the positive signs indicates that in general the readings M_1 and M_2 are more consistent than the readings M_1 and C, but two of the most significant differences are negative.

As there is no marked contrast between the two sets of standard deviations, it will be profitable to apply to them Professor Karl Pearson's recently devised P_{λ_n} test for randomness in sampling*. The distributions of the individual differences $(M_1 - C)$ and $(M_1 - M_2)$ appear to be approximately normal in form in the case of every character, and we will suppose that the probability integrals of the normal curve can be applied approximately in considering the differences between the standard deviations of the two sets of differences†. The test gives a measure of the probability that the whole system of the differences in question is random, and from it we may judge whether it is reasonable to suppose that they are due to chance, or whether there is a real distinction between the two sets. For the total 33 characters, the more stringent—i.e. the lower—of the two values of Q_{λ_n}

* "On a Method of determining whether a Sample of Size n supposed to have been drawn from a Parent Population having a known Probability Integral has probably been drawn at random," *Biometrika*, Vol. xxv. (1933), pp. 379—410. See also: "On the P_{λ_n} Test for Randomness: Remarks, further Illustrations, and Table of P_{λ_n} for given Values of $-\log_{10}\lambda_n$," by Florence N. David, *Ibid.*, Vol. xxvi. (1934), pp. 1—11.

† This is only an approximation as we are only dealing with samples made up by 50 individual values. In determining the probability integrals the correlations between the differences $(M_1 - C)$ and $(M_1 - M_2)$ have to be taken into account, as in the formula for the probable error of the difference of the two standard deviations given in the text.

obtained is ·0000. Omitting two or three of the characters showing most significant differences makes no appreciable increase in this value, and if the seven showing most significant differences are omitted it is only raised to ·0125 for the remaining 26 characters. It must be concluded that the two sets of standard deviations are essentially unlike, since the differences between them cannot be attributed to chance. The readings M_1 and M_2 tend to be more consistent than the readings M_1 and C, but this is not uniformly true for all characters.

The standard deviations for the total male Kerma series, found from Miss Collett's measurements, are given in the penultimate column of Table I, and σ_{M_1-C} divided by these constants in the last column*. The ratios range from ·07 to ·75 (!) and it is clear that several of them are so large that marked unreliability is indicated, even for some characters which show insignificant mean differences.

We may decide, quite arbitrarily, to select for future use those characters for which $\Delta/\text{p.e.}\ m_C$ is not greater than 0·7, and also for which σ/Σ_C is not greater than 0·20. If both sets of differences are considered, so that a measurement is rejected if it has either ratio for either set greater than the limit assigned, then the characters retained will be

(i) ml, rl, $g_o g_o$, $c_y h$, $c_y l$, w_1, $R\angle$, $M\angle$, ih, w_2, $c_p l$ and zz.

Such a stringent test excludes 21 of the total 33 measurements. If the test is applied to the differences $(M_1 - C)$ only four additional measurements may be included ($m_2 h$, rb', $c_r c_r$ and $c_r h$), and if to the differences $(M_1 - M_2)$ only three additional measurements may be included ($c_y c_r$, $g_n d_f$ and G_2'). Other considerations must be taken into account before making a final choice of measurements recommended for future use. As the original list is to be considerably reduced, it is undesirable that some pairs should be included which are really measuring the same character in rather different ways. For this reason it may be decided, after consulting the data in Table I, that the list (i) should be further reduced by omitting w_2, since the other bigonial breadth ($g_o g_o$) is included, and by omitting the projective heights of the condyle ($c_y h$) and incisura (ih), since the length of the ramus (rl) with which they must be highly correlated is included. It is unfortunate

* It may be suggested, again, that the ratios $\sigma_{M_1-C}/\frac{1}{2}(\Sigma_C+\Sigma_{M_1})$ could have been used more appropriately than the ratios σ_{M_1-C}/Σ_C (Σ_{M_1} denoting Morant's standard deviations for the total male Kerma Series). Both quantities were actually calculated in the case of seven characters giving:

	$c_r c_r$	h_1	$p_1 h$	$p_a d_f$	$C'\angle$	$d_1 h$	$p_a d_1$
σ_{M_1-C}/Σ_C	·073	·194	·213	·248	·248	·339	·348
$\sigma_{M_1-C}/\frac{1}{2}(\Sigma_C+\Sigma_{M_1})$	·074	·197	·217	·257	·227	·317	·359

The agreement here is so close, particularly in the case of the lower values, that there would have been little practical advantage in using the second ratio in place of the first. Actually, the present writer had not taken all possible measurements of the total male series in the case of every character, as his purpose was only to make 50 comparisons with Miss Collett's measurements in the case of each series.

that the nine measurements left do not relate to some features of the mandible which are of considerable interest. The four which are allowed when the test of reliability is applied to the differences $(M_1 - C)$, but which are excluded when the differences $(M_1 - M_2)$ are also considered, could all be included with advantage and their retention is hence recommended. It is not surprising to find that all the small chords taken in the symphyseal plane have been found to be unreliable, but it would be desirable to include at least one symphyseal height. Judging from the data in Table I, the most reliable of these measurements are $g_n d_f$, $d_i h$ and h_1 and preference may be given to the last. The "mental angles" C and C', also taken in the symphyseal plane, are admittedly unreliable measurements, but the adoption of the more reliable one may be advocated owing to the importance of obtaining some measure of the projection of the chin. The only direct measurement of the length of the dental arcade in the original technique is that between the alveolar margins at the second molar and first premolar ($m_2 p_1$); this has been rejected in selecting the more reliable measurements, but it appears to be almost as reliable as some included and its continued use may be recommended. The finally selected list is

(ii) w_1, $g_o g_o$, $c_r c_r$, zz, $c_y l$, $ml.\ c_p l$, rb', $m_2 p_1$, h_1, $m_2 h$, $c_r h$, rl, $M\angle$, $R\angle$ and $C'\angle$.

Definitions of these measurements, fuller than those originally provided, are given in the Appendix: see p. 116 *et seq.* The mental angle (C') is known to be more subject to errors of personal equation than the others, and the symphyseal height (h_1) and length of the dental arcade $(m_2 p_1)$ are rather less reliable than the remaining measurements.

The foregoing attempt to estimate the effect of personal equation on mandibular measurements is based on the analysis of data obtained by two workers in the same laboratory, one having learned the technique employed from the other. Smaller differences would almost certainly have been found if more readings on defective bones had been omitted, and it is hoped that the fuller definitions here given will also have the effect of lessening "errors" in measurement. Meanwhile it may be considered safe to neglect the influence of personal equation in making comparisons for the selected group of more reliable measurements, between constants for series dealt with in the same laboratory. There is still a need for an estimate of the personal equations of independent observers who are only guided by written instructions, and further investigation of the whole question would be desirable if it should become possible in future to deal with samples much larger than any at present available.

According to the technique used in this paper, bilateral measurements are taken on the left side only, when possible, but they may be taken on the right instead if the left is defective. All the readings are then used in calculating means and other constants for the series. It might be impossible to justify this procedure if the normal mandible were asymmetrical. Only three of the bilateral measurements can be used to examine the question of asymmetry, since the others depend on adjusting the bone in the standard horizontal position by exerting a vertical

pressure, if necessary, on the second left molar. These three chords were measured on both right and left sides of the same mandibles in the case of the two longest series available and bilateral comparisons are made in Table II. The results are remarkably consistent. While few of the mean differences are statistically significant, they suggest forcibly that in the type the length of the dental arcade from first premolar to second molar (m_2p_1) and the breadth of the ramus (rb') are greater on the left side than on the right, but for the length of the condyle (c_yl) the position is reversed. Such conclusions cannot be reconciled with the statement frequently made that the right side of the mandible is normally greater than the left in every respect*. The mean differences in Table II are small compared with the standard deviations of the measurements on the left side found for the different series, and there can be little harm in making use of the length on the right side if the left is defective.

TABLE II.

Constants of the Differences of Bilateral Mandibular Measurements for two Egyptian Series.

	Means ($L-R$)			Standard Deviations		
	m_2p_1	c_yl	rb'	m_2p_1	c_yl	rb'
Qau ♂	+0·34±·12 (45)	−0·47±·11 (32)	+0·13±·10 (64)	1·241±·088	0·928±·078	1·231±·073
Kerma ♂	+0·25±·14 (39)	−0·02±·10 (37)	+0·45±·09 (53)	1·269±·097	0·918±·072	0·956±·063
Qau ♀	+0·31±·15 (32)	−0·22±·11 (33)	+0·53±·12 (50)	1·227±·103	0·972±·081	1·232±·083
Kerma ♀	+0·16±·14 (34)	−0·14±·10 (33)	+0·57±·13 (42)	1·192±·097	0·844±·070	1·241±·091

(3) *Description of the Material.* Individual and mean measurements of racial series of adult mandibles, taken in accordance with the technique described in the first paper, are given in the following sources. The sexes of nearly all the specimens were either known definitely (series *e*, *f* and *g*), or they had been given by anatomical appreciation of the sexes of the associated crania.

(*a*) G. M. Morant: "A First Study of the Tibetan Skull," *Biometrika*, Vol. XIV. (1923), pp. 193—260. The Tibetan *B* type, represented by 12 male mandibles, is found in the eastern province of Khams. The Tibetan *A* series, from the southern provinces, was augmented by material dealt with in the following paper.

(*b*) G. M. Morant: "A Study of certain Oriental Series of Crania including the Nepalese and Tibetan series in the British Museum (Natural History)." *Ibid.*, Vol. XVI. (1924), pp. 1—105. Data are given for 19 male Nepalese and 25 male Tibetan *A* mandibles.

* Le Double (*Traité des Variations des Os de la Face*, 1906, p. 308) concludes from a survey of the literature: "Enfin, dans l'un et l'autre sexe et dans toutes les races anciennes et modernes, le maxillaire inférieur est, comme les autres os du squelette, plus développé à droite qu'à gauche." This very sweeping statement does not seem to be supported by any adequate statistics whatever.

(*c*) G. M. Morant: "A First Study of the Craniology of England and Scotland from Neolithic to Early Historic Times, with special reference to the Anglo-Saxon Skulls in London Museums." *Ibid.*, Vol. XVIII. (1926), pp. 56—98. Data are given for 66 male and 62 female mandibles from various Anglo-Saxon cemeteries*. Many of the specimens are defective.

(*d*) Brenda N. Stoessiger: "A Study of the Badarian Crania recently excavated by the British School of Archaeology in Egypt." *Ibid.*, Vol. XIX. (1927), pp. 110—150. Data are given for 39 male and 23 female mandibles belonging to the early predynastic series.

(*e*) Gordon Harrower: "A Biometric Study of One Hundred and Ten Asiatic Mandibles." *Ibid.*, Vol. XX[B]. (1928), pp. 279—293. Data are given for 33 mandibles of male Tamils who died in Singapore.

(*f*) *Ibid.* Data are given for 38 mandibles of male Fukien (Hokien) Chinese who died in Singapore.

(*g*) *Ibid.* Data are given for 39 mandibles of male Chinese from the island of Hylam who died in Singapore. It is possible that the crania of these individuals were artificially deformed, but this does not appear to have affected their facial skeletons as the mean facial measurements are very similar to those of other Chinese series: see *Ibid.*, p. 278.

(*h*) T. L. Woo: "A Study of Seventy-One Ninth Dynasty Egyptian Skulls from Sedment." *Ibid.*, Vol. XXII. (1930), pp. 65—93. Data are given for 36 male and 25 female mandibles.

(*i*) Doris Dingwall and Matthew Young: "The Skulls from Excavations at Dunstable, Bedfordshire." *Ibid.*, Vol. XXV. (1933), pp. 147—157. Data are given for 42 male mandibles and the means are quoted in Table IV below.

Data relating to two additional Egyptian series are given in the present paper for the first time. Various constants for these series are in tables in the text. The series are:

(*j*) The Kerma for which the crania have been described by Miss Collett ("A Study of Twelfth and Thirteenth Dynasty Skulls from Kerma (Nubia)." *Ibid.*, Vol. XXV. (1933), pp. 254—284). The crania were sexed anatomically. There are 67 mandibles associated with male crania for which measurements have been published (having serial numbers between 1 and 147 inclusive), and 11 mandibles associated with crania which were not measured owing to their fragmentary state or to posthumous distortion (having serial numbers between 279 and 306 inclusive). Of the total 78 male mandibles 10 were excluded from the series because one or

* The measurements and constants for two additional series of Anglo-Saxon mandibles have been published since the present paper was written. These are in: "The Anglo-Saxon Skulls from Bidford-on-Avon, Warwickshire, and Burwell, Cambridgeshire, with a Comparison of their principal Characters and those of the Anglo-Saxon Skulls in London Museums," by J. C. Brash, Doris Layard and Matthew Young, *Biometrika*, Vol. XXVII. (1935), pp. 373—407.

both sides had lost two or more molars before death*. The measured series of male mandibles thus consists of 68 specimens, and all these except five are practically complete apart from the loss of teeth. There are 68 mandibles associated with female crania for which measurements have been published (having serial numbers between 149 and 275 inclusive); eight associated with crania which were not measured owing to their fragmentary state or to posthumous distortion (having serial numbers 184, 273 and between 282 and 299 inclusive), and one (No. 254) associated with a female cranium which was not measured because it is scaphocephalic. Of the total 77 female mandibles 20 were excluded from the series because one or both sides had lost two or more molars before death. The measured series of female mandibles thus consists of 57 specimens, and all these with the exception of six are practically complete apart from the loss of teeth.

(*k*) The last series consists of 141 mandibles from cemeteries at Qau, Upper Egypt, excavated in 1924 by Mr Guy Brunton and Professor Sir Flinders Petrie for the British School of Archaeology in Egypt. Of these, 137 are of 4th—11th Dynasty date, one belongs to the 12th, two to the 17th and one to the 18th Dynasty. Sexes are given for the majority of the specimens in the report†. These were determined primarily from the skeletons, but confirmation from the grave furniture was often possible. It is said (p. 5):

"The sexing of skeletons is not always an easy matter. The presence of the pelvis is almost essential; and some previous experience or tuition is necessary, otherwise the record may be unreliable in this respect. We were fortunate enough to be visited in camp, soon after the work at Qau started, by Professor Douglas Derry. He very kindly gave detailed instructions on sexing to my assistants on the spot; and I think that the determinations of sex given by them in the Tomb Register are therefore accurate in the main."

The possibility of obtaining a high percentage of correct sexes when the skull only is available must at least be admitted. Of the total 141 mandibles, 131 are associated with crania, but with no other bones of the skeleton in the laboratory. The skulls were sexed by anatomical appreciation before the report of the excavations was seen. Among the 131 for which mandibles have been preserved there are 91 having sexes given in the archaeological report. For 77 of these (84·6 per cent.) the sexes found in the field and in the laboratory agree, and there is disagreement in the other 14 cases. The field sexing was accepted for 12 of these 14 skulls and the laboratory sexing for the other two. The laboratory sexing of the 40 skulls which were not sexed in the field also had to be accepted. Finally, for the 10 mandibles not associated with crania field sexing is only given in one case and sexes were assigned to the other nine by anatomical appreciation based on the mandibles alone, a process which is probably subject to larger errors than the other two. In all, there are 74 bones supposed male and 67 supposed female. The loss

* In selecting these it was assumed that the absence of a socket for the third molar denoted that the tooth had been lost before death, but in a few of these cases the third molar may never have erupted.

† *Qau and Badari I.* By Guy Brunton, with chapters by Alan Gardiner and Flinders Petrie, 1927.

of teeth during life is followed by absorption of the alveolar margin and usually by some distortion of the remaining teeth and their sockets. It is generally assumed that the ante mortem loss of several teeth also results in distortion of other parts of the mandible. Molars are usually the first teeth to be lost and it is of interest to compare the mean measurements of specimens having lost two or more molars from one side or the other, or both sides, with those of the remainder of a series. In computing the constants in Table III it was assumed that the absence of a socket for the third molar denoted that the tooth had erupted and been lost before death, though in some of these cases the third molar may never have erupted. We should anticipate that, apart from its effect on the alveolar region, the loss of molars would affect most markedly measurements of the mandibular angle ($M\angle$), of the length and breadth of the ramus (rb' and rl), and of the bigonial breadth ($g_o g_o$) and length of the corpus ($c_p l$). The means are in Table III and it is clear from these that the characters in question are only affected to a slight extent, if

TABLE III.

Comparison of Mean Measurements for Mandibles with Molars lost before Death and Mandibles with Molars retained.

		$M\angle$	rb'	rl	$g_o g_o$	$c_p l$
Qau ♂	Molars lost	123°·9 (13)	33·7 (13)	60·7 (13)	93·9 (13)	75·8 (13)
	Others	124°·1 ± ·55 (59)	31·9 ± ·20 (60)	61·5 ± ·44 (57)	89·7 ± ·53 (57)	74·7 ± ·38 (59)
Kerma ♂	Molars lost	123°·8 (6)	32·9 (10)	58·8 (6)	84·9 (5)	75·6 (6)
	Others	120°·9 ± ·53 (55)	33·9 ± ·24 (67)	61·7 ± ·55 (56)	88·9 ± ·67 (53)	77·7 ± ·41 (56)
Qau ♀	Molars lost	127°·0 (12)	31·3 (13)	55·4 (12)	82·3 (10)	72·0 (11)
	Others	124°·4 ± ·49 (52)	31·7 ± ·21 (54)	55·4 ± ·36 (52)	83·8 ± ·62 (37)	71·0 ± ·41 (45)
Kerma ♀	Molars lost	125°·9 (13)	32·1 (18)	54·9 (13)	80·4 (12)	72·9 (13)
	Others	127°·4 ± ·72 (49)	32·1 ± ·26 (54)	54·2 ± ·47 (47)	81·1 ± ·57 (45)	72·3 ± ·45 (49)

at all, by the loss of molars. In particular, we fail entirely to confirm the common belief that there is a marked increase in the mandibular angle following such a loss. The data suggest that there is no need to exclude ageing mandibles from a racial series. In the case of the Kerma those bones which had lost before death two or more molars from one side or the other, or both sides, were actually excluded, but none was rejected from the Qau series for the same reason. There are no edentulous, and very few nearly edentulous, specimens in the Qau series.

(4) *Sexual Differences of Mean Values.* Table IV gives the male and female means for the Qau and Kerma Egyptian series, which are the two longest available, and the only other male and female series of sufficient length for the present

TABLE IV.

Mean Mandibular Measurements and their Probable Errors.

Character	Male			Female	
	4th—11th Dynasty Egyptians: Qau	12th and 13th Dynasty Nubians: Kerma	English: Dunstable	4th—11th Dynasty Egyptians: Qau	12th and 13th Dynasty Nubians: Kerma
w_1	113·8 ± ·52 (56)	113·8 ± ·50 (51)	121·0 ± ·69 (29)	108·0 ± ·53 (47)	105·6 ± ·63 (37)
$c_r c_r$	91·7 ± ·46 (59)	92·3 ± ·52 (46)	99·3 ± ·77 (27)	85·9 ± ·46 (49)	86·1 ± ·51 (38)
$g_o g_o$	90·5 ± ·47 (70)	88·9 ± ·67 (53)	98·4 ± ·74 (40)	83·5 ± ·55 (47)	81·1 ± ·57 (45)
G_2'	44·6 ± ·29 (57)	43·8 ± ·27 (59)	47·6 ± ·24 (41)	42·3 ± ·22 (49)	41·6 ± ·30 (43)
zz	43·9 ± ·22 (70)	44·3 ± ·23 (65)	45·4 ± ·25 (42)	43·0 ± ·22 (66)	42·6 ± ·20 (50)
$c_y l$	20·3 ± ·16 (67)	20·8 ± ·14 (57)	21·3 ± ·18 (35)	18·8 ± ·12 (56)	18·6 ± ·17 (48)
rb'	32·2 ± ·19 (73)	33·9 ± ·24 (67)	32·8 ± ·30 (42)	31·6 ± ·19 (67)	32·1 ± ·26 (54)
$c_y c_r$	34·4 ± ·23 (71)	35·2 ± ·26 (55)	35·7 ± ·43 (37)	32·2 ± ·23 (59)	34·1 ± ·21 (53)
$g_n g_o$, L	82·4 ± ·35 (70)	84·9 ± ·38 (56)	85·0 ± ·41 (40)	77·9 ± ·39 (54)	78·8 ± ·31 (46)
$g_n g_o$, R	83·3 ± ·34 (72)	85·8 ± ·39 (57)	84·9 ± ·45 (40)	78·6 ± ·36 (53)	78·8 ± ·32 (47)
$m_2 p_1$	28·0 ± ·15 (60)	28·4 ± ·16 (56)	28·0 ± ·14 (39)	27·8 ± ·11 (50)	27·5 ± ·17 (45)
$h_1 (g_n d_t)$	33·8 ± ·25 (67)	33·6 ± ·28 (46)	32·5 ± ·27 (39)	31·0 ± ·26 (49)	32·1 ± ·31 (35)
$p_a d_t$	27·5 ± ·25 (62)	27·4 ± ·33 (44)	28·3 ± ·28 (39)	25·5 ± ·25 (49)	26·4 ± ·29 (35)
$p_a g_n$	9·5 ± ·11 (74)	9·1 ± ·15 (62)	7·3 ± ·18 (42)	8·7 ± ·10 (66)	9·0 ± ·14 (52)
$p_a d_f$	24·1 ± ·26 (61)	25·0 ± ·33 (40)	25·0 ± ·34 (35)	22·8 ± ·30 (44)	23·0 ± ·32 (34)
ml	105·2 ± ·40 (69)	106·0 ± ·46 (55)	106·8 ± ·54 (34)	100·7 ± ·45 (54)	101·5 ± ·47 (42)
$c_p l$	74·8 ± ·35 (72)	77·7 ± ·41 (56)	76·2 ± ·54 (38)	71·2 ± ·37 (56)	72·3 ± ·45 (49)
rl	61·4 ± ·40 (70)	61·7 ± ·55 (56)	64·7 ± ·47 (38)	55·4 ± ·32 (64)	54·2 ± ·47 (47)
$c_y h$	53·7 ± ·52 (72)	56·3 ± ·59 (61)	58·6 ± ·62 (42)	49·2 ± ·42 (64)	47·1 ± ·63 (47)
$c_r h$	65·1 ± ·47 (72)	66·7 ± ·51 (60)	65·0 ± ·59 (38)	58·7 ± ·36 (63)	56·8 ± ·53 (47)
ih	45·3 ± ·40 (72)	47·8 ± ·46 (63)	48·4 ± ·56 (37)	41·6 ± ·31 (66)	39·4 ± ·44 (48)
ih'	13·1 ± ·13 (66)	12·9 ± ·11 (55)	13·4 ± ·19 (34)	11·8 ± ·12 (59)	11·8 ± ·19 (45)
$m_2 h$	25·7 ± ·21 (66)	26·5 ± ·20 (60)	26·6 ± ·24 (41)	24·3 ± ·25 (48)	24·1 ± ·24 (44)
$p_1 h$	31·4 ± ·20 (63)	31·3 ± ·27 (40)	30·7 ± ·25 (40)	29·6 ± ·25 (46)	29·2 ± ·29 (29)
$d_t h$	36·9 ± ·27 (59)	36·4 ± ·24 (47)	35·5 ± ·25 (39)	34·6 ± ·30 (47)	35·3 ± ·37 (30)
$g_o p_a g_o$	188·2 ± ·76 (69)	195·7 ± ·89 (56)	193·4 ± ·97 (40)	181·2 ± ·84 (48)	182·8 ± ·80 (44)
$M \angle$	124°·1 ± ·50 (72)	120°·9 ± ·53 (55)	120°·9 ± ·64 (39)	124°·8 ± ·44 (64)	127°·4 ± ·72 (49)
$R \angle$	75°·7 ± ·69 (64)	76°·6 ± ·65 (52)	68°·7 ± ·66 (37)	70°·9 ± ·54 (57)	70°·3 ± ·67 (46)
$C' \angle$	71°·5 ± ·47 (53)	72°·2 ± ·60 (42)	69°·0 ± ·63 (36)	71°·4 ± ·57 (43)	72°·7 ± ·68 (37)
$G \angle$	66°·2 ± ·35 (69)	62°·8 ± ·41 (53)	70°·9 ± ·56 (40)	64°·9 ± ·45 (46)	61°·8 ± ·53 (45)
$100\, c_r h/ml$	62·1 ± ·50 (68)	63·3 ± ·59 (53)	59·3 ± ·68 (31)	58·1 ± ·42 (53)	55·8 ± ·61 (41)
$100\, c_r c_r/ml$	87·1 ± ·56 (57)	87·3 ± ·65 (41)	93·5 ± ·75 (25)	85·1 ± ·49 (44)	84·3 ± ·71 (34)
$100\, g_o g_o/c_p l$	121·7 ± ·84 (68)	114·7 ± ·94 (52)	129·7 ± 1·43 (38)	117·5 ± 1·07 (47)	112·5 ± 1·23 (45)
$100\, rb'/rl$	52·9 ± ·41 (69)	55·1 ± ·55 (56)	*51·0 ± ·56 (38)	57·5 ± ·44 (64)	59·9 ± ·71 (47)
$100\, g_o g_o/c_r c_r$	98·4 ± ·50 (58)	95·8 ± ·92 (41)	100·0 ± 1·25 (27)	97·5 ± ·67 (39)	94·1 ± ·85 (36)
$100\, c_y h/c_r h$	82·7 ± ·55 (71)	84·5 ± ·60 (57)	90·0 ± ·68 (38)	84·4 ± ·65 (61)	82·8 ± ·61 (46)
$100\, ih'/c_y c_r$	38·0 ± ·39 (66)	36·7 ± ·38 (51)	37·6 ± ·55 (34)	36·7 ± ·43 (58)	34·9 ± ·60 (45)
$100\, d_t h/c_r h$	57·4 ± ·59 (59)	54·9 ± ·61 (46)	55·3 ± ·56 (36)	59·4 ± ·48 (46)	63·4 ± 1·17 (29)

* Given incorrectly as 49·55 (38) by Dingwall and Young.

purpose are those of Anglo-Saxons. Sexual differences of the absolute measurements are best examined by comparing their sex ratios, i.e. the male means divided by the corresponding female means. For all the mandibular characters in the case of the three series considered, as for almost all measurements of the human body in the case of any racial series, the ratios are greater than unity, but they do not tend to be constant for different characters. A few selected values are shown

in Table V and it is clear from these that the ratio is peculiar to the particular character, the orders for the three series obviously being highly correlated. The heights show a marked tendency to have higher ratios than either the breadths or antero-posterior lengths and the length of the condyle ($c_y l$) also shows high values. The sexual differences of the eight indices and four angles can be compared more directly by considering whether they are statistically significant. The following are the ratios (Δ/p.e. Δ) for the Kerma series which exceed 3·0:—100 $c_r c_r/ml = 3{\cdot}1$, 100 $rb'/rl = 5{\cdot}3$, 100 $d_t h/c_r h = 6{\cdot}4$, $R\angle = 6{\cdot}7$, $M\angle = 7{\cdot}3$ and 100 $c_r h/ml = 8{\cdot}8$; and for the Qau series—100 $g_o g_o/c_p l = 3{\cdot}1$, $R\angle = 5{\cdot}5$, 100 $c_r h/ml = 6{\cdot}1$ and 100 $rb'/rl = 7{\cdot}6$. The most significant differences are thus the same for the two series except that the Qau ratios do not indicate significance in the case of

TABLE V.

Sex Ratios for Mandibular Characters.*

Character	Qau Egyptians	Kerma Egyptians	Anglo-Saxons
$m_2 p_1$	1·007	1·033	1·018
zz	1·021	1·040	1·027
$c_p l$	1·051	1·075	1·042
$g_o g_o$	1·084	1·096	1·081
$c_y l$	1·080	1·118	1·136
$c_r h$	1·109	1·174	1·110

* All means used in computing the ratios shown are based on 30 or more mandibles.

100 $d_t h/c_r h$ and $M\angle$. The last has been given for a considerable number of sexed series, though different workers have measured it in different ways, and it has been found almost invariably that the mean female mandibular angle is appreciably greater than the corresponding male mean. The Qau series appears to be the most exceptional in this respect, since the female angle for it is only slightly greater than the male angle. It may be noticed that all the indices which show well-marked sexual differences involve a rameal height (rl or $c_r h$), and differences in $M\angle$ and $R\angle$ are obviously associated with differences in these characters. The most outstanding differences between normal male and female mandibles belonging to the same race relate to the ramus.

It is of interest to compare cranial and mandibular sexual differences in the case of the same series, and the Kerma Egyptian can be used for this purpose. The mean measurements for the crania have been given by Miss Collett* and they lead to the distribution of 41 sex ratios given in our Table VI, comparison being made there with 26 ratios derived from the mandibular means in Table IV above. There is a striking difference between the forms of the two distributions, the proportions of high ratios—over 1·1, say—being much greater for the mandibular

* *Loc. cit.*, Table II.

TABLE VI.

The Cranial and Mandibular Distributions of Sex Ratios for the Kerma Series.*

	1·000—1·040	1·040—1·080	1·080—1·120	1·120—1·160	1·160—1·200	1·200—1·240	1·240—1·280	1·280—1·320	Totals
Cranial ...	14·5	24·5	0	1†	0	0	0	1‡	41
Mandibular	5·5	10·5	6	1	2	1	0	0	26

* All means used in computing the ratios are based on 30 or more specimens and all absolute measurements available are included.

† For the capacity.

‡ For the simotic subtense.

than for the cranial measurements. The same distinction between the two parts of the skeleton is brought out by a comparison of indices and angles. While most of these measurements of shape fail to distinguish the means of a male and female series representing the same racial type in the case of the cranium, there are a few which normally show significant sexual differences for series of approximately the same length as the Kerma. But there is a larger proportion of the mandibular indices and angles which effects this purpose*. It will be safe to conclude that sexual differences are more marked for the mandible than for the cranium.

The last conclusion encourages the hope that it may be possible to devise an effective method of "mathematical sexing" for the mandible, i.e. of assigning sexes to a sample of male and female bones by considering their measurements only. Our present purpose is restricted to a consideration of the choice of characters which could be used most effectively in sexing "mathematically." It will be safe to assume that for a series representing a single racial type the male and female distributions of a particular character will be approximately normal distributions with approximately equal standard deviations. The characters most suitable for the purpose in view will be:

(*a*) those for which the difference between the means bears the largest ratio to the standard deviations of the two distributions,

(*b*) those which will form a group such that all the intra-racial correlations between them are low, and

(*c*) those which can be taken on the largest numbers of imperfect specimens, so that as few mandibles as possible would have to be rejected owing to the fact that one or other measurement required in sexing could not be found for them.

These three conditions may now be examined with reference to the Kerma series. The intra-racial correlations are dealt with in a later section of this paper

* Owing to the difficulty of sexing, it has been a common practice in recent years to make racial comparisons between the mean measurements of shape—indices and angles—of unsexed series. This is an inexact and unjustifiable procedure, at least in the case of the mandible, since several of the measurements in question show clear sexual differences.

and it may be assumed, for the moment, that a lowly correlated group will be selected if we choose from the absolute measurements only one transverse breadth, only one antero-posterior length and only one height. Table VII includes all the absolute measurements for which the difference between the means exceeds the standard deviations of both male and female distributions. The first three characters are transverse breadths and from these $g_o g_o$ may be chosen, since it can generally be taken on a larger number of imperfect specimens than either w_1 or

TABLE VII.

Sexual Differences of Means and Standard Deviations for the Kerma Series.

Character	Difference of Means	Male σ	Female σ
w_1	8·2	5·34	5·70
$g_o g_o$	7·8	7·18	5·63
$c_r c_r$	6·2	5·18	4·65
$c_y l$	2·2	1·54	1·72
$c_p l$	5·4	4·51	4·34
$c_r h$	9·9	5·90	5·37
rl	7·5	6·04	4·76
$M\angle$	6°·5	5°·79	7°·48
$R\angle$	6°·3	6°·95	6°·72
$100\, c_r h/ml$	7·5	6·33	5·77

$c_r c_r$. The lengths of the condyle ($c_y l$) and corpus ($c_p l$) may be accepted, the latter being the only antero-posterior length in the table. There are two heights and $c_r h$ seems to be a more useful character for our purpose than rl. It will also be an advantage to include some measurements of shape, and among these the most significant sexual differences were found in the case of the Kerma series for $M\angle$, $R\angle$ and $100\, c_r h/ml$. The last cannot be accepted as it is almost certain to be highly correlated with $c_r h$. Judging from the constants in Table VII, the two angles do not separate the male and female distributions as effectively as do several of the absolute measurements, but they may still be included with advantage. The characters which can be best used, as a group, for purposes of sexing thus appear to be $g_o g_o$, $c_y l$, $c_p l$, $c_r h$, $M\angle$ and $R\angle$ and there might be little gain in adding others. The process by which they have been selected is admittedly a crude one, and it depends on the experience gained from a single series, but while the material is so scanty refined statistical methods are not likely to lead to a more profitable result.

(5) *Sexual and Racial Differences in Variability.* Table VIII gives the standard deviations for all characters and the coefficients of variation for the absolute measurements in the case of the Qau and Kerma series. Comparison between the sexes may be made first. Using the coefficients of variation for the absolute measurements and the standard deviations for the indices and angles, it is found

TABLE VIII.

Variabilities of Mandibular Measurements for the Qau and Kerma series.*

Character	Standard Deviations				Coefficients of Variation			
	Male		Female		Male		Female	
	Qau	Kerma	Qau	Kerma	Qau	Kerma	Qau	Kerma
w_1	5·73 ± ·37	5·34 ± ·36	5·39 ± ·37	5·70 ± ·45	5·03 ± ·32	4·69 ± ·31	4·99 ± ·35	5·40 ± ·42
$c_r c_r$	5·20 ± ·32	5·18 ± ·36	4·74 ± ·32	4·65 ± ·36	5·67 ± ·35	5·61 ± ·40	5·52 ± ·38	5·40 ± ·42
$g_o g_o$	5·89 ± ·34	7·18 ± ·47	5·58 ± ·39	5·63 ± ·40	6·51 ± ·37	8·08 ± ·53	6·68 ± ·47	6·94 ± ·50
G_2'	3·20 ± ·20	3·11 ± ·19	2·28 ± ·16	2·89 ± ·21	7·17 ± ·46	7·10 ± ·44	5·39 ± ·37	6·95 ± ·51
zz	2·67 ± ·15	2·70 ± ·16	2·61 ± ·15	2·08 ± ·14	6·08 ± ·35	6·09 ± ·36	6·07 ± ·36	4·88 ± ·34
$c_y l$	1·90 ± ·11	1·54 ± ·10	1·34 ± ·09	1·72 ± ·12	9·36 ± ·55	7·40 ± ·47	7·13 ± ·46	9·25 ± ·64
rb'	2·36 ± ·13	2·96 ± ·17	2.26 ± ·13	2·78 ± ·18	7·33 ± ·41	8·73 ± ·51	7·15 ± ·42	8·66 ± ·57
$c_y c_r$	2·55 ± ·14	2·88 ± ·19	2·57 ± ·16	2·26 ± ·15	7·41 ± ·42	8·18 ± ·53	7·98 ± ·50	6·63 ± ·44
$g_n g_o$, L	4·32 ± ·25	4·17 ± ·27	4·25 ± ·28	3·10 ± ·22	5·24 ± ·30	4·91 ± ·31	5·46 ± ·36	3·93 ± ·28
$g_n g_o$, R	4·23 ± ·24	4·40 ± ·28	3·90 ± ·26	3·25 ± ·23	5·08 ± ·29	5·13 ± ·32	4·96 ± ·33	4·12 ± ·29
$m_2 p_1$	1·73 ± ·11	1·77 ± ·11	1·19 ± ·08	1·66 ± ·12	6·18 ± ·38	6·23 ± ·40	4·28 ± ·29	6·04 ± ·43
$h_1 (g_n d_t)$	3·02 ± ·18	2·78 ± ·20	2·72 ± ·19	2·68 ± ·22	8·93 ± ·52	8·27 ± ·59	8·77 ± ·60	8·35 ± ·68
$p_a d_t$	2·95 ± ·18	3·25 ± ·23	2·62 ± ·18	2·55 ± ·21	10·73 ± ·66	11·86 ± ·86	10·27 ± ·71	9·66 ± ·79
$p_a g_n$	1·34 ± ·07	1·80 ± ·11	1·24 ± ·07	1·48 ± ·10	14·11 ± ·80	20·00 ± 1·26	14·25 ± ·85	16·44 ± 1·12
$p_a d_f$	2·96 ± ·18	3·06 ± ·23	2·92 ± ·21	2·76 ± ·23	12·28 ± ·76	12·29 ± ·94	12·81 ± ·94	12·00 ± 1·00
ml	4·91 ± ·28	5·07 ± ·33	4·91 ± ·32	4·52 ± ·33	4·67 ± ·27	4·78 ± ·29	4·88 ± ·32	4·45 ± ·33
$c_p l$	4·35 ± ·24	4·51 ± ·29	4·10 ± ·26	4·34 ± ·30	5·82 ± ·33	5·80 ± ·37	5·76 ± ·37	6·00 ± ·41
rl	4·90 ± ·28	6·04 ± ·38	3·80 ± ·23	4·76 ± ·33	7·98 ± ·46	9·79 ± ·63	6·86 ± ·41	8·78 ± ·62
$c_y h$	6·50 ± ·37	6·84 ± ·42	4·97 ± ·30	6·45 ± ·45	12·10 ± ·69	12·15 ± ·75	10·10 ± ·61	13·69 ± ·97
$c_r h$	5·94 ± ·33	5·90 ± ·36	4·27 ± ·26	5·37 ± ·37	9·12 ± ·52	8·85 ± ·55	7·27 ± ·44	9·45 ± ·66
ih	4·97 ± ·28	5·43 ± ·33	3·72 ± ·22	4·57 ± ·31	10·97 ± ·62	11·38 ± ·69	8·94 ± ·53	11·60 ± ·81
ih'	1·54 ± ·09	1·26 ± ·08	1·40 ± ·09	1·78 ± ·13	11·76 ± ·70	9·77 ± ·63	11·86 ± ·75	15·08 ± 1·10
$m_2 h$	2·56 ± ·15	2·33 ± ·14	2·54 ± ·17	2·36 ± ·17	9·96 ± ·60	8·79 ± ·55	10·45 ± ·73	9·79 ± ·71
$p_1 h$	2·31 ± ·14	2·53 ± ·19	2·47 ± ·17	2·32 ± ·21	7·36 ± ·44	8·08 ± ·61	8·34 ± ·59	7·95 ± ·71
$d_t h$	3·04 ± ·19	2·45 ± ·17	3·04 ± ·21	2·99 ± ·26	8·24 ± ·52	6·73 ± ·47	8·79 ± ·62	8·47 ± ·74
$g_o p_a g_o$	9·36 ± ·54	9·84 ± ·63	8·62 ± ·59	7·82 ± ·56	4·98 ± ·29	5·03 ± ·32	4·76 ± ·33	4·28 ± ·31
$M\angle$	6°·32 ± ·36	5°·79 ± ·37	5°·19 ± ·31	7°·48 ± ·51	—	—	—	—
$R\angle$	8°·15 ± ·49	6°·95 ± ·46	6°·01 ± ·38	6°·72 ± ·47	—	—	—	—
$C'\angle$	5°·10 ± ·33	5°·77 ± ·42	5°·52 ± ·40	6°·11 ± ·48	—	—	—	—
$G\angle$	4°·27 ± ·25	4°·44 ± ·29	4°·52 ± ·32	5°·24 ± ·37	—	—	—	—
$100\, c_r h/ml$	6·14 ± ·36	6·33 ± ·41	4·52 ± ·30	5·77 ± ·43	—	—	—	—
$100\, c_r c_r/ml$	6·22 ± ·39	6·13 ± ·46	4·80 ± ·35	6·18 ± ·51	—	—	—	—
$100\, g_o g_o/c_p l$	10·23 ± ·59	10·08 ± ·67	10·87 ± ·76	12·26 ± ·87	—	—	—	—
$100\, rb'/rl$	5·04 ± ·29	6·10 ± ·39	5·24 ± ·31	7·20 ± ·50	—	—	—	—
$100\, g_o g_o/c_r c_r$	5·66 ± ·35	8·77 ± ·65	6·20 ± ·47	7·54 ± ·60	—	—	—	—
$100\, c_y h/c_r h$	6·90 ± ·39	6·67 ± ·42	7·52 ± ·46	6·15 ± ·43	—	—	—	—
$100\, ih'/c_y c_r$	4·68 ± ·27	4·02 ± ·27	4·91 ± ·30	6·00 ± ·43	—	—	—	—
$100\, d_t h/c_r h$	6·73 ± ·42	6·17 ± ·43	4·80 ± ·34	9·32 ± ·83	—	—	—	—

* The numbers of individuals on which the constants in this Table are based are given in Table IV.

for the Qau series that 21 of the male constants exceed the corresponding female values, while for the remaining 17 the position is reversed: differences which appear to be definitely significant (Δ/p.e. $\Delta > 3{\cdot}5$, say) are only observed for $R\angle$ (Δ/p.e. $\Delta = 3{\cdot}5$), $100\, c_r h/ml$ (3·5), $100\, d_t h/c_r h$ (3·6) and $m_2 p_1$ (4·0), and for

each of these characters the male constant is the greater. For the Kerma series the male variabilities are the greater in 20 cases and the female in the remaining 18, while the only significant differences are for $100\,ih'/c_y c_r$ (3·9) and ih' (4·2), the female constants being the greater for both these characters. It cannot be supposed that there is any evidence to show that one sex is more variable than the other when all the measurements are taken into account. This conclusion for the mandible is thus the same as that for the cranium and most other parts of the body, and sexual differentiation in variability, if it exists, could only be revealed by far larger samples than those available at present.

Racial differences in variability may be considered from the data in Table VIII, still comparing coefficients of variation of the absolute measurements and standard deviations of the indices and angles. For the male series, the Kerma constant is the greater in 20 cases and the Qau in the remaining 18: only two differences exceed 3·5 times their probable errors, viz. those for $p_s g_n$ (Δ/p. e. $\Delta = 3{\cdot}9$) and $100\,g_o g_o/c_r c_r$ (4·2), the Kerma values being the greater for both. For the female series, the Kerma constant is the greater in 24 cases and the Qau in the remaining 14: only two differences exceed 3·5 times their probable errors, viz. those for $M\angle$ (3·8) and $100\,d_t h/c_r h$ (5·0), the Kerma values again being the greater for both. The Kerma male series thus appears to be slightly more variable than the Qau male, and the female Kerma series appears to be appreciably more variable than the Qau female. These conclusions may be compared with the more exact ones which follow from the application of the P_{λ_n} test. Standard deviations of all the characters were used for this purpose. The more stringent value of Q_{λ_n} resulting from the comparison of the 38 in the case of the two male series is ·1142*, and if the character ($100\,g_o g_o/c_r c_r$) showing the most significant difference is omitted the value for the remaining 37 is raised to ·3265. We must conclude that there is no evidence of the male Kerma being more variable than the male Qau series. The lower value of Q_{λ_n} based on the total 38 characters for the two female series is ·0000, and if the four characters which show most significant differences are omitted—viz. $100\,d_t h/c_r h$, $M\angle$, $100\,rb'/rl$ and $m_3 p_1$, the Kerma constant being greater than the Qau in all these cases—the value for the remaining 34 is ·0061. For the character showing the next most significant difference ($g_n g_o$, L) the Qau constant is greater than the Kerma, and if it is also omitted the probability for the remaining 33 is reduced to ·0038. It is clear that we are dealing with a real difference in variability, and if all the characters are considered together the female Kerma must be considered more variable than the female Qau series.

The only other series of mandibles for which variabilities have been given is that of the male bones from Dunstable. Coefficients of variation are not provided for this, but by comparing standard deviations for all the characters it is suggested

* The values of Q_{λ_n} given were obtained by using probability integrals of the normal curve. The normal curve is only an approximation, however, to the distribution of the differences of standard deviations for two populations which each follow the normal curve, in the case of samples, such as those compared, giving σ's based on approximately 60 individuals.

that the Dunstable series is rather less variable than the Qau and decidedly less variable than the Kerma. The differences in variability shown by the three series are probably due, in part at least, to conditions which are of no racial significance.

It has often been stated that the mandible is far more variable than the cranium, though no adequate statistical evidence seems to have been advanced in support of this view. An intra-racial sample by aid of which the question may be examined is now available in the Kerma series. The method by which the comparison of cranial and mandibular variabilities should be made, however, is by no means clear. The coefficient of va.iation does not give a measure of variability which is independent of the absolute size of the character measured. The data in Table IX relate to all cranial and mandibular absolute measurements available

TABLE IX.

Comparison of Coefficients of Variation for Cranial and Mandibular Measurements of the same Male (Kerma) Series.

Male Mean Measurements		Coefficients of Variation						
		2—5	5—8	8—11	11—14	14—17	17—20	20—23
Less than 40 mm.	Cranial ...	1	4	2	—	—	1	—
	Mandibular	—	3	6	2	—	0·5	0·5
40—80	Cranial ...	2	6	—	—	—	—	—
	Mandibular	—	3	2	2	—	—	—
80—120	Cranial ...	8	2	—	—	—	—	—
	Mandibular	3	2	1	—	—	—	—
Greater than 120	Cranial ...	6	—	—	—	—	—	—
	Mandibular	—	—	—	—	—	—	—

for the male Kerma series except the arcs and two cranial subtenses. The characters represented are thus all chords, whether found with calipers or by projection. It will be seen that in the case of each group of measurements there is a clear negative correlation between the coefficient of variation and the absolute size of the character. There is a clear suggestion, too, that for measurements of approximately the same size the mandibular coefficients of variation tend to be greater than the cranial. Judging from this comparison, it may be concluded that mandibular characters tend, on the average, to be distinctly more variable than cranial characters intra-racially when allowance is made for differences in absolute size. The inter-racial situation may, however, be quite different from this.

(6) *Intra-Racial Correlations of Mandibular Measurements.* No correlations of mandibular measurements appear to have been published in any earlier study, and the best material now available is still insufficient to give more than first approximations to these constants. The longest series which can be used for the purpose is that of the 74 male mandibles from Qau, and this series gives the coefficients in Tables X—XII. We may hope to gain from these constants some knowledge of the factors which govern the growth of the immature, and hence the architecture of the adult mandible, and they are also needed for some special purposes as in aiding the selection of characters which can be used most effectively in racial comparisons. It is known, however, that the correlations of anthropometric characters often differ significantly—if not widely—from series to series, and hence no very great importance should be attached to the values derived from a single series.

There are 90 correlations between selected pairs of 16 absolute measurements in Tables X and XI. Before attempting to reduce these to order it may be noted that the measurements can be grouped conveniently as transverse breadths, heights and antero-posterior lengths. The length of the condyle ($c_y l$) may be considered a transverse breadth, though it is a different kind of measurement from the other five (in Table X), all of which are taken between bilateral points. The projective length of the ramus (rl) may be classed with the heights, though it differs from

TABLE X.

Intra-Racial Correlations of Mandibular Breadths for the Male Qau Series.

	w_1	$g_o g_o$	$c_r c_r$	G_2'	zz	$c_y l$
w_1	—	+·631±·055 55	+·488±·073 49	+·464±·080 44	+·298±·084 53	+·423±·075 54
$g_o g_o$	+·631±·055 55	—	+·556±·061 58	+·598±·058 55	+·412±·068 68	+·439±·068 64
$c_r c_r$	+·488±·073 49	+·556±·061 58	—	+·474±·076 47	+·424±·073 57	+·324±·082 54
G_2'	+·464±·080 44	+·598±·058 55	+·475±·076 47	—	+·263±·085 55	+·310±·085 51
zz	+·298±·084 53	+·412±·068 68	+·424±·073 57	+·263±·085 55	—	+·043±·085 63
$c_y l$	+·423±·075 54	+·439±·068 64	+·324±·082 54	+·310±·085 51	+·043±·085 63	—

TABLE XI. *Intra-Racial Correlations of Mandibular Heights, Antero-Posterior Lengths and Transverse Breadths for the Male Qau Series.*

		h_1	m_2h	ih'	rl	c_rh	c_yh	rb'	m_2p_1	c_pl	ml
Heights	h_1	—	+·642±·051 60	+·113±·085 61	+·063±·085 63	+·174±·080 66	−·072±·083 65	+·134±·081 67	−·057±·091 54	+·094±·083 65	+·597±·055 63
	m_2h	+·642±·051 60	—	+·187±·085 59	+·377±·073 62	+·536±·060 65	+·359±·073 65	+·298±·077 64	−·156±·086 58	+·432±·069 63	+·446±·069 61
	ih'	+·113±·085 61	+·187±·085 59	—	+·491±·064 61	+·658±·047 65	+·492±·063 66	+·216±·079 66	+·139±·089 55	+·195±·080 65	+·092±·083 65
	rl	+·063±·085 63	+·377±·073 62	+·491±·064 61	—	+·687±·043 68	+·854±·022 69	+·157±·079 69	+·161±·087 57	+·340±·071 70	+·132±·081 67
	c_rh	+·174±·080 66	+·536±·060 65	+·658±·047 65	+·687±·043 68	—	+·744±·036 71	+·273±·074 72	+·152±·085 60	+·352±·071 69	+·132±·080 68
	c_yh	−·072±·083 65	+·359±·073 65	+·492±·063 66	+·854±·022 69	+·744±·036 71	—	+·358±·068 71	+·106±·087 59	+·552±·056 71	+·018±·081 69
Ant.-post. lengths	rb'	+·134±·081 67	+·298±·077 64	+·216±·079 66	+·157±·079 69	+·273±·074 72	+·385±·068 71	—	+·040±·087 60	+·656±·046 71	+·164±·080 68
	m_2p_1	−·057±·091 54	−·156±·086 58	+·139±·089 55	+·161±·087 57	+·152±·085 60	+·106±·087 59	+·040±·087 60	—	+·169±·085 59	+·239±·084 57
	c_pl	+·094±·083 65	+·432±·069 63	+·195±·080 65	+·340±·071 70	+·352±·071 69	+·552±·056 71	+·656±·046 71	+·169+·085 59	—	+·384±·069 69
	ml	+·597±·055 63	+·446±·069 61	+·092±·083 65	+·132±·081 67	+·132±·080 68	+·018±·081 69	+·164±·080 68	+·239±·084 57	+·384±·069 69	—
Breadths	w_1	−·028±·093 52	−·059±·096 49	+·321±·082 54	+·268±·084 56	+·306±·082 55	+·268±·084 56	+·167±·088 55	+·114±·099 45	+·129±·089 56	+·115±·089 56
	ss	+·220±·081 63	+·412±·072 61	−·061±·085 63	+·242±·078 66	+·374±·070 68	+·262±·076 68	+·278±·075 69	+·076±·090 56	+·352±·072 68	+·204±·080 65
	c_yl	+·125±·086 60	+·322±·079 58	+·485±·056 61	+·582±·056 64	+·511±·062 65	+·500±·062 66	+·081±·082 66	+·221±·087 54	+·199±·080 66	+·160±·082 64

the other five (in Table XI) which are all vertical projections*. Two measurements which are, according to this grouping, of the same kind—such as two breadths—may be called "like measurements," or if they are of different kinds—such as a breadth and a height—"unlike measurements." The characters may also be divided into two groups according as they relate to the corpus or to the rami, so that two measurements may be said to be of the "same part" or of "different parts." The gonion may be supposed to mark the division of the corpus and ramus, so the bigonial breadth ($g_o g_o$) belongs to both parts. The distinction between them cannot be made clearly in the case of some of the antero-posterior lengths. The breadth of the ramus (rb') must be considered a rameal measurement, but it is partly "covered" by the projective length of the corpus ($c_p l$). Finally, the total projective length of the mandible (ml) embraces both the corpus and the rami, and hence it was excluded in compiling the data in the table below. A summary is given there of the remaining 78 correlations between the absolute measurements.

Correlations between	Insignificant	Significant and $< +0{\cdot}4$	$+0{\cdot}4 - +0{\cdot}6$	$> +0{\cdot}6$
Unlike measurements of different parts	14 (45·1 %)	8 (38·1 %)	1 (5·3 %)	0 (0·0 %)
" " the same part	9 (29·0 %)	7 (33·3 %)	6 (31·6 %)	0 (0·0 %)
Like measurements of different parts	7 (22·6 %)	4 (19·0 %)	4 (21·1 %)	1 (14·3 %)
" " the same part	1 (3·2 %)	2 (9·5 %)	8 (42·1 %)	6 (85·7 %)
Totals	31	21	19	7

The insignificant correlations range from $-{\cdot}156$ to $+{\cdot}220$—all these differing from zero by less than three times their probable errors—and of the total 31 values only six are negative. Intra-racial correlations of mandibular absolute measurements are all insignificant or significant and positive. This state of affairs may be accounted for by supposing that there is a general growth factor affecting all parts of the mandible, and thus inducing positive correlations between all measurements. In other words, a large adult bone tends to be large in all respects and a small bone tends to be small in all respects†. At the same time the relative growth of different parts may be unequal, and our purpose is to discover inequalities of this kind. The insignificant coefficients can be of some aid in this

* The breadth of the ramus (rb') is an antero-posterior measurement, and it is usually not parallel to the standard horizontal plane, but it may be called an antero-posterior *length* for the present purpose.

† It has been demonstrated by Professor Karl Pearson and Miss A. G. Davin ("On the Biometric Constants of the Human Skull," *Biometrika*, Vol. XVI. (1924), pp. 328—363) that the same is true for the cranium. They provide 85 male and 85 female correlations between pairs of absolute measurements of 1800 crania from a single Egyptian cemetery of the 26th—30th Dynasties. Among these male constants only one is negative, and there are 51 (60 per cent.) less than $+0{\cdot}4$ and 34 greater than that value. For our 90 correlations (including those with ml) between absolute measurements of the mandible there are 62 (69 per cent.) less than $+0{\cdot}4$ and 28 greater than that value. A different choice of characters correlated might alter these percentages appreciably, but it may be supposed that they are approximately the same for the cranium and mandible.

search. The only one of these which is between "like" measurements of the "same part" is for the length of the dental arcade from second molar to first premolar (m_2p_1), and the projective length of the corpus (c_pl). But m_2p_1 is entirely "covered" by c_pl, and a significant, if not high, correlation would have been anticipated in such a case. It may also be noted that m_2p_1 is the only measurement which shows no significant correlation with any other measurement in Table XI, while of the six negative coefficients found two are with this same length. The length of the dental arcade thus appears to be almost independent of growth changes which modify other characters of the mandible, and such a conclusion is in entire agreement with a result which had previously been reached by other means. The fact that there is no increase in the length of the dental arches after eight years of age has recently been demonstrated statistically by Dr Matthew Young on adequate living material*. Other parts of the mandible obviously continue to grow until a much later age, and hence the absence of correlation between their measurements and the length of the dental arcade is sufficiently explained. One other feature of interest may be appreciated from the insignificant correlations. Of the 16 coefficients between "like" measurements of "different parts" summarized in the table above, nine are significant and seven are insignificant. One of these seven is for m_2p_1 and the breadth of the ramus (rb'), and four others are the correlations between the symphyseal height (h_1), on the one hand, and the rameal heights (ih', rl, c_rh and c_yh), on the other. It is seen from Table XI that h_1 shows only one significant coefficient, which is with the other height of the corpus measured at the second molar (m_2h)†, while all the other characters, with the exception of m_2p_1 and ml, show five or more significant coefficients. The correlation between h_1 and m_2h is as high as ·64, and the latter height shows seven other significant coefficients†. The growth of the symphysis in height appears to be largely independent of the growth of most other parts of the mandible, though it is not so peculiar in this respect as the length of the dental arcade. It is not known that the symphyseal height ceases to grow at an earlier age than other parts of the mandible.

The correlations which differ significantly from zero may now be considered. Unfortunately many of these are not capable of revealing in a straightforward way any differences there may be between the growth processes of different parts of the mandible owing to the way in which the measurements are defined. There are, for example, three transverse breadths of the rami (w_1, c_rc_r and g_og_o) which are really only measuring the same feature of the mandible in rather different ways. Quite high positive correlations are to be expected between pairs of these breadths owing to "mathematical necessity." The effect of this irrelevant factor

* In "Facial Growth in Children with special reference to Dentition," Part I by Corisande Smyth, Part II by Matthew Young, *Medical Research Council, Special Report Series*, No. 171 (1932). The length of the dental arches is dealt with on pp. 40—44. The peculiarity in the growth of this character was first noted by John Hunter (1771) and it was subsequently investigated by several anatomists whose researches are referred to in the above memoir.

† Remembering that correlations with ml are excluded.

may also be expected to affect, in varying degrees, the correlations of "like" measurements of "different parts" of the bone. Significant correlations between "like" measurements may be capable of revealing the existence of true organic correlations, but such will be masked and no attempt to interpret such data will be made here. Before leaving them, however, it may be noted that six of the seven coefficients greater than 0·6 are between "like" measurements of the "same part," and the remaining one (for rb' and $c_p l$) is between "like" measurements arbitrarily supposed to belong to "different parts," though actually the breadth of the ramus is partly "covered" by the length of the corpus. Correlations of "like" measurements are usually of greatest interest when they are insignificant, but for "unlike" measurements the converse is true. It only remains to consider the significant correlations between "unlike" measurements. The table on p. 105 above shows that of the 22 "unlike" measurements of the "same part," 13 (59 per cent.) are significant, while of the 23 "unlike" measurements of "different parts" nine (39 per cent.) are significant. A significant correlation between "unlike" measurements is thus almost as likely to be found when they relate to "different parts" as when they relate to the "same part." The 22 significant coefficients between "unlike" measurements can be grouped in the following way*:

(*a*) All heights except that of the symphysis (h_1) (viz. $m_2 h$, ih', $c_r h$, $c_y h$ and rl) with all the transverse breadths with which they have been correlated (viz. w_1, zz and $c_y l$) are included except $m_2 h$ with w_1 and ih' with zz. This accounts for 13 of the coefficients.

(*b*) All heights except those of the symphysis (h_1) and incisura (ih') (viz. $m_2 h$, $c_r h$, $c_y h$ and rl) with all the antero-posterior lengths except $m_2 p_1$ (viz. rb' and $c_p l$) are included except rb' with rl, in which case the coefficient is $+ \cdot 157 \pm \cdot 079$. This accounts for seven of the coefficients.

(*c*) The remaining two significant correlations between "unlike" measurements are for the breadth between the mental foramina (zz) with the length of the corpus ($c_p l$) and the breadth of the ramus (rb'—an antero-posterior measurement), respectively. The other breadths (w_1 and $c_y l$) do not show significant correlations with any of the antero-posterior lengths ($m_2 p_1$, rb' and $c_p l$).

The total projective length of the mandible is not included in the above summary because it differs from the other measurements in being of a composite nature by embracing both the corpus and ramus. The projective length of the corpus ($c_p l$) actually forms the greater part of ml, but the correlation between them ($+ \cdot 384 \pm \cdot 069$) is not high, and ml "covers" the length of the dental arcade ($m_2 p_1$), but an insignificant value is found between them. Insignificant correlations are also found between ml and most of the characters with which $c_p l$ is significantly correlated, though the two show almost equal values with the height at the second molar ($m_2 h$). It is curious to find that ml shows a high correlation (·60) with the symphyseal height (h_1), in spite of the fact that the latter character is uncorrelated with the length of the corpus, or with any of the other measurements except the

* See footnote † on page 106.

height of the corpus (m_2h). No obvious explanation of this state of affairs can be offered.

Table XII (see pp. 110—111) gives for the Qau Series a number of correlations between indices and angles, on the one hand, and some of the absolute measurements, on the other, and also between pairs of the indices and angles with one another. These need not be considered in detail. Several examples may be noticed of high spurious correlations between an index and one or other of the absolute measurements from which it is formed. The comparisons involving an angle are of more interest. The mandibular angle ($M\angle$) is seen to be correlated with most of the absolute measurements, but not with the length of the dental arcade (m_2p_1). The high negative coefficients with the heights of the ramus (rl and c_rh) and the positive with the total projective length of the mandible (ml) would have been anticipated, but not the other significant values. The positive coefficient with the symphyseal height (h_1) and the negative with the breadth of the ramus (rb') are noteworthy. The mandibular angle shows quite high correlations with two of the indices, and significant values for these would have been anticipated. The angle between the plane tangential to the coronoid and condyloid processes and the plane tangential to the posterior border of the ramus ($R\angle$) varies more independently, but it shows an unexpectedly high correlation with the breadth-length index 100 c_rc_r/ml. The projection of the chin is a character of great interest since it is distinctive for upper palaeolithic and modern man. As measured by $C'\angle$, a smaller angle denotes a more projecting chin, and hence the negative correlation with the total projective length of the mandible (ml) and the positive with the index 100 c_rh/ml would have been anticipated. But no other significant coefficient is found between the angle and any measurement of shape or size. The proportion of the length to the breadth of the corpus is measured directly by the index 100 g_og_o/c_pl and indirectly by the angle subtended at the gnathion by the two gonia ($G\angle$); the correlation between these two (+·93) is the highest found. The index is quite uncorrelated with the other three angles, and the significant values found with two other indices would have been anticipated from "mathematical necessity." Owing to this factor, it is not possible to say from the indices available whether the shapes of different parts of the mandible vary independently or not. Differently designed measurements would be required to decide this question.

(7) *Comparisons by the Method of the Coefficient of Racial Likeness.* Professor Karl Pearson's method of the coefficient of racial likeness gives a single measure, based on a number of characters, of the probability that two samples represent the same population. It has been used—chiefly in a modified form—as a measure of the divergence of two samples which cannot be supposed to represent the same population*. This method has been applied quite extensively to cranial samples,

* If M_s is the mean of the sth character based on n_s individuals in the case of the first series, and $M_{s'}$ and $n_{s'}$ are the corresponding constants for the second series, then the form of the crude coefficient of racial likeness used in practice is:

$$\frac{1}{m}\Sigma\left(\frac{n_s n_{s'}}{n_s+n_{s'}}\times\frac{(M_s-M_{s'})^2}{\sigma_s^2}\right)-1\pm\cdot67449\sqrt{\frac{2}{m}}=\frac{1}{m}\Sigma(a)-1\pm\cdot67449\sqrt{\frac{2}{m}},$$

but only hitherto to English series of mandibles*. Its effectiveness may be supposed to depend primarily on a suitable choice of characters being made. If the coefficient is only to be used to estimate the probability that the two samples compared represent the same population, then the most suitable characters to use will be those which are lowly correlated intra-racially with one another. The assumption that these correlations will be the same for all series as for the longest available must be made, though actually it is unlikely to be exactly true. But if the use of the criterion is extended to give estimates of the divergences in type of series which cannot be considered to represent the same population—as in the case of the reduced coefficient of racial likeness—then *inter-racial* correlations should, ideally, be considered in choosing the most effective group of characters. The mandible measurements dealt with in this paper are actually only available for 12 male series, and several of these are very small, so it is quite impossible at present to obtain even the crudest possible first approximations to inter-racial constants. Other conditions to be considered are that the number of characters used in computing the coefficients must be as large as possible, as there can be little hope of obtaining a reliable classification based on a few only, and that there is no advantage in including characters which are constant for all racial types, if any such can be found. The ideal conditions which should be taken into account in selecting characters to be used in computing the coefficients of racial likeness may now be considered as far as the very limited material available permits. Choice may be made from among the 13 absolute measurements, three angles and five indices which were selected as being the most reliable measurements (see pp. 90—91 above). There are five transverse breadths, including the length of the condyle ($c_y l$), and the intra-racial correlations between most pairs of these are known to be high (Table X). Inter-racially four of them appear to be highly correlated with one another (see Table XIV), but the fifth ($c_y l$) does not appear to be correlated with these four. Three of the breadths—viz. those at the condyles (w_1) and mental foramina (zz) and the length of the condyle ($c_y l$)—may be chosen for the purpose in view. There are four antero-posterior lengths and if one—the projective length of the corpus ($c_p l$)—is excluded the intra-racial correlations between the others are not high (Table XI), and they are lowly correlated with the selected breadths. Inter-racially these correlations also appear to be of a low order, so the three lengths—viz. the total projective length of the mandible (ml), the breadth of the ramus (rb') and the length of the dental arcade ($m_2 p_1$)—may be

where m characters are compared and σ_s is the standard deviation of the sth character for the longest series available. If $\bar{n}_1$ and $\bar{n}_2$ are the mean numbers of individuals available for the m characters in the case of the first and second series in the comparison, respectively, then the reduced coefficient is defined to be:

$$50 \times \frac{\bar{n}_1 + \bar{n}_2}{\bar{n}_1 \bar{n}_2} \left\{ \frac{1}{m} \Sigma (\alpha) - 1 \right\} \pm 50 \times \frac{\bar{n}_1 + \bar{n}_2}{\bar{n}_1 \bar{n}_2} \times \cdot 67449 \sqrt{\frac{2}{m}}.$$

The reduced coefficient is designed to give an approximation to the value which would be obtained for the crude coefficient if all the means used were based on 100 individuals instead of on the numbers of specimens actually available. On this assumption it provides a measure of the divergence of the types which does not depend on the sizes of the samples.

* In the paper by Brash, Layard and Young cited above (footnote p. 93).

TABLE XII.

Intra-Racial Correlations of Mandibular Absolute Measurements and Indices and Angles for the Male Qau Series.

	$M\angle$	$R\angle$	$G\angle$	$C'\angle$	$100\,c_r h/ml$	$100\,c_r c_r/ml$
w_1	$-\cdot173\pm\cdot087$ 56	$+\cdot102\pm\cdot092$ 53	$+\cdot306\pm\cdot083$ 54	$-\cdot172\pm\cdot098$ 45	—	$+\cdot354\pm\cdot084$ 49
ss	$-\cdot277\pm\cdot076$ 68	$+\cdot248\pm\cdot080$ 62	—	$+\cdot009\pm\cdot094$ 51	—	—
$c_y l$	$-\cdot305\pm\cdot075$ 66	$+\cdot094\pm\cdot087$ 59	—	$-\cdot233\pm\cdot093$ 47	—	—
ml	$+\cdot278\pm\cdot075$ 69	$-\cdot098\pm\cdot085$ 62	—	$-\cdot320\pm\cdot085$ 51	$-\cdot305\pm\cdot074$ 68	$-\cdot618\pm\cdot055$ 57
rb'	$-\cdot429\pm\cdot065$ 71	$+\cdot056\pm\cdot084$ 64	—	$+\cdot008\pm\cdot093$ 53	—	—
$m_2 p_1$	$-\cdot032\pm\cdot088$ 59	$+\cdot188\pm\cdot089$ 53	—	$+\cdot144\pm\cdot095$ 46	—	—
h_1	$+\cdot348\pm\cdot074$ 65	$+\cdot062\pm\cdot087$ 59	—	$+\cdot199\pm\cdot089$ 53	—	—
rl	$-\cdot580\pm\cdot053$ 70	$+\cdot097\pm\cdot085$ 62	—	$-\cdot037\pm\cdot095$ 50	$+\cdot557\pm\cdot057$ 66	—
$c_r h$	$-\cdot518\pm\cdot059$ 70	$+\cdot709\pm\cdot042$ 64	—	$+\cdot136\pm\cdot091$ 53	$+\cdot890\pm\cdot017$ 68	—
$M\angle$	—	$-\cdot344\pm\cdot075$ 63	—	$-\cdot096\pm\cdot093$ 52	$-\cdot639\pm\cdot048$ 68	$-\cdot444\pm\cdot072$ 57
$R\angle$	$-\cdot344\pm\cdot075$ 63	—	—	$+\cdot144\pm\cdot094$ 49	$+\cdot726\pm\cdot040$ 62	$+\cdot588\pm\cdot059$ 56
$C'\angle$	$-\cdot096\pm\cdot093$ 52	$+\cdot144\pm\cdot094$ 49	—	—	$+\cdot336\pm\cdot084$ 51	$+\cdot192\pm\cdot096$ 46
$100\,g_o g_o/c_y l$	$+\cdot223\pm\cdot078$ 68	$-\cdot103\pm\cdot085$ 62	$+\cdot929\pm\cdot011$ 67	$+\cdot139\pm\cdot093$ 51	$+\cdot183\pm\cdot081$ 65	$+\cdot351\pm\cdot079$ 56

TABLE XII (*continued*).

	$100\,g_o g_o/c_p l$	$100\,g_o g_o/c_r c_r$	$100\,rb'/rl$	$100\,ik'/c_y c_r$	$100\,c_y h/c_r h$	$100\,d_1 h/c_r h$
w_1	+·358 ± ·079 55	+·207 ± ·093 48	—	—	—	—
zz	+·083 ± ·082 66	+·057 ± ·090 56	—	—	—	—
$c_y l$	+·274 ± ·079 63	—	−·433 ± ·069 63	—	—	—
ml	−·248 ± ·078 66	—	−·061 ± ·083 66	—	—	—
rb'	−·336 ± ·073 67	+·066 ± ·088 58	+·385 ± ·069 69	−·031 ± ·083 66	—	—
$m_2 p_1$	+·047 ± ·091 55	−·008 ± ·098 47	—	—	—	—
h_1	+·085 ± ·085 62	+·016 ± ·092 54	—	—	—	—
rl	+·037 ± ·083 66	+·053 ± ·090 56	−·731 ± ·038 69	+·380 ± ·072 64	+·444 ± ·066 68	−·447 ± ·073 55
$c_r h$	+·108 ± ·082 66	—	−·737 ± ·037 68	—	—	—
$M\angle$	+·223 ± ·078 68	—	+·132 ± ·080 69	—	—	—
$R\angle$	−·103 ± ·085 62	+·255 ± ·084 56	−·052 ± ·085 62	+·322 ± ·077 62	—	—
$C'\angle$	+·139 ± ·093 51	−·062 ± ·099 46	+·013 ± ·095 50	+·089 ± ·095 50	—	—
$100\,g_o g_o/c_p l$	—	—	−·377 ± ·072 65	—	—	—

chosen. There are four heights, and two of these must be excluded to avoid high correlations, leaving the symphyseal height (h_1) and the projective length of the ramus (rl). Of the three angles which might be used, the one which measures the projection of the symphyseal section ($C'\angle$) is too unreliable a measurement for the purpose, and the mandibular angle ($M\angle$) is almost constant* for all the available types (see p. 96 above). This leaves the angle between the plane tangential to the coronoid and condyloid processes and the plane tangential to the posterior border of the ramus ($R\angle$) which has been included. Finally, of the five indices one ($100\, g_o g_o / c_r c_r$) is almost constant* for all the available types (see p. 96 above), three others are highly correlated intra-racially (and probably inter-racially as well) with selected absolute measurements, and the fifth, which is the breadth-length index of the corpus ($100\, g_o g_o / c_p l$) may be included. Ten measurements were thus chosen for the purpose of calculating coefficients of racial likeness, viz. w_1, zz, $c_y l$, ml, rb', $m_2 p_1$, h_1, rl, $R\angle$ and $100\, g_o g_o / c_p l$. The intra-racial correlations between these for the male Qau series are given in Tables X—XII: 40 of the 45 coefficients are less than ·3, the highest coefficient is + ·597 and no single character has more than two coefficients greater than ·3.

The coefficients of racial likeness computed for the 10 characters between the 12 male and five female series are given in Table XIII. The Qau standard deviations given in Table VIII were used as this is the longest series as yet available. It must be noted, in the first place, that several of the series are very small, and it is known from experience in the case of the cranium, at any rate, that samples composed of less than 30 individuals are of very little value for purposes of racial classification. Three of the male and two of the female series of mandibles are made up by less than 30 individuals. It would be absurd to expect to obtain any reliable classification of 12 racial types from the evidence of a total of only 400 mandibles, and the most that can be hoped for is that the data available will give some indication of the use of more extended material. The samples at any rate are large enough to differentiate most of the types. Of the 76 crude coefficients of racial likeness there are only six which differ from zero by less than three times their probable errors, so the differences between different racial types are sufficiently great to be made apparent when only small numbers of specimens are represented. The question whether the coefficients can be used to provide a reasonable classification of the types may now be considered. An extended experience of these constants computed for cranial series has shown repeatedly that the most suggestive arrangements are always arrived at if only the lowest orders of reduced coefficients are considered, while all above an arbitrarily fixed limit are neglected entirely. Such a condition does not necessarily indicate any defect in the method of comparison, since it may be supposed to be occasioned by the nature of the material. The inter-relationships of the races of modern man are, doubtless, extremely complex, and it is not surprising to find that no quantitative criterion—whether based on measurements of a single character or on those of a number of characters

* [The "almost constant" feature of both the angle and index appears to need further explanation. Ed.]

TABLE XIII. *Coefficients of Racia*

		Sex	$\bar{n}$ †	Predynastic Egyptians (Badari)	Dynastic Egyptians (Qau)	Dynastic Egyptians (Sedment)	Dynastic Egyptians (Kerma)
Reduced Coefficients	Predynastic Egyptians (Badari)	♂	33·5		7·28	3·53	5·68
		♀	18·9		1·57	1·56	−0·24
	Dynastic Egyptians (Qau)	♂	66·4	16·35 ± ·68		1·24	2·63
		♀	55·7	5·58 ± 1·07		1·71	1·01
	Dynastic Egyptians (Sedment)	♂	32·4	10·70 ± ·92	2·84 ± ·69		0·78
		♀	21·2	7·79 ± 1·51	5·57 ± ·98		3·13
	Dynastic Egyptians (Kerma)	♂	55·7	13·58 ± ·72	4·34 ± ·50	1·90 ± ·74	
		♀	44·7	−0·91 ± 1·14	2·03 ± ·61	10·87 ± 1·05	
	Anglo-Saxons	♂	42·4	55·80 ± ·81	18·52 ± ·58	21·75 ± ·82	21·55 ± ·63
		♀	41·2	39·01 ± 1·16	19·58 ± ·64	30·33 ± 1·08	34·82 ± ·70
	English (Dunstable)	♂	37·3	55·32 ± ·85	18·22 ± ·63	24·28 ± ·87	24·65 ± ·67
	Tamils	♂	33·0	31·84 ± ·91	15·92 ± ·69	23·15 ± ·92	23·68 ± ·73
	Nepalese	♂	18·9	31·63 ± 1·25	11·79 ± 1·03	22·65 ± 1·26	19·76 ± 1·07
	Tibetans *A*	♂	24·9	35·53 ± 1·06	16·52 ± ·83	27·21 ± 1·07	24·14 ± ·88
	Tibetans *B*	♂	11·9	62·21 ± 1·72	25·25 ± 1·49	31·34 ± 1·73	20·94 ± 1·58
	Fukien Chinese	♂	37·5	66·40 ± ·85	22·35 ± ·63	32·60 ± ·87	32·30 ± ·67
	Hylam Chinese	♂	38·8	47·04 ± ·84	20·15 ± ·63	23·82 ± ·85	25·98 ± ·66

* All coefficients in this Table are based on the same 10 c

† The $\bar{n}$'s are the mean numbers of mandibles available f

‡ The probable error of a crude coefficient is 0·30.

TABLE XIII. *Coefficients of Racial Likeness based on Mandibular Measure*

					Crude Coefficients ‡
nastic yptians Qau)	Dynastic Egyptians (Sedment)	Dynastic Egyptians (Kerma)	Anglo-Saxons	English (Dunstable)	Tamils
·28 ·57	3·53 1·56	5·68 −0·24	20·89 10·11	19·54 —	10·58 —
	1·24 1·71	2·63 1·01	9·59 9·27	8·70 —	6·98 —
4± ·69 7± ·98		0·78 3·13	7·99 8·49	8·42 —	7·57 —
4± ·50 3± ·61	1·90± ·74 10·87±1·05		10·37 14·92	11·01 —	9·81 —
2± ·58 8± ·64	21·75± ·82 30·33±1·08	21·55± ·63 34·82± ·70		−0·07 —	9·66 —
2± ·63	24·28± ·87	24·65± ·67	−0·18± ·76		6·98
2± ·69	23·15± ·92	23·68± ·73	26·04± ·81	19·94± ·86	
9±1·03	22·65±1·26	19·76±1·07	31·22±1·15	25·23±1·20	1·83±1·25
2± ·83	27·21±1·07	24·14± ·88	28·60± ·96	22·43±1·01	−0·70±1·06
5±1·49	31·34±1·73	20·94±1·58	14·34±1·62	18·67±1·67	36·70±1·72
5± ·63	32·60± ·87	32·30± ·67	20·13± ·76	23·24± ·81	30·09± ·86
5± ·63	23·82± ·85	25·98± ·66	11·79± ·74	12·01± ·79	10·34± ·85

icients in this Table are based on the same 10 characters, viz. w_1, zz, c_1l, ml, m_1P_1, rb', h_1, rl, R
are the mean numbers of mandibles available for the 10 characters used in computing the coefficie
able error of a crude coefficient is 0·30.

ular Measurements.*

efficients‡

Tamils	Nepalese	Tibetans *A*	Tibetans *B*	Fukien Chinese	Hylam Chinese
10·58 —	7·64 —	10·15 —	10·92 —	23·49 —	16·91 —
6·98 —	3·47 —	5·98 —	5·10 —	10·72 —	9·72 —
7·57 —	5·41 —	7·66 —	5·45 —	11·33 —	8·41 —
9·81 —	5·57 —	8·31 —	3·98 —	14·47 —	11·88 —
9·66 —	8·16 —	8·98 —	2·66 —	8·01 —	4·78 —
6·98	6·33	6·70	3·37	8·69	4·57
	0·44	−0·20	6·42	10·56	3·69
1·83 ± 1·25		0·48	3·37	5·43	2·77
−0·70 ± 1·06	2·22 ± 1·40		4·26	8·17	3·05
36·70 ± 1·72	23·08 ± 2·07	26·45 ± 1·87		1·71	5·01
30·09 ± ·86	21·61 ± 1·20	27·32 ± 1·01	9·45 ± 1·67		2·92
10·34 ± ·85	10·89 ± 1·19	10·05 ± ·59	27·50 ± 1·66	7·66 ± ·79	

p_1, rb', h_1, rl, $R\angle$ and $100\, g_o g_o / c_p l$.
ating the coefficients.

to face p. 112

considered in conjunction—can give a measure of all degrees of relationship. It may still be possible to unravel the tangle of relationships by considering only close degrees of similarity such as may be taken to indicate close relationship. Turning to the reduced coefficients of racial likeness in Table XIII, it will be seen that the larger values fail to provide an arrangement of the types which can be supposed to indicate their inter-relationships. The Tibetan *B* series, for example, has decidedly lower coefficients with the two English than with four of the Asiatic series. If it is decided to take no account of any reduced coefficients greater than an arbitrarily chosen value, then this limit may be chosen so that it leads to the most reasonable arrangement of the types. If it be fixed at 11 we find:

(*a*) that three Egyptian series are linked to one another in every possible way, while the fourth—which is the Badarian having a cranium diverging appreciably from dynastic Egyptian types—is linked to one of the three but to no other series;

(*b*) the two English series are linked only to one another;

(*c*) the six Asiatic series are linked to one another, though not in every possible way, particularly close connections being found between the Tamil, Nepalese and Tibetan *A* series, while the two Chinese and the Tibetan *B* diverge appreciably from this group and from one another.

This arrangement of the 12 series appears to be a reasonable and suggestive one, and it does not conflict with the scheme of relationship suggested by the cranial or—as far as is known—by any other characters. If reduced coefficients only slightly greater than 11 are also taken into account some unexpected connections are found. If the limit is fixed at 15, for example, the Anglo-Saxon series becomes linked to the Tibetan *B* and Hylam Chinese. It would clearly be an advantage to fix an upper limit less than 11 to the reduced coefficients which may be used to estimate racial affinity, and this should become possible when more series are available for comparison. Meanwhile there is a distinct hope that mandibular measurements will prove to be of value in aiding the solution of the problem of racial classification.

Coefficients of racial likeness are available for several pairs of the cranial series corresponding to the series of mandibles dealt with above, and, in general, there are more crania than mandibles available, though the differences in number are not great. The cranial coefficients are nearly all decidedly larger than the corresponding mandibular coefficients. This difference—which will depend, partly at least, on the choice of characters used—is most interesting in the cases where the mandibular coefficient differs insignificantly from zero. This is so for male Anglo-Saxons with the English series from Dunstable (cranial reduced coefficient $= 13{\cdot}66 \pm {\cdot}45$), and for the three male series of Tamils, Nepalese and Tibetans *A* with one another. The male reduced cranial coefficients for these last are: Tamils and Nepalese $16{\cdot}21 \pm {\cdot}45$, Tamils and Tibetans *A* $40{\cdot}89 \pm {\cdot}50$, Nepalese and Tibetans *A* $12{\cdot}60 \pm {\cdot}42$*. It is clear that the mandibular measurements used are

* These values are taken from *Biometrika*, Vol. XXIV. (1932), p. 121 and Table VII facing p. 126.

not capable of making distinctions between types which are clearly distinguished by cranial measurements. A comparison of the measurements of the mandible not used in calculating the coefficients suggests that this conclusion is not due to the selection of characters made. The existence of an insignificant coefficient of racial likeness, or the absence of any statistical differentiation between all the mean measurements available for two series of mandibles, cannot be accepted as evidence for identity of racial type if the series are no longer than the ones at present available. It is possible that much larger samples would be capable of differentiating closely allied racial types, or it may be that mandibular characters—unlike cranial characters—are in fact incapable of effecting this purpose. They may still be of considerable value in aiding racial classification.

It may be noticed that the majority of the corresponding male and female reduced coefficients of racial likeness in Table XIII differ very significantly. If the material were more adequate a closer approach to equality would be expected, and the divergences noted must be supposed due to the small sizes of the samples, to the fact that the corresponding male and female samples may not represent precisely the same racial type in every case, and possibly, also, to inaccurate anatomical sexing of a few specimens.

(8) *Comparison of Single Characters.* The value of different characters considered singly for purposes of racial classification may now be dealt with. In computing a coefficient of racial likeness the α (see footnote to p. 108 above) found is approximately the square of a quantity which is the difference of two means divided by its standard error. An α greater than 10 may be taken to indicate that the difference is significant. The α's found in the comparison of the 12 male series, and for the 10 characters used in computing the coefficients, range from zero to 87, and 180 of the total 660 are greater than 10. The α's were also calculated for three additional characters—viz. $M\angle$, $C'\angle$ and $100\, g_o g_o/c_r c_r$—before a final selection was made of the list which could best be used for the coefficients of racial likeness. For any particular one of these 13 measurements there is a total of 66 α's, and for $C'\angle$, 35 of these are greater than 10, for w_1 the number of differences supposed significant is 34, for $100\, g_o g_o/c_p l$, 24, for h_1, 23, for zz, 18, for $c_y l$, 17, for rl, 16, for ml, 16, for $m_2 p_1$, 15, for rb', 10, for $R\angle$, 7, for $M\angle$, 3, and for $100\, g_o g_o/c_r c_r$, 2. There is reason to believe that several of the significant differences found between the mean $C'\angle$'s may be due to the differences in technique of different observers, but this is not so for the other measurements. They evidently differ enormously in their ability to differentiate the various series, and some appear to be valuable for the purpose in view, while others are practically constant for the few racial types represented. The mandibular angle ($M\angle$) and the index $100\, g_o g_o/c_r c_r$ were not used in computing the coefficients owing to the fact that they show very few significant differences (see footnote, p. 112).

All the measurements available may be dealt with in groups. There are five transverse breadths—including the length of the condyle—and it has been seen that the bi-condylar (w_1) shows far more significant differences than the bi-fora-

minal (zz). For every breadth except $c_y l$ the Badarian Egyptian series has the smallest mean, and for three of the four the Anglo-Saxons have the largest mean. These characters appear to be highly correlated inter-racially, as would have been anticipated (see Table XIV), and for two of them absolute distinctions are made

TABLE XIV.

Means of Selected Mandibular Measurements for Male Series.

	$g_o g_o$	$c_r c_r$	w_1	zz	$c_y l$
Predynastic Egyptians: Badari	83·9 (31)	86·8 (29)	109·5 (30)	43·4 (36)	20·3 (36)
Dynastic Egyptians: Sedment	88·1 (29)	92·1 (22)	114·3 (26)	44·0 (34)	20·8 (30)
" " Kerma	88·9 (53)	92·3 (46)	113·8 (51)	44·3 (65)	20·8 (57)
" " Qau ...	90·5 (70)	91·7 (59)	113·8 (56)	43·9 (70)	20·3 (67)
Tamils	91·8 (33)	94·5 (33)	117·7 (33)	45·1 (33)	19·5 (33)
Nepalese	92·2 (19)	93·4 (19)	115·0 (19)	46·2 (19)	18·9 (19)
Tibetans *A*	92·8 (24)	93·8 (25)	117·0 (25)	45·7 (25)	18·8 (25)
Hylam Chinese	94·6 (39)	98·2 (39)	122·5 (39)	46·7 (39)	19·4 (39)
Fukien Chinese	95·4 (38)	97·2 (38)	121·9 (38)	46·8 (38)	20·1 (38)
Tibetans *B*	96·5 (11)	93·9 (12)	122·0 (12)	47·0 (12)	19·9 (12)
English: Dunstable	98·4 (40)	99·3 (27)	121·0 (29)	45·4 (42)	21·3 (35)
Anglo-Saxons	100·4 (33)	100·3 (27)	123·7 (25)	45·3 (57)	21·7 (38)

between the Egyptian, Asiatic and English series. The only index providing the ratio of two breadths is $100\, g_o g_o / c_r c_r$, and this only shows two significant differences in 66 comparisons. The length of the condyle ($c_y l$) is a character of racial significance and the two largest means are for the English series, the Egyptian follow next and all the Asiatic series have smaller means. The data in Table XIV suggest that this character is not highly correlated inter-racially—if at all—with the other transverse breadths. The remaining absolute measurements, angles and indices fail to arrange the types in suggestive orders. The position is probably obscured, of course, by the fact that the means for the short series available may diverge appreciably from the true means for the sampled populations, but after examining these characters one by one it is surprising to find that the coefficients of racial likeness are capable of reducing the apparent jumble of characters in such a way that they can be supposed to provide a measure of racial affinity.

(9) *Conclusions.* The main purposes of this paper are to present the measurements of two series of Egyptian mandibles and, with the aid of the information they supply, to explore the possible use of measurements of the bone for estimating racial affinities. The technique used is that employed in the Biometric and Galton Laboratories since 1923. By examining differences between the readings of two observers on the same specimens it was possible to select the more reliable measurements from the list originally defined, and it is suggested that only those

(given on p. 91) should be recorded in future studies. Measurements taken in accordance with the same technique are now available for twelve male and five female series*, but several of these are made up by less than 30 individuals. The most marked sexual differences are found between heights of the ramus, indices and angles associated with such heights and the length of the condyle. Several of these are large enough to suggest that when considered in conjunction they may make it possible to devise an effective method of sexing a racial series of mandibles by considering its measurements alone. Sexual differences appear to be relatively greater for the mandible than for the cranium. The very limited material available suggests that there is sexual equality in the variability of mandibular characters. The different series show a few significant, but very small absolute, differences in variability. Mandibular characters tend, on the average, to be more variable intra-racially than cranial characters when allowance is made for differences in absolute size. Intra-racial correlations are given for the longest male series available. Those between absolute measurements are either insignificant, or significant and positive, indicating that there is a general growth factor affecting almost all parts of the bone. The length of the dental arcade is uncorrelated with other measurements, however, and growth here is known to cease well before maturity is reached. The lack of association between the projection of the chin and other characters is a noteworthy fact. Ten measurements are selected for the purpose of calculating coefficients of racial likeness, and these criteria lead to a reasonable arrangement of the types which encourages the hope that a similar comparison of more extended material would furnish valuable aid in estimating racial relationships. At the same time the fact that samples of the sizes at present available are not differentiated cannot be accepted as a test of racial identity. The different measurements vary greatly in their ability to differentiate the series from one another, and when considered singly the only ones which arrange the series in suggestive orders are the transverse breadths and the length of the condyle.

APPENDIX.

Definitions of Mandibular Measurements.

The biometric technique for measuring the human mandible was first described in *Biometrika* in 1923 (Vol. XIV. pp. 253—260). Fuller definitions are given below of the 16 measurements which were selected (as described in Section 2 of the present paper) chiefly on account of the fact that they can be taken with greater accuracy than the others. It is recommended that only this selected group should be used in future studies. All readings of lengths are given to the nearest 0·1 mm. and all of angles to the nearest 0°·5. If a measurement is uncertain owing to the damaged condition of a specimen it is queried, or omitted entirely if it is felt that the reading is not a very close approximation to that which would have been obtained on the complete bone. It is difficult to specify the extent of defect which makes it necessary to exclude a particular measurement, but remarks on this question are given in the definition of each measurement. A reading may also be queried owing to the anatomical peculiarity of a specimen.

* Not counting the Anglo-Saxon series described by Brash, Layard and Young (*loc. cit.*).

Definitions of "points' are given most conveniently in the accounts of the measurements. The apparatus required is first a pair of small calipers and a mandible board, and secondly a scriber and goniometer described below.

It is first necessary to define a "standard horizontal plane." If a mandible be placed on a horizontal surface with the teeth uppermost, it will normally rest in a unique position, but occasionally it will be found capable of a slight rocking displacement. The plane of reference is defined to be that horizontal plane with which contact is made at three or more points when a vertical pressure is applied to the second left molar tooth or to its cavity. This is known as the standard horizontal, or basal plane, and, when pressed vertically downwards on the second left molar, the mandible is said to be in the standard horizontal position. In about 75 per cent. of cases for most series the pressure, exerted by the finger, will not change the position of the bone; in about 20 per cent. it will definitely rotate the bone "backwards" or "forwards," and in the remaining 5 per cent. the operator may feel uncertain as to whether the pressure should rotate the bone "backwards" or "forwards," or maintain it in an intermediate and somewhat unstable position. The last case is the only one which presents any difficulty in practice, and when it is met with the intermediate position should be accepted, and the observer should resist the inclination to favour one which ensures greater stability. It is often advisable to fix the mandible in the standard horizontal position with the aid of plasticine before taking certain measurements. Unilateral measurements are taken on the left side, but if that is defective they may be taken, alternatively, on the right. But in this case the pressure is still exerted on the second *left* molar if the measurements in question are to be referred to the standard horizontal plane. The standard transverse vertical plane may be defined as one perpendicular to the standard horizontal plane which makes contact with both condylar processes: it is represented by the vertical rameal wing of the mandible board in Plate II (*b*). The standard sagittal plane of the symphysis is defined as that one passing through the intradental (a point defined below), and which is perpendicular to both standard horizontal and transverse planes. These three planes are what we refer to when using the adjectives horizontal, transverse and sagittal in the definitions of the measurements and elsewhere.

The first seven measurements are taken with small calipers. The form of these (*Tasterzirkel*) made by P. Hermann, Rickenbach und Sohn is as convenient to use as any, but the tenths of a mm. have to be estimated on them and it is desirable that anyone unaccustomed to doing this should have preliminary practice with calipers provided with a vernier scale. The first two measurements are maximum projections taken by using the ends of the bars with flat sides and it is understood, of course, that the line joining the points of contact with the bone is parallel to the scale, though this necessity is sometimes overlooked by beginners. All the other caliper measurements are taken by using the tips of the bars.

w_1. Maximum breadth outside condyles. This maximum projection may be taker in any direction: it is not necessarily horizontal or transverse, though both these conditions will be fulfilled in the case of a perfectly symmetrical bone. The maximum breadth at the condylar processes will usually have points of contact which are unquestionably on the condyles themselves, though not necessarily on their articular surfaces. Occasionally there are excrescences on the outer surfaces of the condylar processes: usually the maximum breadth obtainable will not fall on these, but it may do so if the excrescences are exceptionally large and in such cases the excrescences are avoided in taking the measurement. If the condyles are slightly broken it may be possible to obtain a queried measurement, which should not be recorded if it is suspected that it may be more than 1 mm. in error.

$c_y l$. Maximum length of the left condyle avoiding excrescences on the process as in the case of w_1. The measurement is normally neither horizontal nor transverse. If the left condyle is defective it may be taken on the right, and this fact is noted, or the right may be used instead of the left if for any other reason it provides a more accurate measurement. A queried reading is not given if it is suspected that it may be more than 0·5 mm. in error.

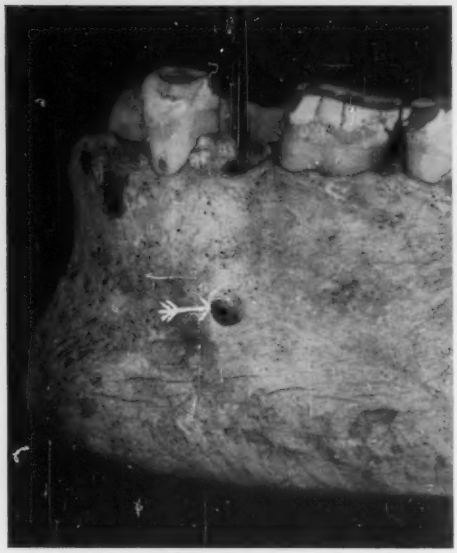
rb'. Minimum antero-posterior "breadth" of the left ramus at any inclination to the horizontal. The normal ramus has a "waist" at about the level of the molar teeth and the minimum breadth is found there. Occasionally, and more frequently for female bones, the posterior border runs downwards from the condyle without sufficient inflection to provide a minimum at that level, so that the ramus decreases continuously in breadth until the angle is reached. In such cases it is necessary to fix a point on the posterior border which may be considered to mark the union of the body of the ramus and the angle, and the minimum breadth is taken to this point, which should always be at least 13 mm. distant from the gonion (defined below). The procedure is arbitrary but it leads to reasonably consistent results in practice (see data for *rb'* in Table I above). If the left ramus is defective the measurement can be taken on the right side and this fact is noted. A queried reading is not given if it is suspected that it may be more than 0·5 mm. in error.

m_2p_1. Chord between the points on the outer left alveolar margin at the middle of the second molar (or its cavity) and at the middle of the first premolar (or its cavity). The positions of the points are first indicated by the extremities of pencil lines drawn downwards from the alveolar margins. In the case of the molar point it is often advisable to proceed in the following way: the positions of the ends of the socket are first indicated by vertical pencil lines, and with the aid of dividers a point is found on the alveolar margin equi-distant from the two points where the lines meet the alveolar margin. In the case of this tooth the alveolar margin is defined as that margin which is seen when the mandible is viewed in *norma lateralis*. If the tooth is *in situ* the middle point on the margin is determined without regard to the point of bifurcation of the anterior and posterior roots. The measurement is taken on the right side if greater accuracy is thus obtained. The alveolar margins of excavated mandibles are frequently broken to some extent, but in such cases it is often possible to obtain close approximations to the true measurements, since these are taken in directions which are roughly antero-posterior. A measurement is not given if the positions of the first premolar or second molar appear to have been modified by the ante-mortem loss of any tooth. A queried reading is not given if it is suspected that it may be more than 0·5 mm. in error.

h_1. Symphyseal height from intradental to the point farthest removed from it in the symphyseal plane. The plane in which this measurement is taken is determined by anatomical appreciation, and in the case of an asymmetrical specimen it may not coincide with the standard sagittal plane of the symphysis defined above (p. 117). A line can be drawn in pencil on the bone, starting from the process between the two central incisors, which will indicate the union of the two halves of the mandible. In tracing this line across the inferior border the *fossae digastricae* and the *spinae mentales* will be taken into account. The highest point on it, in the case of a normal mandible—i.e. the tip of the process between the central incisors—is defined to be the intradental, and the measurement is the maximum from this point to the lower border of the mandible and on the pencil line indicating the symphyseal plane*. The process on which the intradental is located is subject to considerable variation in form. If it terminates in a horizontal edge, or narrow horizontal surface, the anterior point, or mid-anterior point, on this is the accepted terminal for the measurement. If there are two tips instead of one—a rare occurrence—the point is defined to be one "in the air" mid-way between them. In all doubtful cases, such as arise when the intradental process has been partly absorbed owing to disease, or when it has been broken post-mortem, the measurement should not be recorded. A reading is not recorded if it is suspected that it may be more than 0·5 mm. in error.

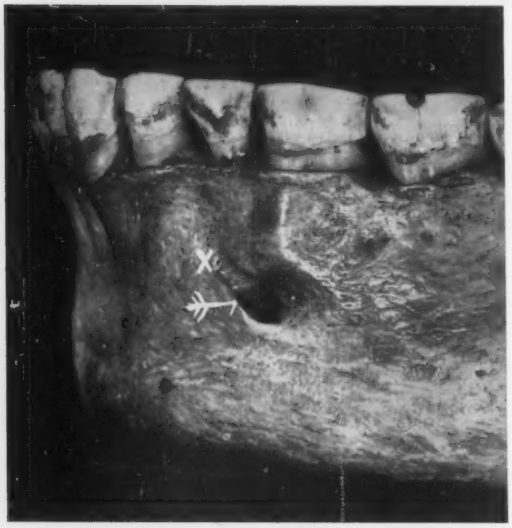
zz. Minimum chord between the anterior margins of the right and left *foramina mentalia*. Normally the mental foramen has a definite and regular anterior margin (see Plates I (α) and II) and there is no difficulty in determining the most anterior point on it which will be a terminal

* This lower terminal is not defined in exactly the same way as the gnathion of the original technique, and it is found in practice to be a more reliable point.

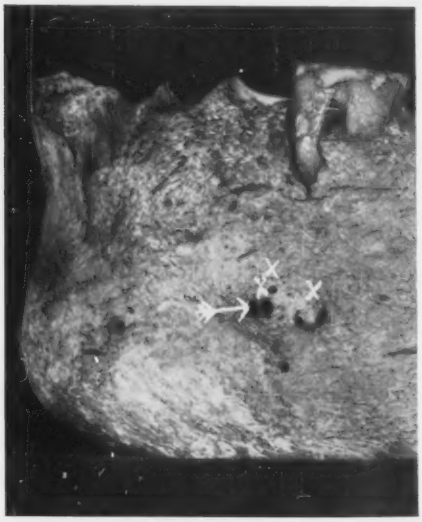


(*a*) Normal foramen.

(*b*) Foramen with depression anterior to it.



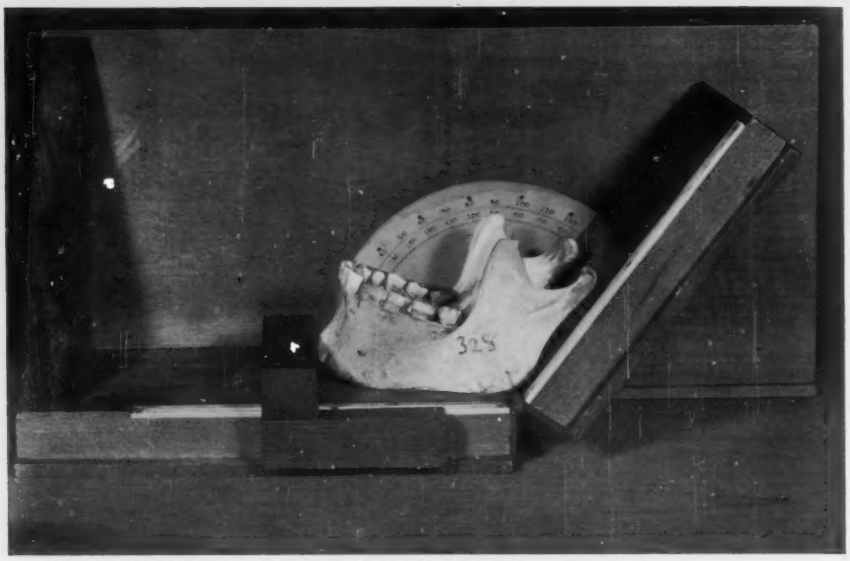
(*c*) Foramen with depression anterior to and above it.



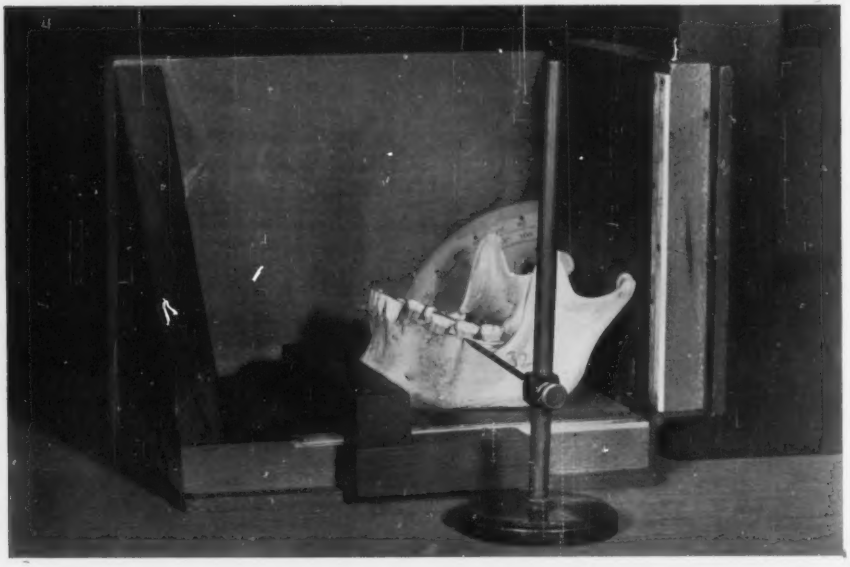
(*d*) Multiple foramina.

The Region of the left Mental Foramen, illustrating difficulties in locating the Point on it.

ate I

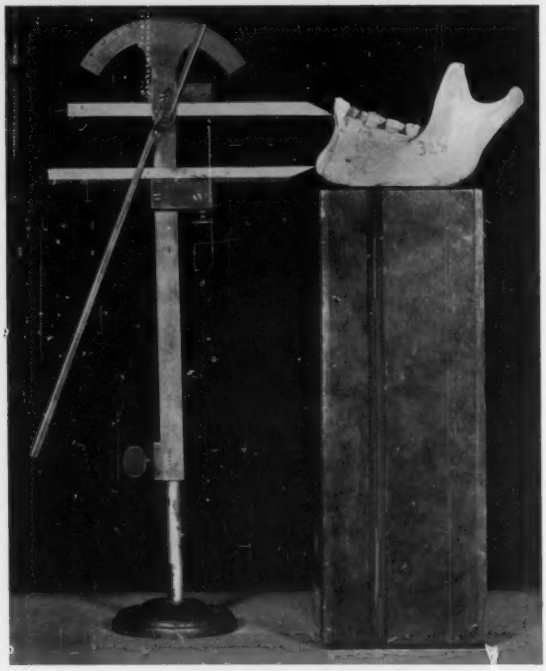


(*a*) First position.



(*b*) Second position.

The Mandible Board with Bone on it in two different Positions.



(*a*) The mental angle ($C'\angle$).

(*b*) The angle between the condylar-coronoidal line and the ramus tangent ($R\angle$).

Illustrating Methods of measuring two Angles of the Mandible.

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of the measurement*. Somewhat arbitrary instructions must be given to cover unusual anatomical variations of the region. Occasionally there is a foramen with a more or less well-marked anterior margin, but with a shallow depression anterior to it which may, or may not, have a definite anterior margin itself (see Plate I (*b*) and (*c*)). In such cases the depression is not considered in locating the terminal of the measurements. If there are two or more foramina on the same side and quite separate the point is found on the largest of these, but if they appear from external examination to be confluent the margin of the most anterior is chosen (see Plate I (*d*)). If it is still uncertain how the measurement should be taken owing to some anatomical peculiarity it is best omitted.

$c_r c_r$. Coronial breadth from right coronion to left coronion. The coronia are located in the following way. The mandible is inverted and held on a sheet of duplicating paper so that contact is made with points on both coronoid processes and with one or both condyles. Owing to asymmetry it is usually found that one condyle will then touch the paper but both will not. The mandible is moved just sufficiently to leave clear impressions on the tips of the coronoid processes and the "centres" of the small areas marked, if these are not sufficiently small to be considered "points," are the coronia. Occasionally two separated impressions are left on one of the coronoid processes: by erasing the carbon marks and repeating the procedure it may be possible to decide that one of these should be given preference, but if they are both still clearly marked the "centre" of the more anterior impression is the accepted coronion. In the case of mandibles which have become stained a dark colour a board on which white chalk has been rubbed will give clearer impressions. If one condyle is missing the other may be used to give points which can be supposed close approximations to the true coronia, but if both condyles are missing the coronia cannot be located with sufficient accuracy. The tips of the coronoid processes should be examined carefully before accepted as intact, as slight damage may make it impossible to locate the points with sufficient accuracy. A measurement is not recorded if it is suspected that it may be more than 1 mm. in error.

All the remaining measurements, except the last, are found with the aid of a mandible board which has to be adjusted in three different positions in dealing with a particular mandible. These are shown in Plate II (*a*) and (*b*) and Plate III (*b*). The board photographed there was designed by the late Professor Sir George Thane, Dr D. E. Derry and Professor Karl Pearson in consultation†. The two white strips are ivory scales having a common zero in the axis of the hinge of the movable flap, which is known as the rameal wing of the instrument. The solid set-square, which can be made to slide along the scales, gives projective lengths from the zero axis. In the first position (Plate II (*a*)) the mandible is adjusted so that the horizontal part of the board represents the standard horizontal plane, while the rameal wing is in contact with the left ramus at the condyle and above the angle and with the right ramus at one or both of these regions. If the posterior surface of the right condyle or angle is defective the position can still be approximated to with sufficient accuracy, but if both are defective to more than a slight

* The points of the calipers should just rest on the margins of the foramina, so that it is not possible to push them right inside without moving them and thus increasing the scale reading. Similarly, in taking the minimum breadth of the ramus (*rb'*) the points of the calipers should not be separated so far that they will pass over the borders of the bone.

† It was first described in a paper by Dr Derry: "A New Mylometer," *Journal of Anatomy and Physiology*, Vol. XLVIII. (1914), pp. 430—431. Copies of this board made since have been modified slightly with advantage. A screw attached to the rameal wing passes through a groove in the vertical back board, and a butterfly nut behind makes it possible to fix the wing in any desired position. In reading the protractor and mm. scales one eye only should be used, and the line of vision should be aligned with the surface of the rameal wing, or solid set-square, to which the measurement is taken. Several different forms of mandible boards have been described which, like the one here used, are all similar in principle to Broca's original instrument. As the writer has had no experience in using the other types, he cannot say whether they will serve effectively to determine some of the measurements defined in this paper or not.

extent no measurements depending on this adjustment are taken. If the right ramus is intact, but the posterior surface of the left condyle or angle is defective, the position can be approximated to by making contact between the rameal wing and the two regions on the right and the one which is intact on the left, while maintaining the standard horizontal position by pressing on the second *left* molar. Having adjusted the mandible in the first position on the board, it is made firm with the aid of plasticine and the three following measurements are taken.

M∠. Mandibular angle, i.e. the angle between the standard horizontal and standard rameal planes, the latter being represented by the rameal wing of the board. The angle is read on the semicircular scale.

$c_p l$. The projective length of the corpus. The solid set-square is brought into contact with the most advanced point in the region of the chin, which is not necessarily a point in the symphyseal plane. Plate II (*a*) shows the set-square in position for measuring $c_p l$.

rl. Projective length of the left ramus. The solid set-square is brought into contact with the left condyle by sliding it along the scale on the rameal wing of the board. If the superior surface of the left condyle is defective the measurement is taken to the right condyle and this fact is noted.

With the mandible in the first position on the mandible board, the points at the angles nearest to the zero axis of the board are defined to be the gonia. These are located by first drawing short pencil lines, roughly bisecting the mandibular angle, on the external surface of the left ramus (see Plate II (*a*)) and (after moving the rameal wing into a horizontal position without moving the bone) on the internal surface of the right ramus. The lines are drawn to indicate that their lower terminals are the points on the left and right angles nearest to the zero axis of the board. After removing and inverting the mandible, other pencil lines are drawn on either side which will generally indicate the join at the angles of the external and internal surfaces of the mandible. These latter lines can generally be traced without difficulty except in the cases of markedly everted or inverted angles, and after different measurers have had sufficient experience with normal forms it is found that they deal with the abnormal cases in ways which are closely similar (see data for $g_o g_o$ in Table I). It should be constantly remembered that the object of tracing the second pair of lines is to determine points nearest to the zero axis of the board and not to mark the union of the external and internal surfaces without reference to this consideration. The points of intersection of the first pair of lines, which may have to be continued slightly, with the second pair of lines are the gonia.

$g_o g_o$. Chord from left gonion to right gonion found with small calipers. If the measurement is uncertain, owing to difficulty in placing the mandible on the board, or to the fact that the angles are defective, it is not recorded if suspected to be more than 1 mm. in error.

In the second position the mandible is in the standard horizontal position and the rameal wing of the board is vertical. The three following measurements are then taken, but the position of the mandible relative to the board is not the same for each.

ml. Maximum projective length of the mandible read on the horizontal scale of the board with both condyles in contact with the vertical rameal wing and the solid set-square in contact with the most advanced point of the chin (see Plate II (*b*)). If either condyle is missing, or broken to a considerable extent, it is not possible to determine this measurement with sufficient accuracy, and if it is suspected that a reading may be more than 1 mm. in error owing to any cause it is not recorded.

$c_r h$. Projective height of the left coronoid process. The mandible is placed so that the left ramus is close and roughly parallel to the vertical rameal wing, on which the solid set-square is moved until its arm is brought into contact with the process. If the left coronoid process is defective—and it should be examined carefully to determine whether it is or not—the measurement is taken on the right side, but pressure is still exerted on the second left molar to give the

standard horizontal position. A reading is not recorded if it is suspected that it may be more than 1 mm. in error.

m_2h. Projective height of the corpus at the middle point of the outer alveolar margin of the second left molar. The point is the same as that used in taking the measurement m_2p, and the remarks given relating to its location need not be duplicated. But in determining m_2h damage to the alveolar margin is of far more consequence, and any distortion of the tooth owing to the ante-mortem loss of the first molar may also make it necessary to exclude the measurement. The tip of a scriber resting on the same drawing board as the mandible board is brought into contact with the point (see Plate II (*b*)) and then the height is read by moving the scriber until its tip touches the scale on the vertical rameal wing. The measurement is taken on the right side if greater accuracy is thus obtained, but pressure on the second left molar still determines the standard horizontal position. A reading is not given if it is suspected that it may be more than 0·5 mm. in error.

$R\angle$. Angle of condylar-coronoidal line with ramus tangent. This is the only measurement taken with the mandible in the third position on the board (see Plate III (*b*)). The bone is inverted so that it rests on the left coronoid and condylar processes and on one, or both, of these two on the right; while maintaining this position, it and the rameal wing are moved until the latter makes contact with the regions of the condyle and angle on the left side (i.e. the farther side as seen in Plate III (*b*)) and with the region of the angle or condyle, or of both these regions, on the right. In practice the bone is supported by the left hand of the measurer and not by plasticine as shown in the plate. The angle is then read on the scale. If the left ramus is intact but the superior extremity of either the coronoid or condylar process is defective on the right, while the posterior border of the right ramus is intact, then a sufficiently close approximation to the angle can be obtained. This is also the case if the left ramus is intact while the damage to the right is confined to the posterior border, either at the angle or at the condyle, but not at both these regions. If the right ramus is intact and the left is defective, it may be possible to determine the angle, a proper interchange of "right" and "left" being made in the instructions above. In this case the angle is said to be taken on the right side and the fact is noted. A reading is not given if it is suspected that owing to any cause it may be more than one degree in error.

$C'\angle$. Angle between the standard horizontal plane and the line joining the infradental to the most anterior point in the standard sagittal plane of the symphysis. Plate III (*a*) shows Ranke's goniometer in position for measuring the angle. The rectangular block shown, on which the mandible is adjusted in the standard horizontal position, is a makeshift*. The infradental is defined to be the mid-point of the common tangent (traced in pencil on the bone) to the two curves of the outer alveolar margins of the central incisors. If one margin is intact and the other broken a sufficiently close approximation to this point can generally be found. The goniometer is adjusted so that the tip of its upper arm is in contact with the infradental, while its arms are perpendicular to the vertical plane tangential to the posterior surfaces of the condyles. The plane in which the angle is read on the goniometer will thus coincide with the standard symphyseal plane of the mandible. The adjustment is made by eye and exact coincidence is not likely to be attained, but this fact should not affect the reading appreciably. The lower arm can still be moved up and down and towards and away from the bone. Its tip is brought into contact with the most projecting point of the chin in the plane of measurement. This point is often found to be on the outline which is seen when the mandible is viewed in

* A specially devised stand would be a great convenience in measuring $C'\angle$. The rectangular block should be permanently attached to a small drawing board on which the goniometer could stand, and two vertical boards extending above the surface on which the mandible is placed would be useful. The condyles would be brought into contact with one of these and the other, perpendicular to it, would have vertical lines close together which would make it easier to determine the most projecting point of the chin in the standard plane of the symphysis.

norma lateralis, but it may be hidden from this aspect, as when there is a median dimple, and then the mandible must be viewed obliquely. The determination of the most projecting point is always difficult and the measurer should repeat his readings of the angle on different occasions until satisfied that his "error" has been made as small as possible. The "mental angle" is the least accurate of the measurements defined above.

LIST OF PLATES.

I. The Region of the left Mental Foramen, illustrating difficulties in locating the Point on it (see p. 119 of text).

- (*a*) Normal foramen. Australian, R.C.S. No. 20·3202, male, 1·8 natural size.
- (*b*) Foramen with depression anterior to it: the anterior margin of the depression is immediately to the right of the cross. Kerma Egyptian, No. 61, male, 1·4 natural size.
- (*c*) Foramen with depression anterior to and above it: the end of the depression, which has the form of a sinus, is to the right of the cross. Australian, R.C.S. No. 20·8461, male, 1·8 natural size.
- (*d*) Multiple foramina, the point being located on the margin of the anterior foramen. The other foramina are below the crosses. Australian, R.C.S. No. 3955·3, male, 1·7 natural size.

II. The Mandible Board with Bone on it in two different Positions.

- (*a*) First position for finding the mandibular angle ($M\angle$), the projective lengths of the corpus ($c_p l$) and ramus (rl), and the positions of the gonia.
- (*b*) Second position for finding the total projective length (ml) and projective heights.

III. Illustrating Methods of measuring two Angles of the Mandible.

- (*a*) The mental angle ($C'\angle$).
- (*b*) The angle between the condylar-coronoidal line and the ramus tangent ($R\angle$).

ON THE CRANIOLOGY OF OCEANIA. CRANIA FROM NEW BRITAIN.

By Dr Med. GERHARDT von BONIN.

The object of this paper is the description of two series of crania from New Britain, one of which is in the Field Museum of Chicago, the other one in the Kaiser Wilhelm Institut für Anthropologie in Berlin-Dahlem.

To the late Dr Berthold Laufer, of the Field Museum, and to Professor Eugen Fischer, Director of the Kaiser Wilhelm Institut, and to Professor O. F. Kampmeier, head of the Department of Anatomy, of the University of Illinois, the writer wishes to express his sincerest gratitude. A grant from the Bache Fund enabled the writer to go to Europe and to examine the series reposing in Berlin-Dahlem, as well as to derive many benefits from personal contact with his friend, Dr G. M. Morant.

Both series came from the Gazelle Peninsula, the north-eastern part of the island*. (See map, p. 124.) The skulls now in the Field Museum formed originally part of Parkinson's collection which was divided after his death between the Field Museum in Chicago and the Museum für Völkerkunde in Berlin. The latter part was described by Wilhelm Müller†. The skulls in the Kaiser Wilhelm Institut were collected by Dr Thurnwald in 1910.

Lack of time and other work in Berlin made it impossible for the writer to include the female skulls, or to take all the measurements which are generally included in a biometric investigation; the usual multiplication of measurements, such as taking H and H'; O_1, O_1' and *Lacr.* O_1 and so on, had to be omitted, as well as those measurements which require a craniophor, such as OH and the profile angle. Yet even with these shortcomings many of the pooled male means for all three series can be based on more than 200 skulls.

The sexing had to be done by appreciation. At first sight this seemed to be a fairly easy job, since the sexual differences appeared to be more pronounced in the crania from New Britain than in those of many other races, e.g. in the Javanese or Andamanese. It is interesting to compare the final results with those for other races by the ratio of male to female mean, i.e. the sex ratio. This is done in Table I.

* For information about the general geographical and racial conditions of New Britain, see R. Parkinson, *Dreissig Jahre in der Südsee*, Stuttgart (1907), and W. Behmann, "Ozeanien" in *Handbuch der geographischen Wissenschaften*. Herausgegeben von Fritz Klute. Potsdam (1930).

† "Beiträge zur Kraniologie der Neu-Britannier," *Mitteilungen aus dem Museum für Völkerkunde in Hamburg. Beiheft zum Jahrbuch der Hamburgischen wissenschaftlichen Anstalten*, Band XXIII (1905), Th. 5.

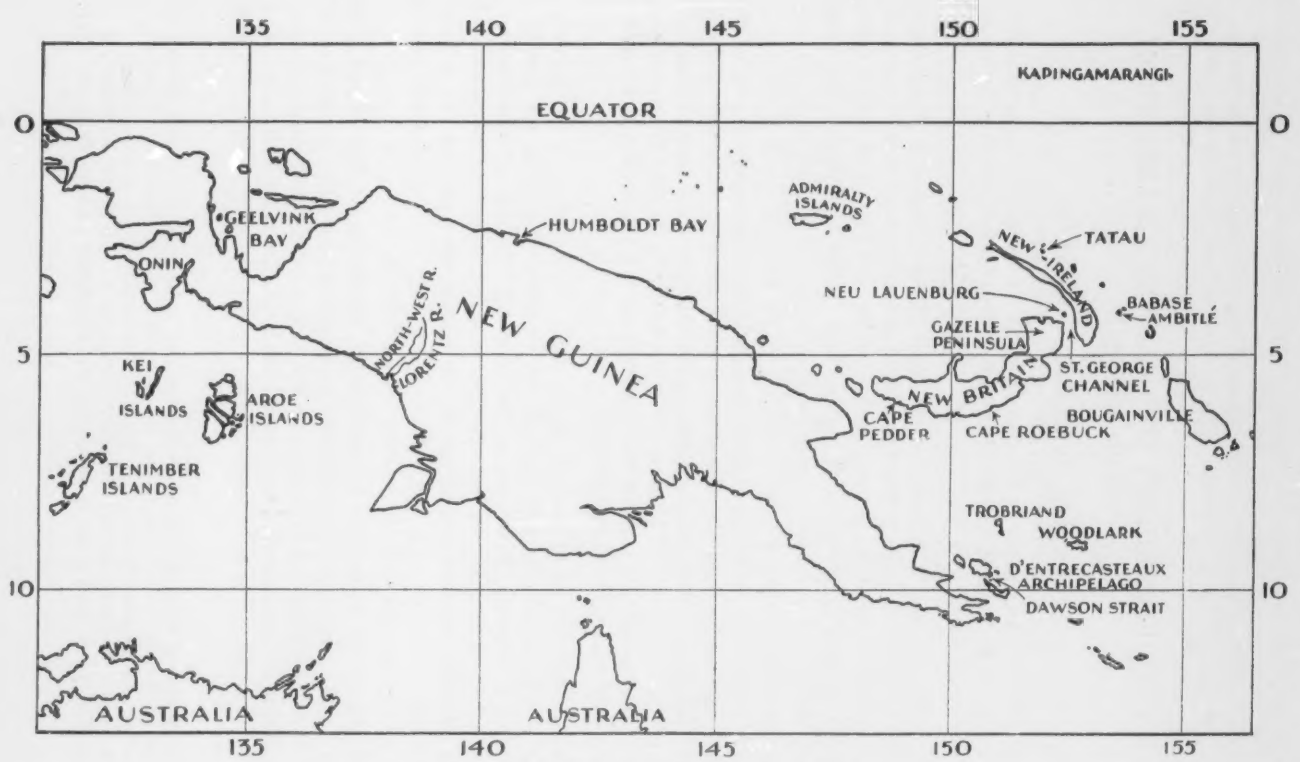


Fig. 1. Sketch Map of New Guinea and New Britain.

TABLE I.

The Sex Ratio (♂ Mean/♀ Mean) for Oceanic and other Series.*

	New British (Müller and v. Bonin)	Australians *A* (pooled)	Easter Islanders (v. Bonin)	Teita Negroes (Kitson)	Egyptians *E* (Pearson and Davin)
L	1·063	1·068†	1·062	1·046	1·047
B	1·035	1·038	1·029§	1·024	1·025
LB	1·060	1·074	1·072	1·051	1·053
B'	1·038	1·048	1·028§	1·039	1·027
H' or *H*	1·054	1·061	1·041	1·038	(*H*) 1·038
S	1·051	1·055‡	1·045†	1·037	1·034
U	1·048	1·055	1·047†	1·037	1·038

* Both means are based on 40 or more crania unless otherwise stated, and all are taken from papers in *Biometrika*.

† Female mean based on 38 crania.

‡ ,, ,, 39 ,,

§ ,, ,, 37 ,,

Interpretation of this table is difficult since the probable errors of the constants cannot be given, but it appears that the sexing has been done in the same way for the series from New Britain, the Australians, and the Easter Islanders. Two of these series have been sexed, at least partially, by the present writer, and it would evidently be reasoning in a circle if one of these were adduced as a support for the correctness with which the other was sexed. The fact, however, that the writer's results agree with those for the Australians, and also with the results of those authors (Müller and Volz) who published the other parts of the material from New Britain and from Easter Island, respectively, seems to indicate that the sexes of the skulls given here are at least approximately correct. The sex ratios for the three Oceanic series are appreciably greater than those found for the Teita Negroes and the Egyptians.

The technique employed here, including the measurement of the capacity and the construction of the type contours, is that used by the Biometric School in London, repeatedly described in the pages of *Biometrika*. An innovation was introduced in constructing the horizontal type contours. Instead of drawing the parallel $2\frac{1}{2}$ the points (*TN*, *R* and *TN*, *L*) in the temporal region nearest to the median sagittal axis (*FO*) were found by drawing on the individual contours and their average positions are shown on the types. A comparison of the direct measurements with those measured on, or computed for, the construction of the type contours is given in Table II. All contour measurements except *OH* have been taken from the sagittal contour; *OH* is obtained as *MA* from the transverse contour, though these two do not correspond exactly. The contour *L* is determined by means of a compass, one arm of which is put on the glabella, while the other seeks the most distant point on the occiput. The lengths *G'H*, *GL* and *LB*

TABLE II.

A Comparison of Mean Caliper (or Craniophor) and Type Contour Measurements for the New Britain Series at Chicago.

Character	Male		Female	
	Type Contour	Caliper	Type Contour	Caliper
L	183 * (62)	184·6 (63)	172 * (37)	173·4 (43)
H'	132·6‡ (62)	135·6 (63)	127·6‡ (35)	128·9 (41)
OH	115·0† (62)	115·6 (60)	110·2† (37)	—
LB	99·9† (62)	101·0 (62)	93·9† (35)	95·0 (42)
G'H	67·3† (62)	67·4 (61)	63·5† (35)	63·9 (39)
GL	103·3† (62)	103·7 (61)	99·1† (33)	99·2 (38)
fml	34·9‡ (62)	34·8 (63)	33·7‡ (35)	32·5 (37)
S_1'	106·7‡ (62)	109·3 (63)	103·8‡ (37)	105·2 (42)
S_2'	117·1‡ (62)	117·3 (63)	111·1‡ (37)	110·7 (42)
S_3'	96·7‡ (62)	96·6 (63)	93·3‡ (37)	93·9 (41)

* Measured on the type contour.
† Used in the construction of the type contour.
‡ Calculated from lengths used in the construction of the type contour.

have actually been measured on the individual contours, and their means have been used to construct the type contour. The remaining measurements can be found by calculation from the co-ordinates used in the construction of the types. Most of the measurements agree as closely as can be desired, yet there is a tendency for the type contour measurements to be smaller than the corresponding mean caliper lengths. This need not be supposed due to any faults in the instruments, as it may rather be taken as a consequence of the fact that the skulls are never absolutely symmetrical. It is sometimes needful to raise or lower the point of the tracer slightly, and in so doing the point on the skull will generally be projected on the paper in a position somewhat inside that which it would have occupied had it been exactly in the mid-sagittal plane. The bad disagreement in the case of H' and S_1' may be due to the fact that the bregma was located differently on the two occasions in the case of some of the skulls.

Our first task is to determine whether the three series mentioned above—they will be called henceforth the Berlin (Parkinson), the Berlin (Thurnwald) and the Chicago series*—can be pooled or not. It was not known whether Parkinson and Thurnwald had obtained samples from the same population, nor was it above doubt whether Müller had followed the same technique as the present author. Müller states that he adhered to v. Luschan's technique which had been published by that

* Or, alternatively, the Berlin (Müller), the Berlin (von Bonin) and the Chicago (von Bonin); see the correspondence of collectors and measurers in the headings to Table III.

author in Neumayer's Handbuch*. It is clear from that account that some of the measurements have been taken in a way different from that employed for the other two series. Thus the capacity has been measured by shot, and we find indeed that, at least for the male series, Müller's values are decidedly higher than those of the present writer. Von Luschan's definitions of the nasal height and palatal length are unusual ones, and hence no comparisons can be made between Müller's values and those determined in accordance with the biometric technique.

The following corrections were made in Müller's tables:

♀ 561: for S read 352 instead of 302.

♀ 569: The indices given have slipped one column to the right, thus 73·99 is not $100H'/L$, but $100B/L$, and so on.

♀? 1658: S_3' (= 184) has been omitted. Since S_3 is given as 104 it seemed unsafe to read 0 instead of 8.

♂ 547 has been omitted altogether, the direct measurements (C=925, L=162) indicate a pathological (microcephalic) skull.

♂ 1634 *fml* (= 25) has been omitted. Misprint for 35?

♂ 532 *Querumfang* (= 383) has been omitted.

♂ 537 *fmb* (= 37) has been omitted. Misprint for 27?

The means of the three male and the two female series are given in Table III, and in computing for Müller's series the skulls he measured at Hamburg were omitted. The series were compared by the method of the Coefficient of Racial Likeness (C.R.L.)†. For this purpose the standard deviations of the Egyptian E series were used. They are almost as low as those of any other series recorded, and it is shown below that they are very close to the values given by the pooled New Britain series. Müller has given two female series, one headed "females" and the other "doubtful females". In the first female column of Table III the means derived from the female group alone are given, in the second the means based on both "certain" and doubtful females. As was to be expected, the latter are almost throughout higher for absolute measurements than the first, but they are also nearer the values obtained from the series measured by the present writer. The values of the function α between this longer series of Müller and the series from Chicago are given in the last column of the table. The C.R.L.s computed are given in Table IV, the lower left divisions of which give the values for the function α which are > 9, i.e. which are commonly regarded as significant.

* *Anleitung zu wissenschaftlichen Beobachtungen auf Reisen*. 3. Auflage (1905). Von Luschan's technique is also given in a paper in the *Korrespondenzblatt der Deutschen Gesellschaft für Anthropologie, Ethnologie und Urgeschichte*, Jahrgang XXXVII. (1906) entitled "Die Konferenz von Monaco." Müller says (S. 73): "Das Messschema ist das von Herrn Prof. von Luschan in Übereinstimmung mit den Herrn Prof. Martin und Manouvrier gelehrte." For several characters the definitions given by these three anthropologists are identical for all practical purposes, but in the case of others—e.g. the orbital breadth and palatal length—they all used different definitions. Where doubt arises it has been assumed that Müller followed von Luschan's definitions.

† This method has been explained repeatedly in the pages of *Biometrika*. The present writer has given the formulae used in his work in a previous paper on the crania of the Easter Islanders, Vol. XXIII. (1931), p. 253.

TABLE III. *Means of Series from New Britain.*

Collection at: Collected by: Measured by:	Males			Females			
	Berlin Parkinson Müller	Chicago Parkinson von Bonin	Berlin Thurnwald von Bonin	Berlin (A) ♀ Parkinson Müller	Berlin (B) ♀+♀? Parkinson Müller	Chicago (C) Parkinson von Bonin	α (B and C)
C	1367·1 (97)	1285·7 (55)	—	1220·7 (30)	1216·6 (44)	1207·7 (30)	·16
L	184·1 (102)	184·6 (63)	184·2 (64)	172·7 (34)	173·3 (49)	173·4 (43)	·01
B	133·0 (102)	132·5 (63)	131·3 (64)	126·8 (34)	127·2 (50)	128·8 (43)	2·90
H'	134·3 (101)	135·6 (63)	134·5 (62)	126·6 (33)	126·9 (48)	128·9 (41)	4·62
B'	93·3 (102)	93·3 (62)	93·4 (64)	90·0 (34)	90·3 (50)	89·4 (43)	1·30
LB	101·3 (100)	101·0 (62)	100·2 (62)	95·1 (33)	95·4 (47)	95·0 (42)	·29
S	374·7 (102)	374·7 (63)	374·3 (63)	355·0 (32)	355·3 (47)	357·9 (41)	1·34
U	508·6 (102)	507·3 (63)	508·0 (64)	483·5 (32)	484·5 (47)	484·7 (42)	·00
Q'	312·7 (101)	308·8 (61)	—	297·4 (32)	298·1 (46)	—	—
Bregmatic *Q'*	—	304·9 (63)	303·4 (64)	—	—	292·6 (42)	—
fml	33·7 (98)	34·8 (63)	34·6 (62)	32·7 (33)	32·4 (47)	32·5 (37)	·04
fmb	28·0 (98)	28·5 (63)	28·4 (62)	26·8 (33)	26·6 (47)	27·2 (37)	1·86
GB	96·3 (97)	97·3 (59)	97·7 (63)	90·7 (29)	91·1 (44)	91·7 (42)	—
G'H	68·3 (97)	67·4 (61)	67·6 (59)	63·0 (30)	63·3 (46)	63·9 (39)	·54
J	134·5 (93)	136·5 (53)	135·4 (57)	124·6 (26)	125·0 (41)	126·3 (38)	1·74
GL	107·1 (96)	103·7 (61)	105·4 (58)	100·8 (29)	101·0 (43)	99·3 (38)	—
NH'	[48·7 (100)]*	48·0 (62)	47·1 (63)	[45·5 (31)]*	[45·5 (47)]*	45·4 (42)	—
NB	26·4 (97)	27·3 (62)	26·9 (63)	25·2 (30)	25·0 (46)	26·2 (42)	11·76
O_1, *R*	—	44·3 (62)	43·6 (64)	—	—	41·9 (41)	—
O_1', *R* or *L*	40·2 (101)	41·4 (50) *R*	—	38·0 (30)	38·4 (46)	39·1 (32) *R*	3·81
O_2, *R* or *L*	33·1 (101)	32·8 (62) *R*	33·4 (64) *R*	32·1 (30)	32·2 (46)	33·0 (41) *R*	4·10
G_1	—	55·7 (56)	56·5 (49)	—	—	52·3 (37)	—
G_1'	—	50·9 (56)	—	—	—	48·5 (38)	—
G_2	41·2 (99)	41·7 (49)	40·6 (48)	38·6 (29)	38·3 (44)	38·7 (37)	·50
100 *B*/*L*	72·3 (102)	71·8 (63)	71·3 (64)	73·5 (34)	73·4 (49)	74·3 (43)	2·84
100 *H'*/*L*	73·1 (101)	73·5 (63)	73·0 (62)	73·4 (33)	73·3 (47)	74·5 (41)	3·88
100 *B*/*H'*	99·5 (101)	97·7 (63)	97·7 (62)	99·7 (33)	99·9 (48)	99·9 (41)	·00
Oc. I.	59·0 (102)	59·1 (63)	59·4 (63)	60·0 (33)	59·7 (46)	60·1 (38)	·34
100 *G'H*/*GB*	70·9 (94)	69·4 (57)	69·3 (58)	69·2 (28)	69·2 (43)	69·7 (39)	·26
100 *NB*/*NH'*	[54·3 (96)]†	57·2 (62)	57·4 (63)	[55·7 (28)]†	[55·3 (44)]†	58·0 (42)	—
100 O_2/O_1, *R*	—	74·0 (62)	76·7 (64)	—	—	78·8 (41)	—
100 O_2/O_1', *R* or *L*	82·4 (101)	79·0 (50) *R*	—	84·6 (30)	83·9 (46)	84·3 (32) *R*	·15
100 G_2/G_1	—	75·2 (49)	71·1 (42)	—	—	73·9 (35)	—
100 G_2/G_1'	—	82·1 (49)	—	—	—	79·6 (36)	—
100 *fmb*/*fml*	{83·1 (98)}‡	82·2 (63)	82·2 (62)	{82·0 (33)}	{82·1 (47)}	83·8 (37)	1·88
A∠	{66°·3 (96)}	68°·6 (61)	66°·7 (58)	{66°·4 (29)}	{66°·5 (43)}	66°·9 (38)	·28
N∠	{75°·5 (96)}	73°·0 (61)	75°·1 (58)	{76°·2 (29)}	{76°·1 (43)}	74°·8 (38)	3·21
Alveolar *P*∠	78°·0 (90)	79°·1 (58)	—	77°·1 (28)	77°·6 (40)	—	—

* Von Luschan's nasal height, recorded by Müller, is from the nasion to the tip of the anterior nasal spine, while the other *NH*''s in the Table are for the chord from the nasion to the "base" of the spine.

† For von Luschan's nasal height.

‡ The indices and angles in curled brackets were found from the mean values of the component lengths instead of from individual values.

It is seen from this table that there is a very low coefficient of racial likeness between the two series at Berlin measured by Müller and by the present writer respectively. No significant differences are found between the 22 characters compared, and this suggests that the measurements were taken by the two workers in ways which may be supposed identical for practical purposes. Neither the capacities nor the orbital breadths and indices could be included in that comparison, however, and these three characters show significant differences between the Berlin (Müller) and the Chicago Series. But it may be seen from Table III that if the dacryal orbital breadth had been found for the Berlin series measured by the present writer the mean would probably not have been significantly different from that given by Müller, and the same supposition may be made in the case of the missing orbital index for the dacryal breadth. The

TABLE IV.

Coefficients of Racial Likeness between Male Series from New Britain (upper right C.R.L.s, *lower left values of* $a > 9$).

	Berlin (Müller)	Chicago (von Bonin)	Berlin (von Bonin)
Berlin (Müller)	—	4·13 ± ·18 (27)†	0·32 ± ·20 (22)
Chicago (von Bonin)	*NB* 9·76, *A*∠ 15·11 100 O_2/O_1' 15·15, O_1' 17·15 *C* 18·04, *N*∠ 19·62	—	1·24 ± ·18 (29)
Berlin (von Bonin)	—*	100 O_2/O_1 9·00 *N*∠ 10·85	—

* No comparisons of capacities or of orbital breadths and indices can be made between the Berlin (Müller) and Berlin (von Bonin) series.

† The female C.R.L. corresponding to this is 0·91 ± ·19 for 25 characters. The *a*'s are given in the last column of Table III and that for *NB* (11·76) is the only one greater than 9.

significant difference found for the mean capacities is almost certainly due to the fact that the character was measured in different ways by Müller and the present writer, since their calvarial lengths, breadths and heights show very small differences. Omitting the capacity, the coefficient between the Berlin (Müller) and the Chicago Series is reduced to 3·63 ± ·19 for 26 characters. We may conclude that the two New Britain Series at Berlin represent the same racial type, while that at Chicago represents another, but closely allied, population. The fact that the female series at Chicago and the female series at Berlin measured by Müller have a very low coefficient emphasizes the close relationship between the two series. It was felt that there was sufficient justification to combine the three New Britain Series, and the pooled means are given in Table V, Müller's capacities having been omitted. The type contours (Figs. 2—7) can only be based on 62 male and 37 female skulls of the Chicago series.

‡ The indices and angles in curled brackets were found from the mean values of the component lengths instead of from individual values.

TABLE V.

Mean Measurements of Crania from New Britain.*

Character	Male	Female	Character	Male	Female
C	1285·7 (55)	1207·7 (30)	O_1, *R*	43·9 (126)	41·9 (41)
L	184·3 (229)	173·3 (92)	O_1', *R*	40·6 (151)	38·7 (78)
F	—	172·6 (43)	Lacr. O_1, *R*	40·2 (49)	38·2 (33)
B	132·4 (229)	127·9 (93)	O_1, *L*	44·4 (60)	41·3 (41)
B''	112·4 (99)	106·1 (48)	O_1', *L*	41·3 (52)	38·8 (31)
B'	93·3 (228)	89·9 (93)	Lacr. O_1, *L*	40·0 (52)	37·8 (31)
EOW	108·6 (59)	101·7 (42)	O_2, *R*	33·1 (227)	32·6 (87)
IOW	102·1 (59)	95·5 (42)	O_2, *L*	32·7 (60)	32·7 (42)
Biasterionic *B*	107·8 (102)	103·2 (49)	G_1	56·1 (105)	52·3 (37)
H'	134·7 (226)	127·8 (89)	G_1'	50·9 (56)	48·5 (38)
H	137·5 (61)	—	G_2	41·2 (196)	38·5 (81)
OH	115·6 (60)	—	*EH*	11·3 (48)	10·5 (31)
LB	100·9 (224)	95·2 (89)	$100\,B/L$	71·9 (229)	73·8 (92)
S_1'	109·7 (229)	104·3 (88)	$100\,H'/L$	73·2 (226)	73·9 (88)
S_2'	117·4 (229)	110·5 (86)	$100\,B/H'$	98·5 (226)	99·9 (89)
S_3'	96·1 (228)	93·7 (84)	$100\,(B-H')/L$	{−1·6 (226)}	{0·0 (89)}
S_1	124·5 (229)	118·6 (86)	*Oc. I.*	59·1 (228)	59·9 (84)
S_2	132·5 (229)	125·1 (86)	$100\,fmb/fml$	{82·6 (223)}	{82·8 (84)}
S_3	117·5 (228)	113·0 (85)	$100\,G'H/GB$	70·0 (209)	69·4 (82)
S	374·6 (228)	356·5 (88)	$100\,NB/NH$, *R*	53·8 (61)	54·1 (40)
U	507·8 (229)	484·6 (89)	$100\,NB/NH$, *L*	[53·5 (61)]	[53·5 (41)]
Glabella *U*	518·7 (63)	490·6 (42)	$100\,NB/NH'$	57·3 (125)	58·0 (42)
Q	311·2 (162)	298·1 (46)	$100\,O_2/O_1$, *L*	73·8 (60)	79·1 (41)
Bregmatic *Q*	304·1 (127)	292·6 (42)	$100\,O_2/O_1$, *R*	75·4 (126)	78·8 (41)
fml	34·3 (223)	32·4 (84)	$100\,O_2/O_1'$, *R*	81·3 (151)	84·1 (78)
fmb	28·3 (223)	26·9 (84)	$100\,O_2$/Lacr. O_1, *R*	81·0 (49)	86·7 (33)
G'H	67·8 (217)	63·6 (85)	$100\,G_2/G_1$	73·3 (91)	73·9 (35)
GL	105·7 (215)	100·2 (81)	$100\,G_2/G_1'$	82·1 (49)	79·6 (36)
J	135·3 (203)	125·6 (79)	$100\,EH/G_2$	27·1 (48)	26·9 (31)
GB	97·0 (219)	91·4 (86)	Alveolar $P\angle$	78°·4 (148)	77°·6 (40)
NH, *R*	51·0 (61)	48·8 (40)	Prosthion $P\angle$	76°·8 (56)	—
NH, *L*	51·0 (61)	49·0 (41)	$N\angle$	{74°·7 (215)}	{75°·5 (81)}
NH'	47·5 (125)	45·4 (42)	$A\angle$	{67°·1 (215)}	{66°·7 (81)}
NB	26·8 (222)	25·6 (88)	$B\angle$	{38°·2 (215)}	{37°·8 (81)}
PH	18·8 (25)	17·7 (38)	Weight	622·1 (100)	499·8 (44)
SC	8·9 (100)	8·2 (46)			

* The means in this table were calculated from the measurements of individual crania provided by Müller and the present writer, the question of which of their measurements may be supposed comparable having been discussed in the text. The indices and angles in curled brackets were found from the individual values given in Appendix II below and the mean values derived from the means of the component lengths calculated for Müller's measurements. The indices in square brackets were found from the means of the component lengths, no individual indices having been calculated.

The description of the New Britain crania will be based primarily on the male type. There can be little doubt that the male type shows racial peculiarities more prominently than the female. Moreover, for most races there are more male than female crania available, hence racial comparison is generally based on male series.

When all the New Britain crania are laid out on the table in the laboratory, two things strike the eye of the observer: they all look very much alike, and there

is a strangely savage look about them. One has at once the impression that one is dealing with a homogeneous population, but with a population which appears to differ markedly from other races. But these are vague impressions; how far is a deeper study able to substantiate them?

It is far easier to test the first point raised, i.e. the question of variability. Although the actual arithmetical work may be laborious, the method is nevertheless clear: the computation of the standard deviations for the various measurements. Table VI gives these of a number of characters for the New Britain Series as well

TABLE VI.

Variability: Standard Deviations of Male Series.

Character	New Britain*	Egyptian *E* Series†	Australians *A*‡
L	5·70 ± ·18	5·72 ± ·09	6·67 ± ·35
B	4·36 ± ·14	4·76 ± ·08	4·95 ± ·19
B'	4·85 ± ·15	4·05 ± ·06	5·36 ± ·22
H'	4·41 ± ·14	5·03 ± ·08§	4·94 ± ·20
LB	3·99 ± ·13	3·97 ± ·06	4·34 ± ·18
U	13·82 ± ·44	13·77 ± ·22	16·50 ± ·67
S	11·80 ± ·38	12·51 ± ·20	13·62 ± ·67
J	5·02 ± ·17	4·57 ± ·08	6·07 ± ·25
G'H	3·96 ± ·13	4·15 ± ·07	4·21 ± ·23
GL	4·46 ± ·20	4·85 ± ·08	5·88 ± ·27
GB	4·28 ± ·14	4·67 ± ·08	—
O_2, *R*	1·84 ± ·06	1·91 ± ·03	2·08 ± ·09 (*R* or *L*)
G_1	3·09 ± ·14	3·33 ± ·06	—
G_2	2·93 ± ·10	2·63 ± ·05	—
fml	2·28 ± ·07	2·47 ± ·04	2·52 ± ·11
fmb	1·93 ± ·06	2·15 ± ·03	2·17 ± ·10
100 *B*/*L*	2·66 ± ·08	2·68 ± ·06	3·88 ± ·19
100 *H'*/*L*	2·37 ± ·08	2·94 ± ·05§	3·70 ± ·19
100 *B*/*H'*	3·95 ± ·12	4·30 ± ·06§	5·41 ± ·21
100 (*B* − *H'*)/*L*	2·85 ± ·09	—	—
Oc. I.	2·28 ± ·08	3·30 ± ·05	—
100 *G'H*/*GB*	4·48 ± ·15	4·96 ± ·08	—
100 O_2/O_1, *R*	4·76 ± ·20	5·05 ± ·08	5·61 ± ·29‖
100 G_2/G_1	5·75 ± ·29	6·79 ± ·13	6·98 ± ·45¶
100 *fmb*/*fml*	5·82 ± ·19	5·79 ± ·09	5·83 ± ·26

* The numbers on which the New Britain standard deviations are based can be seen in Table V.
† Pearson and Davin, *Biometrika*, Vol. XVI. (1924).
‡ Morant, *Ibid.*, Vol. XIX. (1927).
§ For *H* in place of *H'*.
‖ For 100 O_2/*Lacr.* O_1 in place of 100 O_2/O_1, *R*.
¶ For Broca's palatal index in place of 100 G_2/G_1.

as for the long *E* Series of 26th—30th dynasty Egyptian crania, published by Pearson and Davin, and for the Australian *A* collected from the literature by Morant. The series we are dealing with here is somewhat less variable than the Egyptian *E*

Series and decidedly less variable than the Australian. The following significant differences (Egyptian – New Britain Series) are found:

$$B', \Delta = -\cdot80 \pm \cdot161;\ H', \Delta = +\cdot62 \pm \cdot16;\ 100\,H'/L, \Delta = +\cdot57 \pm \cdot083;$$
$$Oc.\ I, \Delta = +1\cdot02 \pm \cdot083.$$

To substantiate the second point—the impression that the New Britain type is a very peculiar one—is impossible. We shall learn, on the contrary, that these crania are, in spite of superficial impressions, not as different from those of other races as might be assumed.

In order to compare in a general way the proportion of face and brain box, we may consider an index formed from the bizygomatic breadth and the maximum breadth of the skull, defined by the formula $100\,J/B$. Individual values of this "transverse cranio-facial index" have not been given for several of the races. It has been calculated from the male means for a number of series, without unduly weighting the European races (Table VII)*.

It appears that this index is capable of differentiating some "primitive," phaenozygous races from the rest, but it leaves this "rest" in a rather bewildering disorder. The value for the inter-racial standard deviation of this sample which, as will be noticed, includes the Neanderthaloid specimen of La Chapelle-aux-Saints, is given at the bottom of the table. The intra-racial standard deviation has been found by Davidson Black for several Chinese series†, and its value has also been worked out for the present series. We find:

Standard deviation of the Index ...	$100\,J/B$,
Inter-racial Standard Deviation ...	$3\cdot57 \pm \cdot24$,
Northern Chinese (Davidson Black)	$4\cdot20 \pm \cdot22$,
New Britain Series	$3\cdot92 \pm \cdot16$.

A short calculation will show that all these values are insignificantly different from each other‡.

There appears to be in the New Britain Series a greater protrusion and a more accentuated setting off of the face than is found in most races. The first point is brought out by the profile as well as by the nasal angle, while the "setting off"

* The means in this table are nearly all taken from earlier papers in *Biometrika*, or from tables to be given in the second part of the present paper. The means for the European and Egyptian series are given by Morant (1928), those for the Asiatic series by the present writer (1931) and Morant (1924), and those for African Negro Series are from *Biometrika*, Vol. VIII. (1912), pp. 298—299. The Eskimo means are taken from *Annals of Eugenics*, Vol. I. (1926), p. 268 and the La Chapelle measurements are from Vol. II. (1927), p. 376 of the same journal. The Moriori means are not those to be given in the second part of the present paper, but they are quoted from *Biometrika*, Vol. XI. (1915), p. 93. All the means in the table are based on more than 30 crania except the Tibetan B, the J in this case being for 15 specimens and the B for 14.

† *Palæontologia Sinica*, Series D, Vol. VI. (1928).

‡ From the coefficients of variation of J, B and $100\,J/B$ (not given in Table VII) we deduce an inter-racial coefficient of correlation between J and B $+\cdot475 \pm \cdot084$. This is again only insignificantly different from the intra-racial $r = +\cdot373 \pm \cdot021$ given by Pearson and Davin for the E Series.

TABLE VII.

Transverse Cranio-Facial Index, computed from Male Means for various Races.

Series	*J*	*B*	100 *J*/*B*
Loyalty Islander	136·6	130·2	104·9
Greenland Eskimo	139·5	134·4	103·8
New Caledonian	136·6	132·2	103·3
New Britain	135·3	132·4	102·1
Australian *A*	133·6	132·2	101·1
Easter Islander	134·3	133·3	100·8
Tibetan *B*	137·5	139·4	98·6
Maori	136·4	138·7	98·3
Pesechem (New Guinea)	124·2	127·1	97·7
Kaffir	134·1	137·4	97·6
Northern Negro	129·5	133·15	97·3
La Chapelle-aux-Saints	152·0 ?	156·2	97·3
Moriori	137·4	141·4	97·2
Aino	137·3	141·2	97·2
Sardinian	127·7	132·3	96·5
Tasmanian	131·0	136·0	96·3
Dayak	132·9	138·2	96·2
Northern Chinese: Peking	132·7	138·2	96·0
Nepalese	127·2	132·6	95·9
Japanese	134·1	139·9	95·9
Gaboon	129·0	135·5	95·2
Predynastic Egyptian: Naqada *A* and *Q*	125·9	132·7	94·9
Tagal (Philippine Islands)	131·7	138·9	94·8
Reihengraeber	132·2	139·8	94·6
Tibetan *A*	131·0	138·7	94·4
Southern Chinese: Fukien	132·6	140·9	94·1
Anglo-Saxon	133·3	141·7	94·1
British Neolithic	130·4	138·9	93·9
Javanese: Bantam and Batavia ...	132·0	140·8	93·8
Guanche	134·5	143·8	93·5
Egyptian: 26th—30th Dynasty... ...	128·7	138·9	92·7
Mediaeval Bohemian	131·5	141·9	92·7
Whitechapel English	130·1	140·7	92·5
Swedish Prehistoric	129·2	139·7	92·5
Serbo-Croat	136·5	147·5	92·5
Aëta (Philippine Islands)	132·8	143·5	92·5
British Iron Age	130·6	141·4	92·4
Etruscan	131·9	143·7	91·8
Congo Negro	126·5	138·5	91·3
Farringdon Street English	131·0	142·4	91·3
French	130·7	143·4	91·1
Coptic	127·9	140·95	90·7
Badensian	134·7	148·6	90·6
Andamanese	123·3	136·4	90·4
Great Russian	128·8	142·65	90·3
Württemberger	133·3	147·9	90·1
Alsatian	133·0	147·7	90·0
Basque	128·8	143·5	89·8
Vorarlberg	131·8	148·0	89·1
Burmese *A*	134·0	143·7	87·8
Inter-racial Mean (50)	132·35 ± ·43	139·58 ± ·52	94·8 ± ·34
„ σ	4·47 ± ·39	5·48 ± ·37	3·57 ± ·24

is demonstrable in the horizontal type contour. The profile angle, measuring the slope of the line connecting the nasion with the alveolar point (or, alternatively, the prosthion) relative to the Frankfurt Horizontal, is rather small while the nasal angle, indicating the slope of the same line in relation to the base of the skull (i.e. the angle *Bas., N, Alv. P.*), but measured in the opposite sense to the profile angle, is correspondingly high. It has been pointed out by Morant in his memoir on the Neanderthaloid skulls that the nasal angle is more capable than any other single measurement he was able to find of arranging the modern races in an order which would appear suggestive.

The horizontal contours (Figs. 4 and 5) show a pronounced narrowing behind the temporal ridges as can readily be seen when the contour types for the New Britain Series are compared with others available in *Biometrika*. Thus the New Britain contours may be compared with the contours of the Sedment Egyptian crania published by Woo* which are of almost the same lengths (*FO* Sedment males 180·6, New Britain males 181·0).

The cranial capacity is low. It has been shown elsewhere† that its value is not predicted correctly by Hooke's regression formula which is reasonably correct for the majority of human cranial series thus far known. A regression formula worked out from the present series was given there; we must regard the "ill-filled" skull as one of the racial characteristics of the natives of New Britain.

We give some of the values for the characters of the frontal bone which can be deduced from measurements taken on the sagittal and horizontal male type contours:

Rotation of base of frontal bone	51°·4
Angle of frontal bone flatness...	25°·3
100 Subtense $N\beta/N\beta$... ...	21·2
Index of frontal flattening ...	10·4
$100\,\frac{\frac{1}{2}(TRx+TLx)}{TRy+TLy}$	18·4

If they are compared with those given by Morant in his memoir on the Neanderthaloid skulls‡, it will be found that they are all within the ranges of distribution previously found for modern races of mankind.

An attempt was made in the case of the male New Britain crania at Chicago to grade the development of the supraorbital ridges in the way indicated by the figures in Martin's *Lehrbuch*, S. 876 (2nd edition). Marks in the table of individual measurements, Appendix II A, were given: 1 for the stages depicted in Martin's uppermost picture, then grade 2, and, when a true *torus supraorbitalis* was present, 3. It is, we suspect, the high development of the supraorbital ridges, more than any other single feature, which has led some of the fathers of Anthropology to the

* *Biometrika*, Vol. XXII. (1930), p. 85.

† *Journal of Comparative Neurology*, Vol. LIX. (1934), pp. 1—28.

‡ *Annals of Eugenics*, Vol. II. (1927), pp. 355, 356 and 367.

belief that the race to which the New Britain crania belong forms a particularly primitive branch of the human family.

The state of the pteric region in 58 male Chicago crania can be tabulated as follows, five of the total 63 specimens being omitted from the count as they have the sutures closed or partly closed*:

	Right only	Right and left	Left only
Os epiptericum (single or multiple) ...	5	1	3
Sut. fronto-temp. or proc. front. ossis temp.	5	4	2
Stenokrotaphie	2	1	1

The cerebral relief in the temporal region, i.e. the protuberances and impressions on the outer surface of the cranium corresponding to the third frontal gyre and the Sylvian fissure of the brain, has been studied in detail by Schwalbe†. The writer has tried to follow Schwalbe in estimating the degree of development of these structures. In the remarks on individual specimens, 1 stands for poor development, 4 for a very pronounced and 2 and 3 for intermediate development. In most New Britain skulls the relief on the outside of the skull is well marked, and a careful observer would detect all the details to which Schwalbe has called attention. It seemed, however, that an observation of the two features just mentioned would suffice to give a general indication of the degree of development of the cerebral relief. The position of the frontal prominence was ascertained first, and then the position of the Sylvian impression found. This impression is not coincident with the point of greatest narrowing behind the temporal crests seen in the contour. It is almost invariably (except for perhaps two or three cases) behind this, at about the place where the ordinate 3 of the horizontal type contour meets the outline. The grades allotted to the individual skulls cannot, of course, be compared with those given by other authors, and in particular with those given by Schwalbe himself, since they are purely arbitrary and uncontrolled. We may, however, compare the right and left sides as is done below.

The configuration of the glenoid fossa calls for a few words. It is well known, particularly since Boule's work, that there is a marked difference between *homo sapiens* and the anthropoids in this region. The apes have a shallow fossa, restricted to the temporal bone in a transverse direction, and bordered by a well-pronounced

* Remarks on the conformation at the pterion of the individual specimens are given in the table of individual measurements (Appendix II A). By a *sutura fronto-temporalis* is meant a suture between the two bones not associated with a definite process of either, and by a *processus frontalis ossis temporalis* a definite process of the temporal bone reaching to the frontal. In both of these cases there is contact between the two bones for an appreciable length. By *stenokrotaphie* is here meant "pterion in *K*," or the condition of a process of the temporal bone just touching the frontal so that it cannot be said that there is a suture between them. In the summary in the text only two crania needed to be entered twice (see the remarks on the individual specimens), but otherwise the entries are single.

† "Über das Gehirn-Relief der Schläfengegend des menschlichen Schädels." *Zeitschrift für Morphologie und Anthropologie*, Bd. x. (1907), S. 1—93.

post-glenoidal apophysis behind; man has a deep fossa, the medial end of which usually lies on the sphenoid bone and is formed by a *spina angularis*, and at the posterior border of which there is only a rudimentary indistinct post-glenoidal apophysis. A glance at the figures given by Boule in his memoir* will bring this back to the reader's mind. In the New Britain Series, the post-glenoidal apophysis is comparatively well developed. The writer must ask the reader to accept this statement, since there appears to be no method applicable to a long series of skulls which could express it numerically. The presence or absence of a *spina angularis* bounding the glenoid fossa has been noted for the individual skulls in the male Chicago series. It was present in the majority of cases, but still there were altogether 21 cases in which this structure was missing on both sides and, in addition to these, five in which it was missing on the right, and five in which it was missing on the left side, that is to say where the sphenoid bone was excluded from the medial border of the glenoid fossa. To make these figures comparable with those given by Sarasin for the New Caledonians†, they are given below for the sum of both sides.

New Britain (Chicago)	...	...	52 out of 126: $41{\cdot}3 \pm 3{\cdot}0$ °/₀.
New Caledonians (Sarasin)	...	...	70 out of 196: $35{\cdot}7 \pm 2{\cdot}3$ °/₀.

The difference is insignificant, but such figures may become useful once they are known for a greater number of series.

Tympanic perforations are rare in the material with which this paper deals; the appended table contains remarks about them in the case of the male New Britain crania at Chicago. They have been observed among these 63 specimens six times on both sides, four times on the left only, and twice on the right side only.

Our notes contain further remarks about the jugular foramen; it has been noted whether it was larger on the right (*JR*) or on the left (*JL*) side. We shall come back to this point later.

Schwalbe first determined the relative proportions of the "three divisions of the face." He defined them by lines drawn through the nasion, the lower margin of both orbits, the lower margin of the nasal aperture and the prosthion. The skulls were orientated in the Frankfurt plane, so that, strictly speaking, a projection of these divisions on a frontal plane perpendicular to the Frankfurt plane was measured. Schwalbe was able to show in this way that there are distinct differences between the Neanderthaloid and modern types of man. This method of investigation was applied by Sarasin to the New Caledonian skulls, and he concluded that they exhibit a more primitive pattern than those of other races. It can be objected that once the Frankfurt plane is defined by the two auricular points and the left sub-orbital point, it may be impossible to determine a horizontal line going through the infraorbital margins of both sides, but, even apart from this theoretical

* "L'Homme fossile de la Chapelle-aux-Saints." *Annales de Paléontologie* (1911).
† *Anthropologie der Neu-Caledonier und Loyalty-Insulaner*. Berlin (1916—1922).

consideration, Schwalbe's method cannot be applied to most of the series for which type contours have been published, since the lower margins of the apertura pyriformis and the prosthion are rarely given. Instead of this, the projections of the sub-orbital and N.S. points on to the line joining the nasion and the alveolar point, i.e. $G'H$, have been measured from type contours with the following results:

Height of Fields: projected on line N-Alv. P.

	Orbital	Nasal	Alveolar	Totals
New Britain Series ...	23·5	25·5	18·1	67·1
	35·0 %	38·0 %	27·0 %	
Middle Javanese	25·9	27·2	17·9	71·0
	36·5 %	38·3 %	25·2 %	
Anglo-Saxon	29·1	23·7	19·2	72·0
	40·4 %	32·9 %	26·6 %	
English: Farringdon Street	30·0	21·3	19·6	70·9
	42·3 %	30·0 %	27·6 %	

This table may suggest that the alveolar field remains more or less constant in its proportions throughout the human races, but that the orbital field tends to gain on the nasal field as one goes from races considered more primitive to higher ones. The unconscious apprehension of this fact is perhaps one of the factors which lead us to call some races "primitive." But to vindicate this scientifically we should require data for a much larger number of modern series, as well as for fossil and anthropoid material. It would first be necessary to investigate the question of the intra-racial variation of these measurements. Since contours are needed for this, we have not gone into it here, reserving it for a later occasion, when it is hoped that a larger series of contours will be at hand. At any rate, the small orbital field of "lower" races should not make one believe that the eyes are actually smaller, but rather that they are placed higher. Whether this indicates that the development of the frontal lobe of the brain is somewhat greater for the higher races, in whom it may have pushed the eyes and orbits further down into the face, must remain hypothetical.

The malar bone is larger in the New Britain Series than in the Egyptian *E* Series which is so far the only one for which comparative values are available. The minimum arc from the point where the malar-maxillary suture crosses the lower border of the orbit to the lowest point on the zygomatic suture, which is still on the lateral surface of the arch, has been taken in accordance with Woo's definition*. Using the right side both in the New Britain and the *E* male series, we have the following mean values for the measurement Ml_1:

New Britain Series: 63·2 (50); Egyptian *E* Series: 59·4 (718); σ, *E* Series: 4·38.

* See *Biometrika*, Vol. XXII. (1931), pp. 324—352. There is an error in Table I; the symbols Ml_1 and Ml_2 should be inter-changed.

If the New Britain standard deviation is supposed the same as the Egyptian, the difference between these means is 8·8 times its probable error. It is suggested that this character should be included in the routine programme of skull measurements.

The general shape of the nasal aperture requires no comment as it is sufficiently described by the nasal index. The only point to which we would wish to direct the attention of the reader is the shape of the inferior border of the apertura nasalis. That there are great racial differences here, which it is practically impossible to describe metrically, has been known for a long time. The best analysis of this region has been given by Gower*, to whose paper the reader may be referred for a general orientation. We have noted whether the border was rounded (*r*), or had the form of a sharp edge (*sh*), and in each case it has been noted whether the anterior (*a*), intermediate (*i*), or posterior (*p*) ridge was taken as the end of the lateral nasal heights, *NH, R* and *NH, L*. There is little doubt in the writer's mind that the height *NH'* to the "base" of the anterior nasal spine is by far the better measurement as it leaves much less room for personal equation.

The dentition in the Chicago series is uniform throughout: third molars are present in all skulls. Caries and loss of teeth during life (as judged by the state of resorption of the alveolar process) have been observed in a minority of cases.

It seemed of interest, particularly after the work of Woo†, to find some data bearing on the asymmetry of the New Britain skull. For this purpose, a few of the measurements defined and used by him have been taken on the Chicago male series. These are in Woo's notation: F_2, P_3, O_7, Ml_1, T_1, T_6, S_3, and Mx_1.

The means for these bilateral measurements are given in Table VIII, and in

TABLE VIII.

Mean Bilateral Measurements for Male New Britain Skulls.

	New Britain			Egyptian *E* Series (Woo) *R* – *L*	*a*
	R	*L*	*R* – *L*		
F_2	83·2 (55)	82·1 (55)	+1·1	+ ·6	4·24
P_3	162·3 (59)	160·1 (59)	+2·2	+2·2	·00
O_7	90·5 (59)	90·6 (59)	– ·1	–1·3	6·51
Ml_1	63·2 (50)	63·3 (50)	– ·1	– ·2	·19
T_1	92·65 (58)	91·7 (58)	+ ·95	+ ·9	·02
T_6	47·5 (57)	48·7 (57)	–1·2	– ·3	6·61
S_3	35·8 (56)	36·2 (56)	– ·4	– ·3	·29
Mx_1	65·3 (53)	65·2 (53)	+ ·1	+ ·2	·62

* *American Journal of Physical Anthropology*, Vol. VI. (1923), pp. 27—36.

† "On the Asymmetry of the Human Skull," *Biometrika*, Vol. XXII. (1931), pp. 324—352.

the fourth and fifth columns the differences between the right and left sides are compared with those found by Woo for the long Egyptian *E* Series.

At first sight it might seem that there is a difference in the degree of asymmetry found. In order to test this, the function α was computed for all the differences between the New Britain and the Egyptian *E* Series, using for this purpose the standard deviations given by Woo. The values found are given in the last column; they show that there is nowhere a significant difference in the degree of asymmetry between the two series. It would be of interest to follow this up for other series and for other measurements.

In accord with what has been observed in other series, the New Britain crania have generally a larger right jugular foramen. This was found in 48 out of 63 cases, that is to say in $76{\cdot}2 \pm 3{\cdot}6$ per cent.

Similarly, we find from the marks given for the average degree of development of the Sylvian fissure and the protuberance of the third frontal gyre:

	R	*L*
Development of Sylvian fissure	2·1	1·7
Protuberance of third frontal gyre	1·9	1·6

As usual, both are more pronounced on the right side, but since there can be little doubt that a deeper fissure will cause a frontal protuberance to stand out more clearly, these figures cannot be accepted as independent data; rather the whole contents of the foregoing table should be looked upon as one piece of evidence, showing a more pronounced cerebral relief on the right side.

All this goes to show that the pattern of the New Britain skull, as far as asymmetry is concerned, is essentially the same as that of other races.

To sum up, we may enumerate once more the outstanding characteristics of the New Britain skulls: They are dolichocephalic, they have a small cranial capacity (due not so much to the length of the main diameters of the skull as to their being "ill filled"), they are phaenozygous, prognathous, and have large zygomatic bones. The orbital field of their faces appears to be rather small and the nasal aperture is ill-defined as regards its lower border.

We will collect here the data and graphs of the type contours.

Measurements of Type Contours (Tables IX—XI).

TABLE IX.

Measurements of New Britain Transverse Type Contours.

	Males (62)		Females (37)	
	R	*L*	*R*	*L*
MA	115·0		110·2	
*M*1	57·3	57·3	54·1	54·1
2	60·7	61·2	58·3	58·5
3	62·6	62·6	60·2	60·5
4	64·3	64·0	61·9	62·4
5	64·1	64·1	61·9	62·5
6	62·8	63·0	60·8	61·2
7	60·6	61·0	58·8	59·1
8	56·3	56·7	54·8	54·8
9	47·5	47·3	46·6	46·1
10	32·4	32·2	32·6	31·6
$A\frac{1}{4}$	16·6	17·3	18·1	16·2
ZRx	3·4	4·5	3·8	3·6
y	60·2	60·9	57·5	57·3

TABLE X.

Measurements of New Britain Horizontal Type Contours.

	Males (62)		Females (36)	
	R	*L*	*R*	*L*
FO	181·0		170·7	
$F\frac{1}{4}$	26·2	25·9	26·7	23·3
$F\frac{1}{2}$	40·2	39·8	38·8	36·8
*F*2	50·3	49·1	46·7	45·1
3	48·1	46·6	47·4	46·0
4	52·9	51·7	51·6	49·9
5	60·6	58·4	59·5	56·7
6	65·2	62·6	63·8	61·0
7	66·0	63·6	64·6	62·0
8	62·1	60·4	60·6	59·1
9	54·5	53·5	53·5	52·8
10	42·4	41·8	41·7	41·7
$O\frac{1}{2}$	25·5	24·9	25·8	25·8
Tx	19·3	18·5	17·1	17·2
y	51·8	50·7	48·0	46·5
*TN***x*	30·0	30·0	25·9	26·1
y	46·6	45·0	45·0	43·6

* This is the point on the section of the temporal region nearest to the *FO* axis.

TABLE XI.

Measurements of New Britain Median Sagittal Type Contours.

	Males	Females
$N\gamma$	176·6 (62)	168·4 (37)
Ordinates above $N\gamma$: 0	26·4 (62)	26·7 (37)
$N\frac{1}{4}$	38·6 (62)	39·9 (37)
1	57·7 (62)	57·9 (37)
2	71·2 (62)	70·1 (37)
3	79·7 (62)	77·4 (37)
4	84·6 (62)	82·5 (37)
5	87·3 (62)	84·9 (37)
6	86·8 (62)	84·3 (37)
7	82·3 (62)	79·9 (37)
8	72·6 (62)	70·8 (37)
9	54·6 (62)	52·7 (37)
$\gamma\frac{3}{4}$	47·0 (62)	—
$\gamma\frac{1}{4}$	29·0 (62)	27·8 (37)
Ordinates below $N\gamma$: 0	57·0 (62)	—
$\frac{1}{4}$	53·9 (62)	50·0 (37)
1	50·5 (62)	47·4 (37)
2	51·5 (61)	47·7 (35)
8	46·7 (62)	44·6 (37)
9	35·9 (62)	33·6 (37)
$\gamma\frac{1}{2}$	—	25·5 (37)
$\gamma\frac{1}{4}$	22·3 (62)	18·4 (37)
$\gamma\frac{1}{8}$	16·4 (62)	—
Vertex: $\overset{x}{y}$ from N	95·0 (62)	90·9 (37)
	87·9 (62)	85·6 (37)
Bregma: $\overset{x}{y}$ from N	66·5 (62)	64·3 (37)
	83·4 (62)	81·5 (37)
Glabella: $\overset{x}{y}$ from N	4·7 (62)	3·0 (37)
	9·6 (62)	10·5 (37)
Occipital point: $\overset{x}{y}$ from γ	0·4 (62)	—
	1·2 (62)	—
Lambda: $\overset{x}{y}$ from γ	5·5 (62)	5·5 (37)
	30·8 (62)	30·2 (37)
Sub-orb. point: $\overset{x}{y}$ from N	8·6 (62)	6·5 (37)
	26·4 (62)	24·9 (37)
Auricular point: $\overset{x}{y}$ from N	89·2 (62)	83·2 (37)
	26·2 (62)	24·9 (37)
Opisthion: $\overset{x}{y}$ from γ	54·2 (62)	52·5 (37)
	52·7 (62)	50·4 (37)
Basion: from γ	100·6 (62)	96·4 (35)
from N	99·9 (62)	93·9 (35)
Alveolar point: from N	67·3 (62)	63·5 (35)
from Bas.	103·3 (62)	99·1 (35)
Sp.: $\overset{x}{y}$ from N	64·3 (62)	60·7 (36)
	31·9 (62)	30·4 (36)
N.S.: $\overset{x}{y}$ from N	10·5 (62)	10·4 (36)
	48·9 (62)	45·1 (36)
Prosthion: $\overset{x}{y}$ from N	15·0 (62)	16·0 (35)
	63·4 (62)	59·8 (35)
Palate: $\overset{x}{y}$ p' from N	43·3 (62)	39·4 (36)
	50·0 (62)	46·1 (36)
Palate: $\overset{x}{y}$ p from Alv. P.	40·8 (60)	39·4 (34)
	14·7 (60)	14·1 (34)
Nose: $\angle LN\gamma$	119°·1 (59)	119°·5 (35)
Nose: NL	20·6 (59)	19·4 (35)
Nose: Subtense: $\overset{x}{y}$ from N	8·4 (59)	—
	2·9 (59)	—
Frontal max. sub. from $N\beta$: $\overset{x}{y}$ from N	47·8 (62)	47·1 (37)
	22·6 (62)	23·6 (37)
Occipital max. sub. from λ Op: $\overset{x}{y}$ from λ	49·9 (62)	45·8 (37)
	28·3 (62)	25·5 (37)

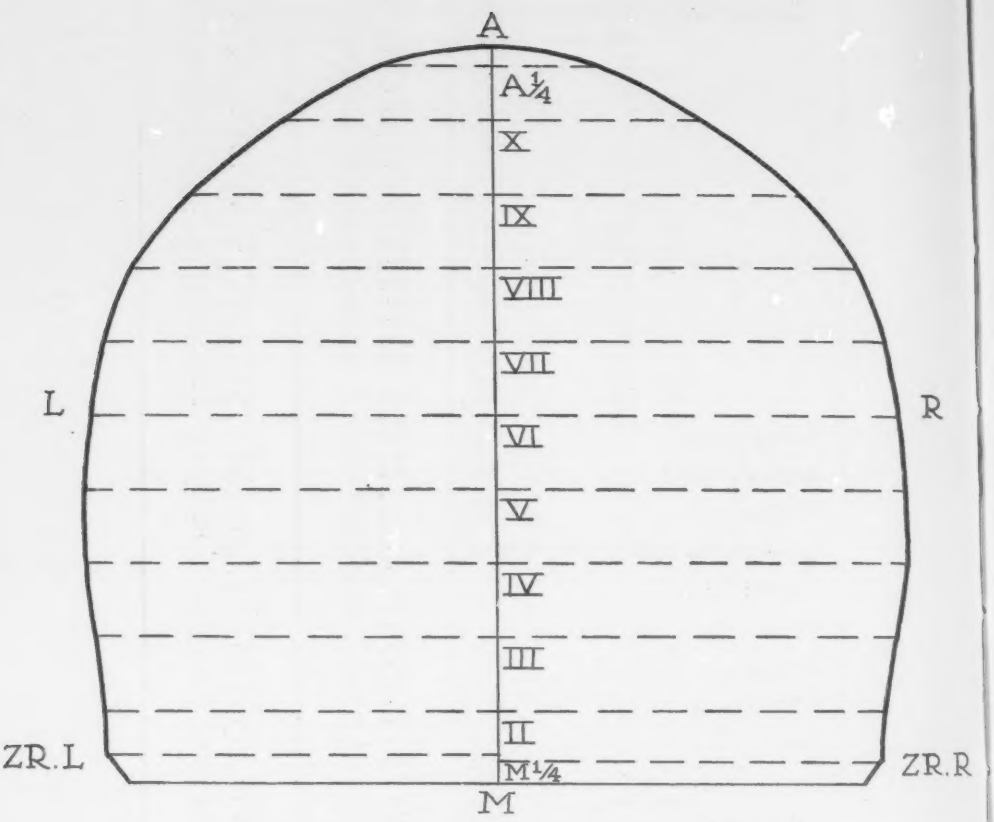


Fig. 2. New Britain Male Transverse Type Contour, based on 62 cases.

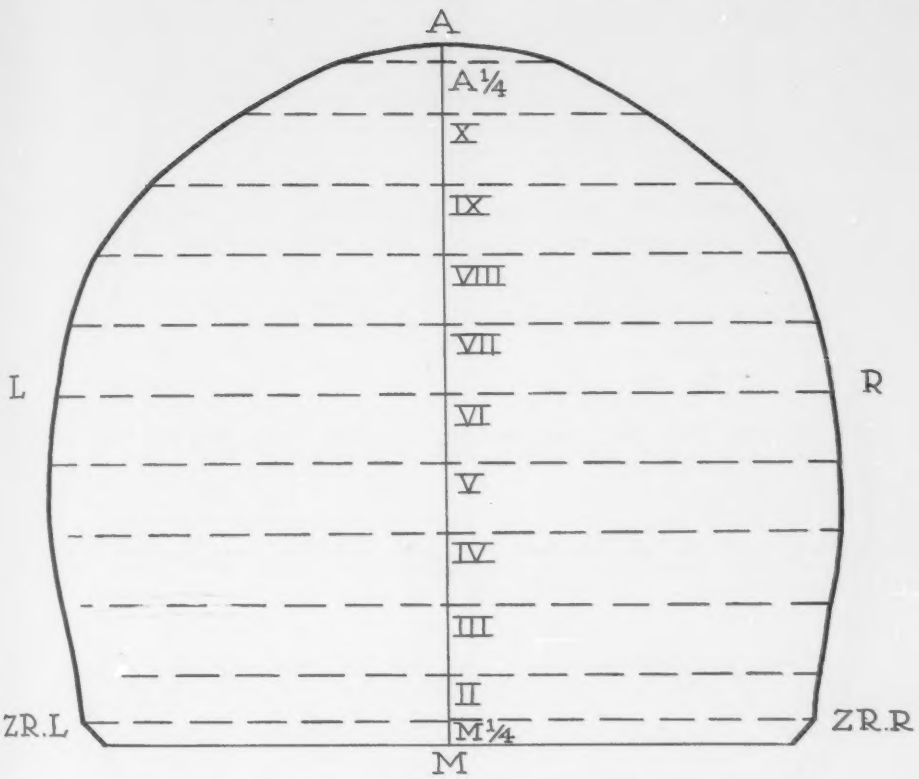


Fig. 3. New Britain Female Transverse Type Contour, based on 37 cases.

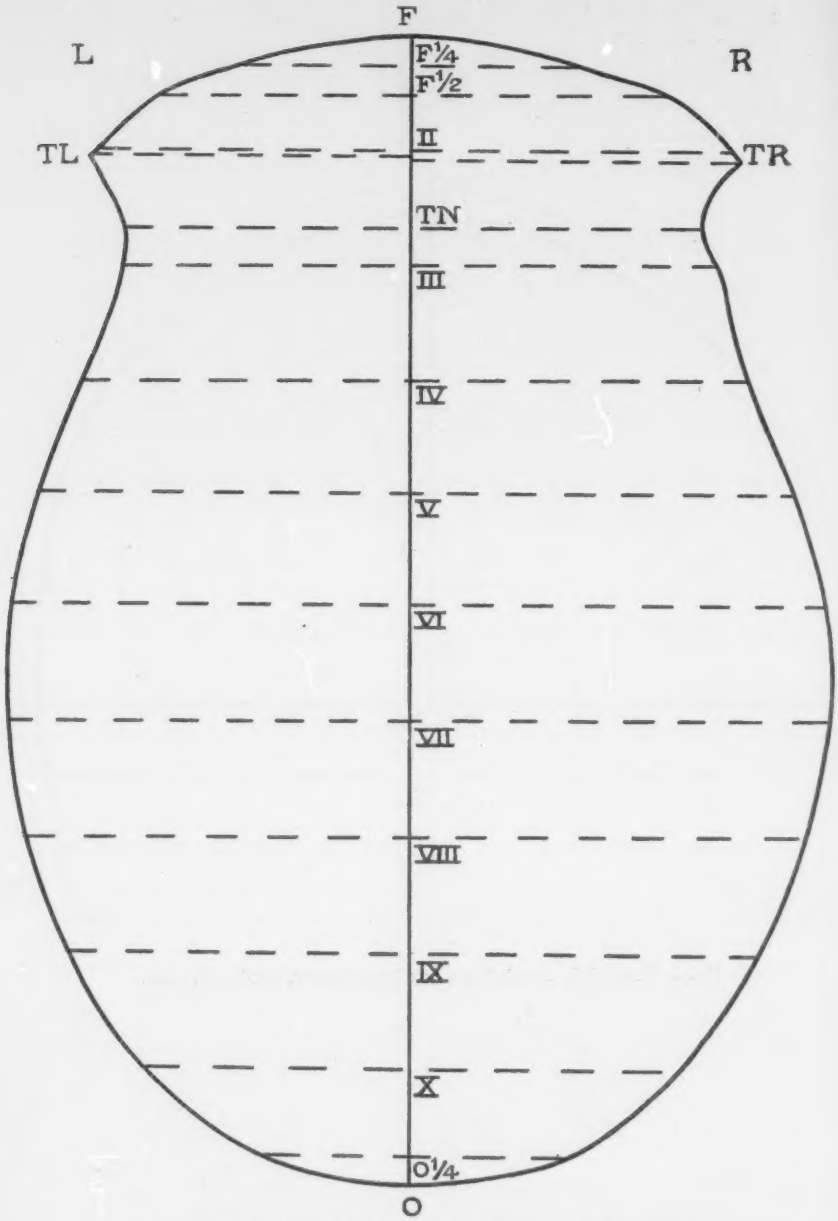


Fig. 4. New Britain Male Horizontal Type Contour, based on 62 cases.

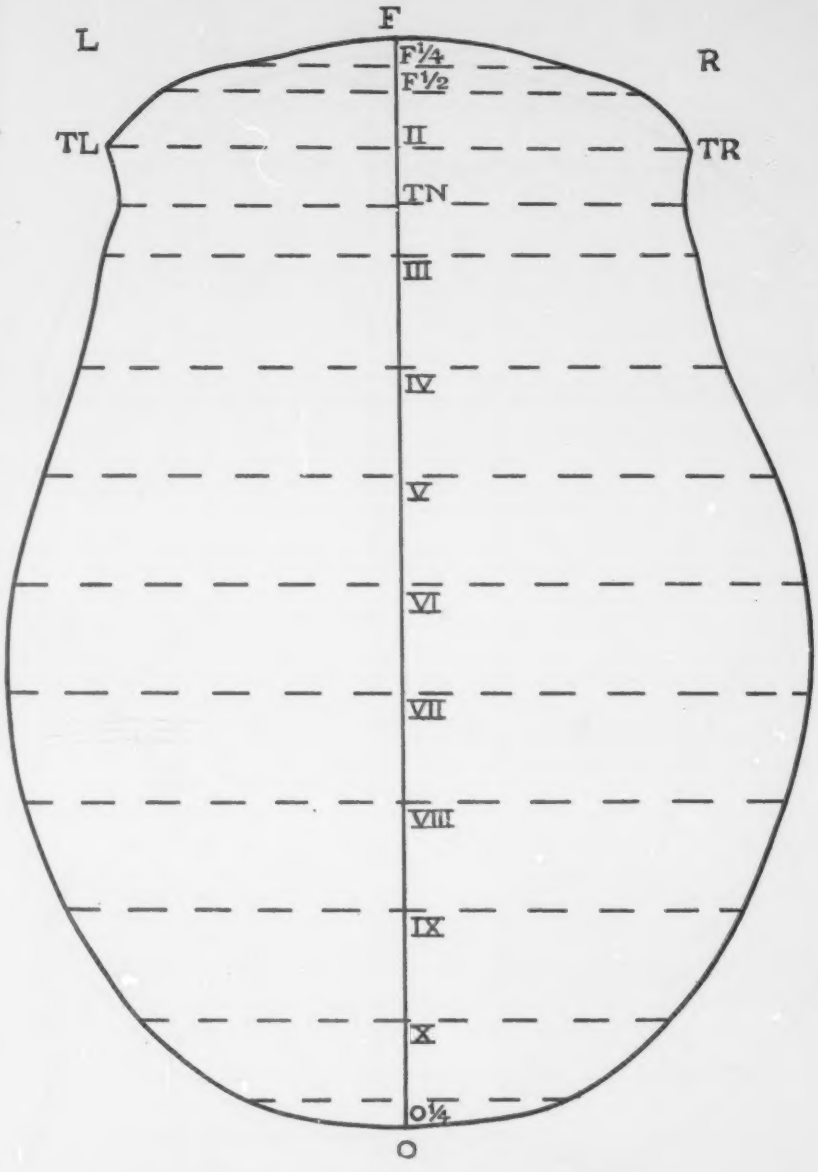


Fig. 5. New Britain Female Horizontal Type Contour, based on 37 cases.

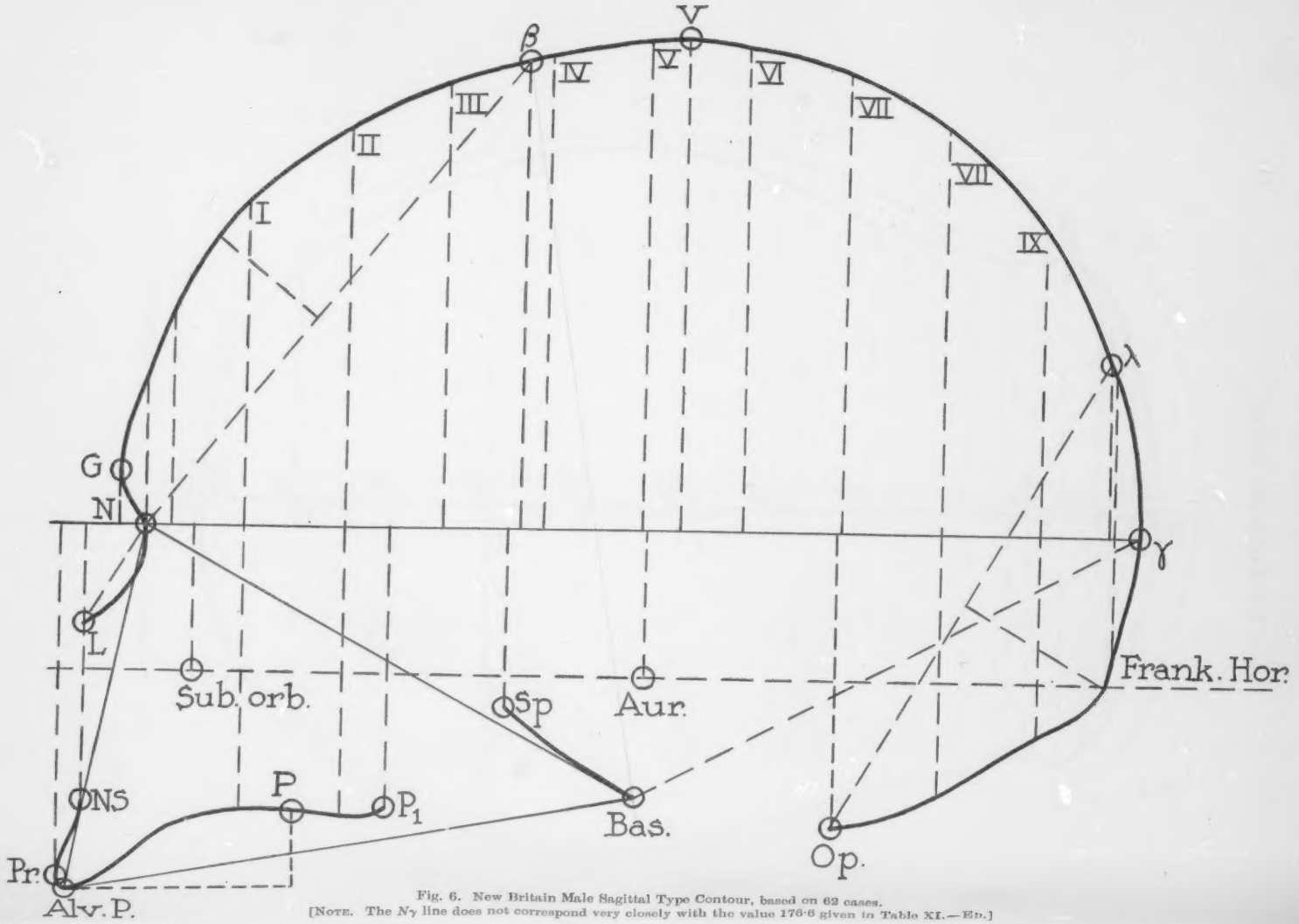


Fig. 6. New Britain Male Sagittal Type Contour, based on 62 cases.
[Note. The $N\gamma$ line does not correspond very closely with the value 176·6 given in Table XI.—Ed.]

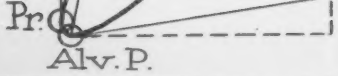


Fig. 6. New Britain Male Sagittal Type Contour, based on 62 cases.
[NOTE. The $N\gamma$ line does not correspond very closely with the value 176·6 given in Table XI.—ED.]

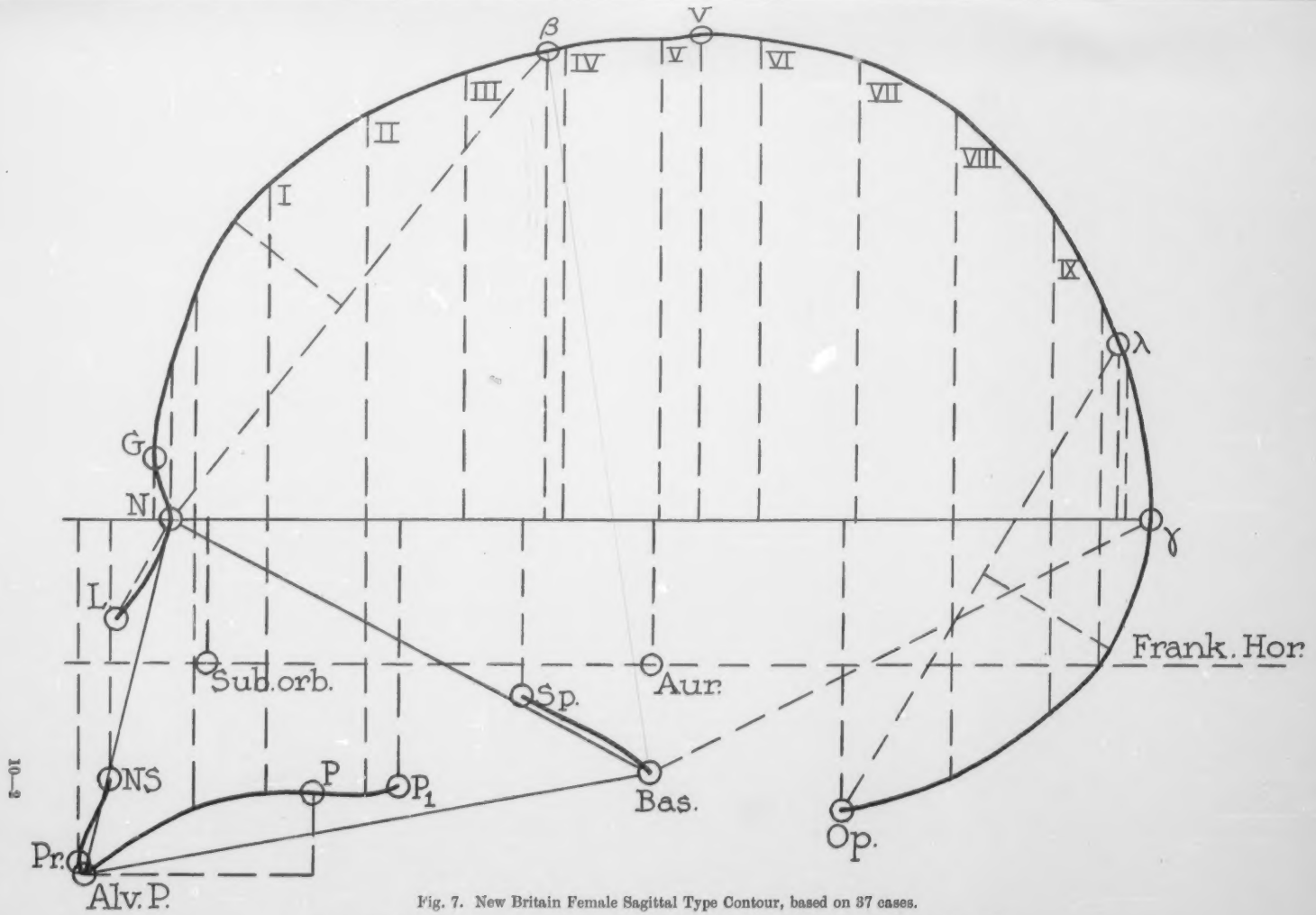


Fig. 7. New Britain Female Sagittal Type Contour, based on 37 cases.
[NOTE. The $N\gamma$ line does not correspond very closely with the value 168·4 given in Table XI.—ED.]

APPENDIX II.

Individual Measurements of New Britain Crania.

The following abbreviations refer to measurements and remarks which are given for the male series at Chicago (Appendix II A) only: F_2, P_3, O_7, Ml_1, Mx_1, T_1, T_6, S_3. See T. L. Woo: "On the Asymmetry of the Human Skull." *Biometrika*, Vol. XXII. (1931), pp. 324—352.

Jugular foramina. *JR*, greater on right; *JL*, greater on left. *J* =, signifies no appreciable difference between right and left.

Supra-orbital ridges. Numerals signify estimated degree of development, 1 poor, 3 greater development. See Martin's *Lehrbuch*, 2nd ed., fig. 390, S. 876.

Pyriform aperture. *r*, round; *sh*, sharp; *a*, border formed by anterior; *i*, border formed by intermediate; *p*, border formed by posterior crest. See text, p. 138.

Glenoid fossa. *sp. ang.*, *spina angularis* of sphenoid, bounding the fossa on both sides unless otherwise indicated; *t*, fossa confined to temporal bone, on both sides unless otherwise indicated.

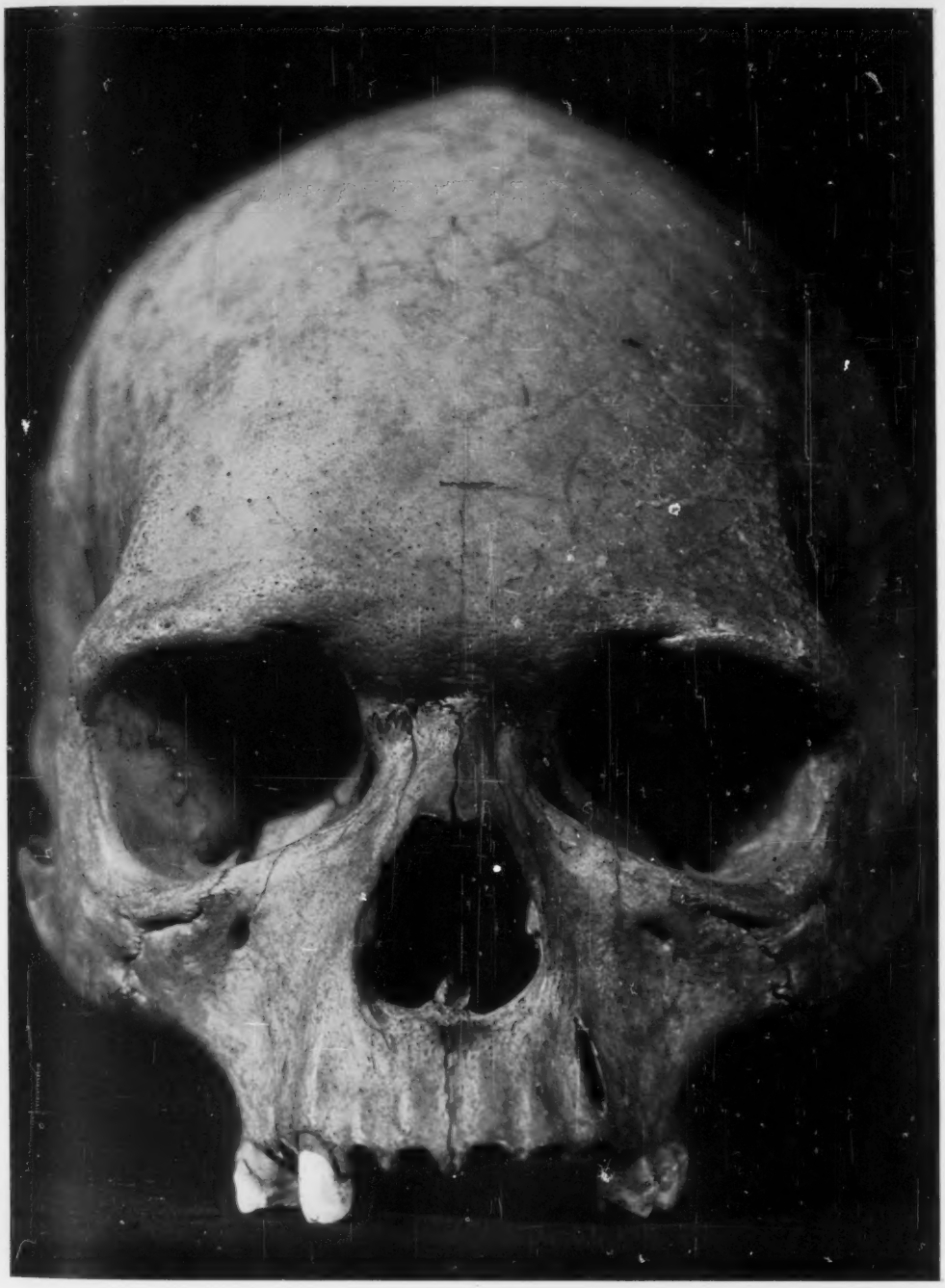
Sylvian fissure and third frontal gyre. Relief of convolutions of brain (*fissura Sylvii* and protuberance of third frontal convolution) on external surface of skull—given numbers according to their degrees of development. See text, p. 135.

Conformation at pterion. *n*, normal, see text, p. 135.

LIST OF PLATES.

Plates I—VIII show four normae of a typical male (Chicago, No. 43143) and a typical female (Chicago, No. 43231) New Britain cranium. They were taken with the focal plane of the camera parallel or perpendicular to the Frankfurt horizontal plane of the specimen.

Plate	I.	Normal male.	*Norma facialis.*	Ca. 1·1 natural size.
,,	II.	,, female.	,, ,,	,, 1·2 ,,
,,	III.	,, male.	,, *lateralis.*	,, 0·9 ,,
,,	IV.	,, female.	,, ,,	,, 0·9 ,,
,,	V.	,, male.	,, *verticalis.*	,, 1·1 ,,
,,	VI.	,, female.	,, ,,	,, 1·0 ,,
,,	VII.	,, male.	,, *basalis.*	,, 1·0 ,,
,,	VIII.	,, female.	,, ,,	,, 1·0 ,,



A Normal Male New Britain Cranium, No. 43143. *Norma facialis.* × *circa* 1·1.

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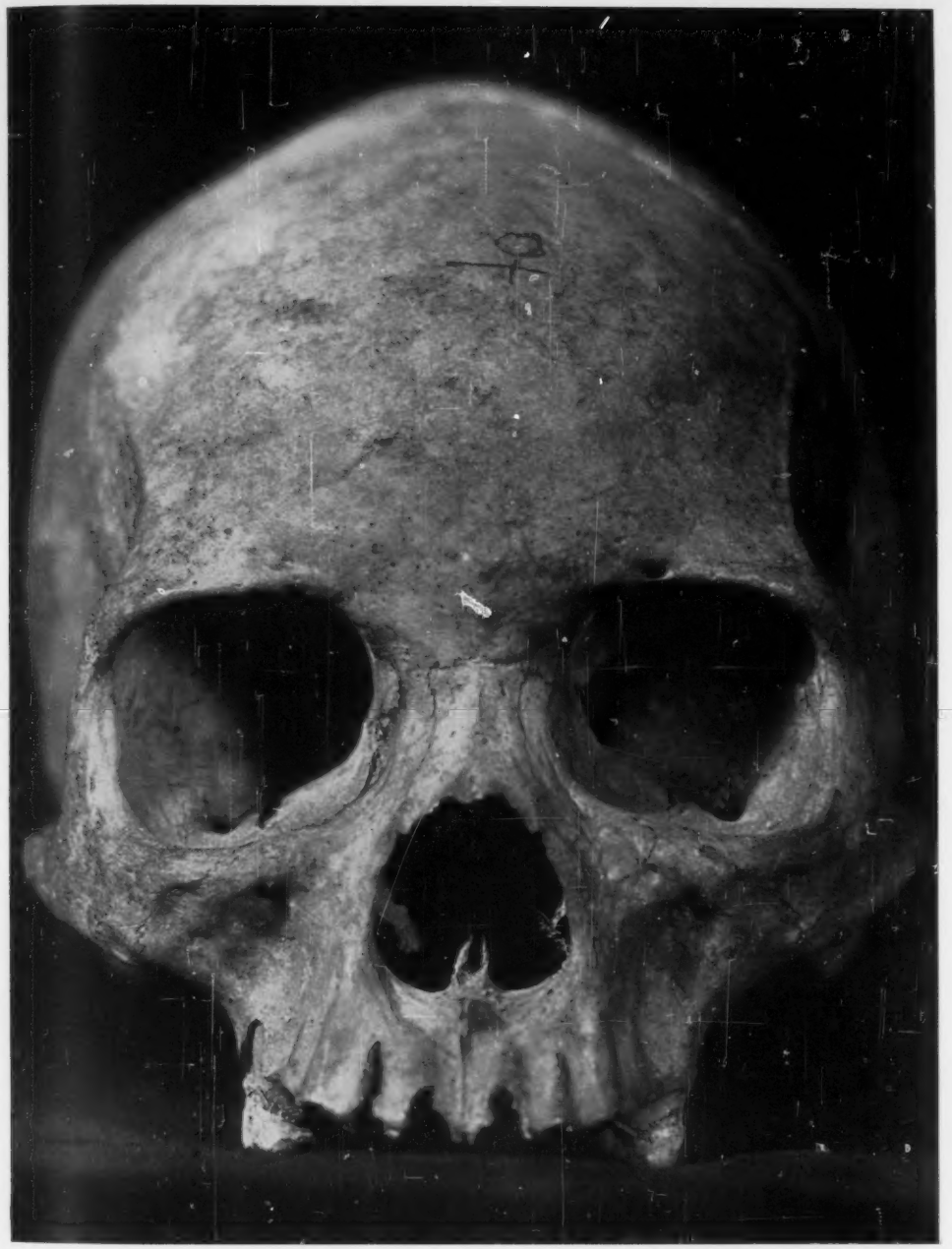

A Normal Female New Britain Cranium, No. 43231. *Norma facialis.* × *circa* 1·2.

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A Normal Male New Britain Cranium, No. 43143. *Norma lateralis* (left profile). × *circa* 0·9.

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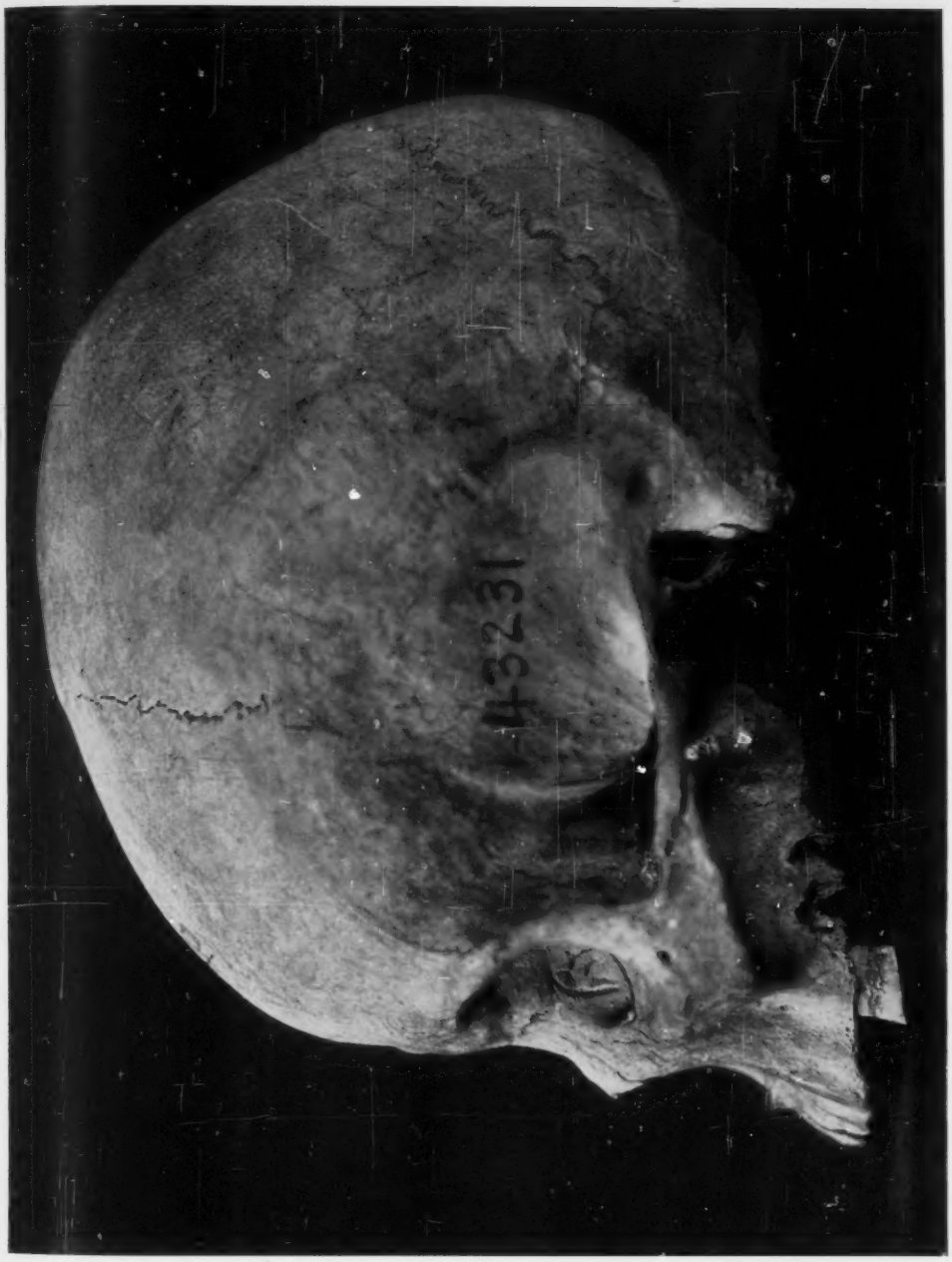

A Normal Female New Britain Cranium, No. 43231. *Norma lateralis* (left profile). × *circa* 0·9.

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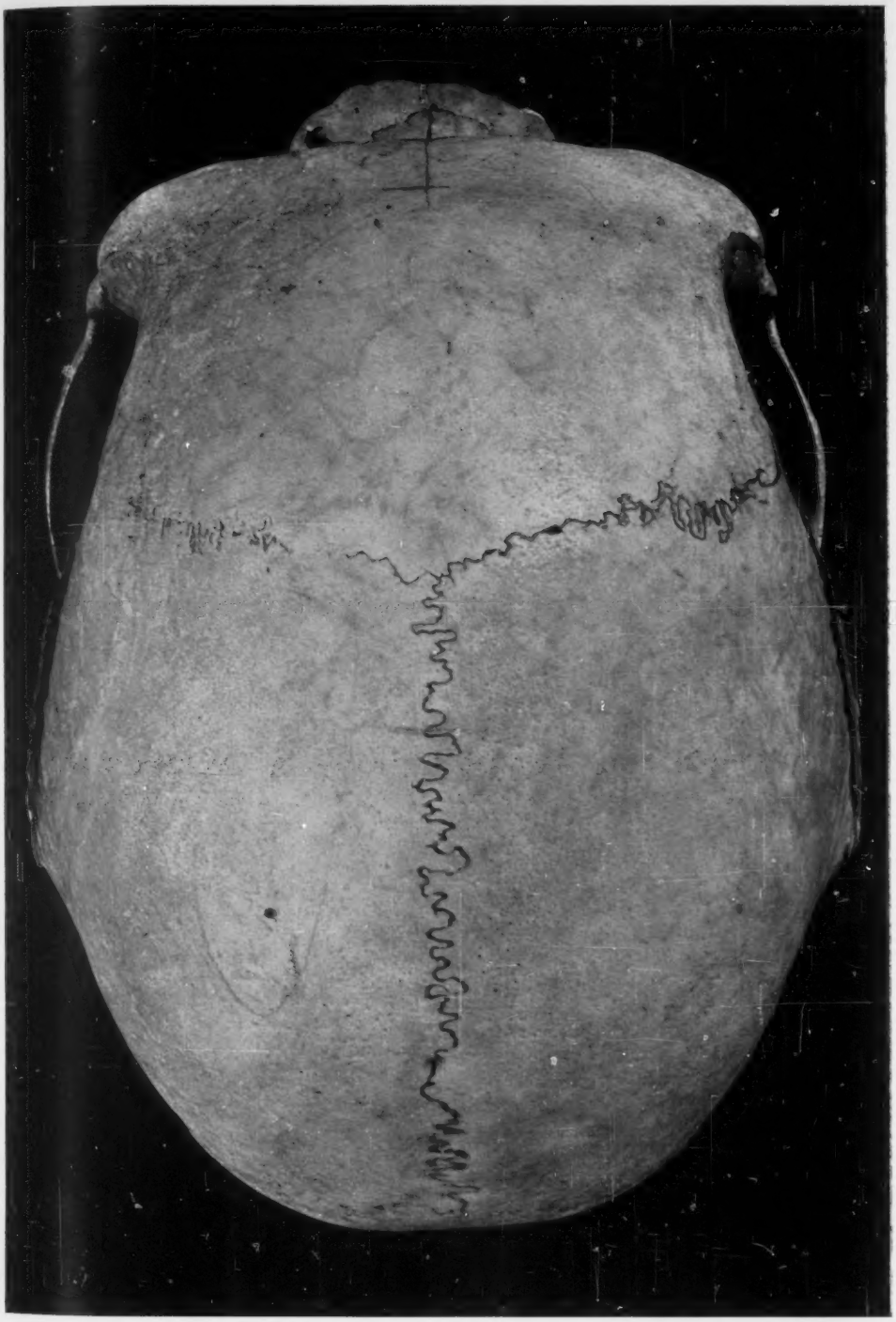

A Normal Male New Britain Cranium, No. 43143. *Norma verticalis.* × *circa* 1·1.

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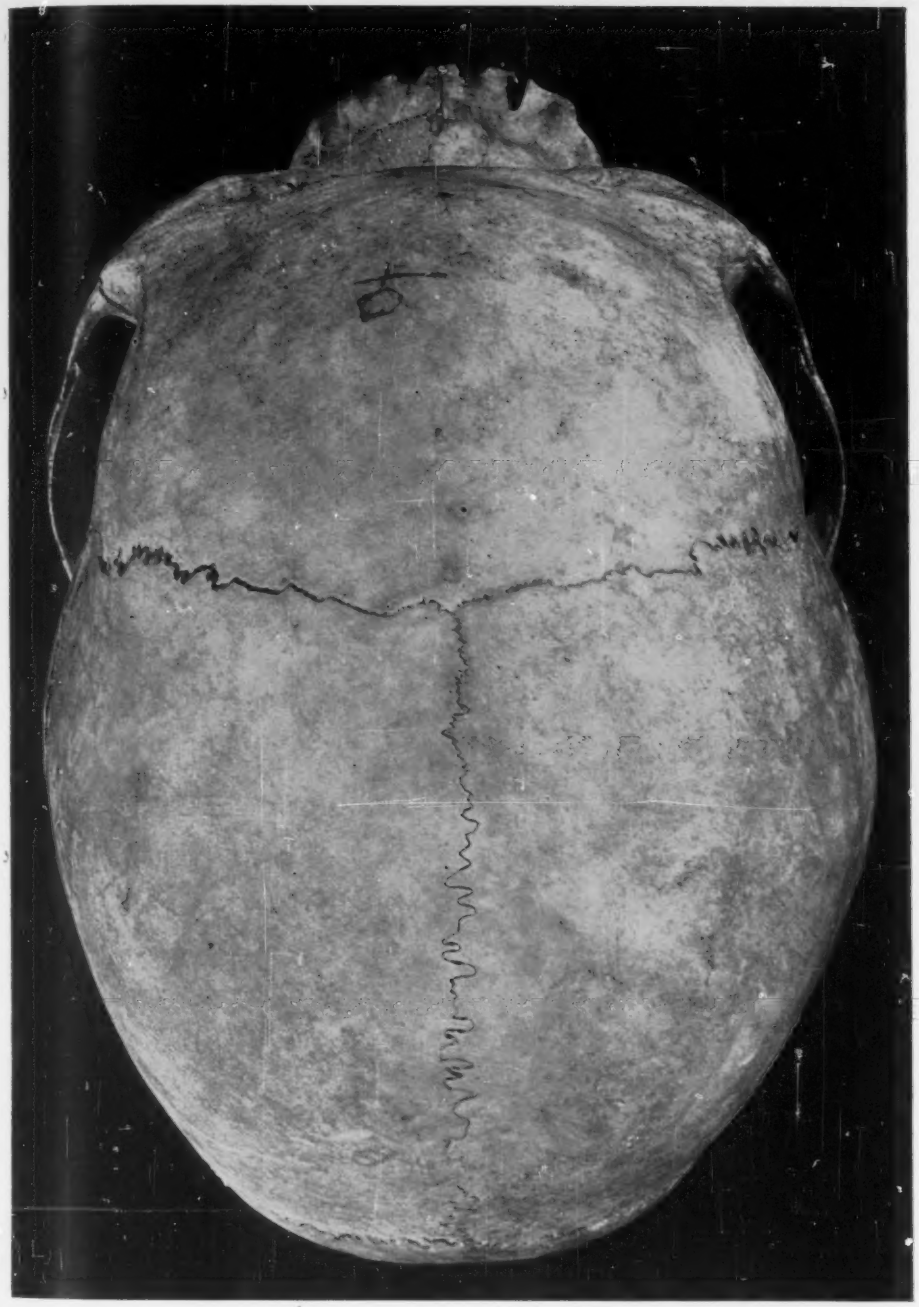
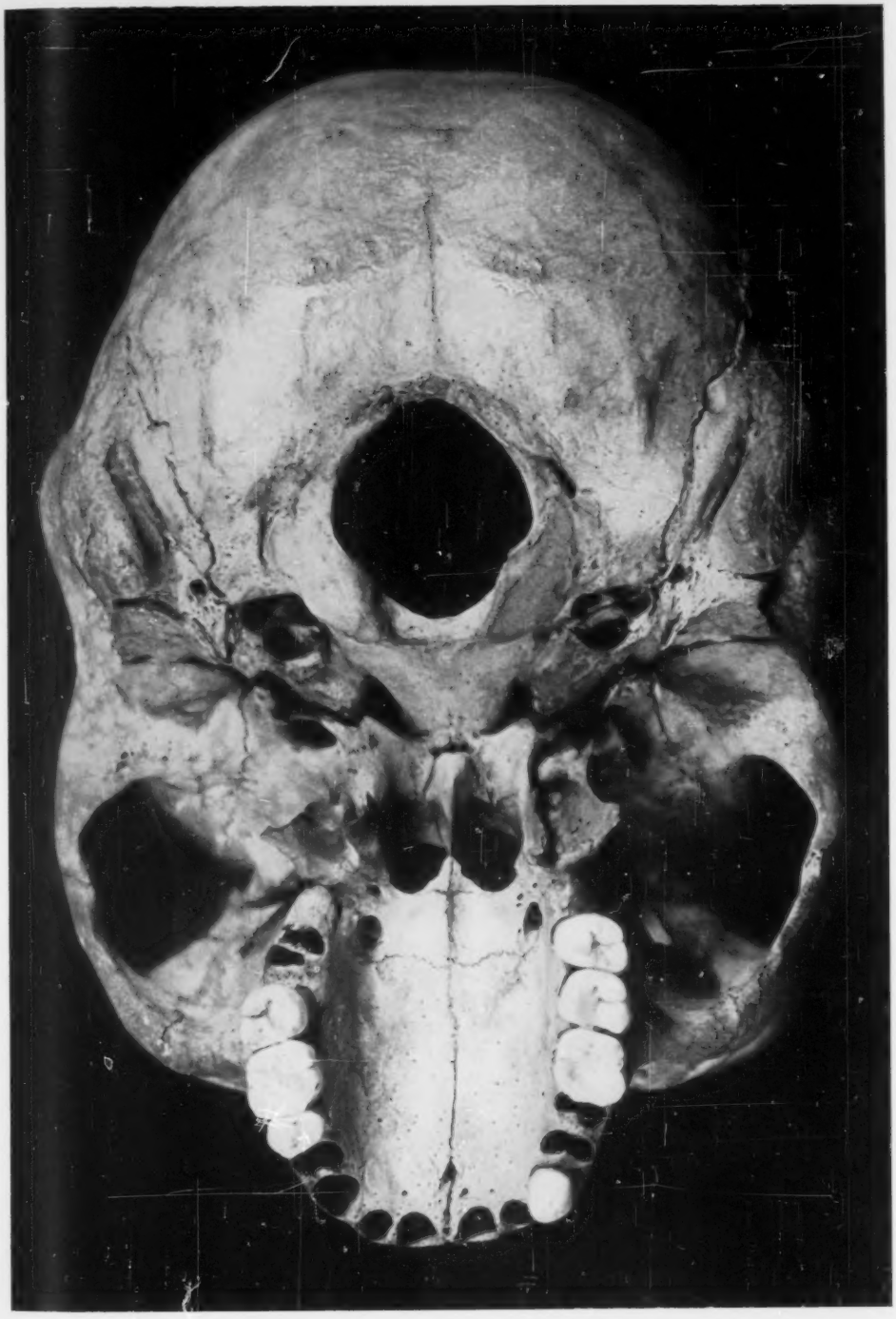


A Normal Female New Britain Cranium, No. 43231. *Norma verticalis.* × *circa* 1·0.



A Normal Male New Britain Cranium, No. 43143. *Norma basalis.* × *circa* 1·0.

A Normal Female New Britain Cranium, No. 43231. *Norma basalis.* ×*circa* 1·0.

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Serial No.	*C*	*L*	*B*	*H′*	*H*	*OH*	*LB*	*B′*	*U*	*Glabella U*	*Q′*	*Bregmatic Q′*	S_1'	S_2'	S_3'
43130	1144·2	180	132	128	130	111	99·5	95·7	492	505	300	299	108·1	111·0	95·1
43133	1253·7	178	131	136·5	141	116	94·0	91·3	498	511	310	305	110·1	120·6	91·0
43134	1323·7	184·5	131·5	135	135·5	116	100·3	98·0	507	520	310	309	108·0	120·4	96·0
43135	1309·6	185	136	133	134·5	114	103·4	95·4	514	525	308	302	111·5	114·2	97·5
43137	1110·4	176	123	129	132	110	95·1	90·3	481	490	291	287	107·3	107·3	95·0
43138	1334·9	185	135·5	137	138	117	101·0	92·9	513	523	313	314	112·3	117·5	102·7
43140	1291·6	186	137	141	141	—	100·0	95·1	519	528	326	326	107·0	132·5	100·4
43141	1323·2	185	134	135·5	135	117	100·0	91·4	509	520	315	312	111·0	114·0	100·1
43143	1197·7	177	129·5	127	129·5	110·5	94·5	88·1	491	505	298	295	103·0	114·0	93·2
43144	1233·8	185·5	133	132·5	138	119	97·0	91·0	509	517	314	302	102·6	117·2	109·2
43147	—	192·5	136	145·5	147	120·5	104·1	106·8	540	546	322	321	110·0	126·5	103·1
43148	1233·8	198	129	134	137·5	115	109·2	90·0	512	534	305	301	110·2	115·0	99·0
43154	1289·9	184	130	134	135	116·5	96·0	98·1	506	516	306	306	113·0	117·2	95·6
43155	1183·3	182·5	123	135	136	111	102·0	93·2	493	506	296	294	108·0	118·0	97·1
43158	1282·1	185·5	133	133	134	108	99·2	92·0	511	523	301	296	100·0	111·5	103·0
43159	1354·6	191	133	133	134	110·5	112·0	102·3	521	524	304	303	115·0	106·2	100·0
43162	1305·7	185	137	136	138	112	104·0	101·1	516	529	305	304	106·4	121·2	91·0
43169	1357·4	195	133·5	138·5	140	121	—	95·2	524	540	322	315	116·2	131·1	105·6
43170	1290·6	188	126	131·5	132	113	102·7	94·0	509	521	300	297	109·3	123·0	89·3
43174	—	178	135	136	—	—	94·9	90·0	497	510	—	305	109·0	115·0	100·0
43175	1291·7	183	129	140	143	117	105·0	96·1	496	512	310	305	110·0	117·5	96·4
43181	—	191	124	144	—	—	105·0	83·6	510	524	—	309	111·0	121·0	98·0
43183	—	180	127·5	135	137	116	103·2	96·0	498	500	308	307	111·4	115·2	87·0
43184	1312·0	182·5	130	135	137	116	101·4	92·0	498	514	310	306	107·0	115·0	95·2
43185	1307·1	185·5	132	135	137	111	102·0	90·1	503	519	300	299	114·0	119·2	94·3
43191	1349·1	181	129	131	133	108	101·2	93·1	498	510	295	292	106·0	111·0	91·0
43192	1180·3	177·5	130	136·5	137	111	104·0	88·0	481	496	300	298	106·3	108·2	97·6
43195	1275·5	179	135	133	135	117	100·3	88·1	491	502	312	310	110·0	118·1	91·6
43197	1362·5	186	137	139	141	121	106·0	102·0	518	525	325	320	112·0	115·3	100·0
43198	1194·2	191·5	126	133·5	135	115	100·0	93·5	516	520	299	297	115·0	123·1	92·1
43200	1393·1	189·5	138	137·5	138	117	99·2	98·3	523	536	319	314	106·2	126·3	99·0
43202	1311·1	186	132	141	142	116	107·1	85·1	512	522	309	308	105·2	120·1	94·0
43203	1188·9	186	127	133	134	112	101·7	89·8	502	514	301	301	109·0	111·7	99·6
43204	1233·1	185	129	140	142	114·5	102·6	90·0	500	513	301	300	111·0	122·1	92·2
43207	1432·1	184	142	140·5	143·5	124	99·2	95·0	517	526	330	324	109·9	119·1	99·0
43208	1355·8	188	134	135	136	114	108·0	100·0	523	530	313	308	101·8	120·3	102·2
43209	1297·6	182	132	133	136·5	117·5	102·0	91·6	497	507	311	304	106·0	113·0	100·3
43210	—	183	132	138	142	122	98·3	86·6	508	514	318	310	112·0	121·1	91·3
43211	—	181	130	126·5	129	109	98·8	84·5	497	506	293	288	103·5	109·0	96·0
43212	1099·6	173	126	133	134	111	98·2	91·0	481	491	294	290	107·1	107·5	93·0
43213	1306·6	190	132	140	139	116	104·1	86·0	517	533	308	305	109·1	123·0	95·2
43214	1261·5	178	133·5	136	137	116	99·0	97·3	500	510	310	307	110·3	114·0	89·0
43216	1429·3	188	137	141	142	120·5	98·0	99·1	517	530	322	317	113·2	118·0	105·0
43217	1296·2	187	133	130·5	135·5	114	101·0	90·8	507	531	300	298	110·0	114·1	95·0
43218	1287·4	185	137	138	141	123	103·0	95·1	507	520	326	312	110·3	117·0	94·2
43221	—	191	133	136	139	116	102·1	93·6	514	530	313	306	112·0	118·0	97·6
43223	1308·8	192	129	138	142·5	119	102·0	91·4	523	534	312	309	108·0	122·0	100·0
43224	1225·8	184	137·5	130·5	134	113·5	95·6	98·3	515	520	312	307	108·0	107·0	105·2
43232	1210·4	182	131	135	136	116	96·0	92·5	497	512	301	300	113·4	119·0	88·0
43235	1326·9	188	137	134	136	117	100·0	97·6	518	527	313	308	112·0	120·4	92·1
43238	1269·3	178·5	132	138	140	116	98·0	89·0	495	505	306	300	108·0	115·2	95·4
43239	1394·1	186·5	135	139	141	120	103·5	85·3	507	520	319	312	111·8	120·1	95·8
43241	1288·6	180	133·5	140	142	119	100·0	97·2	502	510	317	305	109·4	117·6	97·0
43244	1363·2	179	141·5	138·5	140	114	105·0	98·0	515	526	307	309	106·6	115·6	89·3
43248	1302·9	186	135	135	137·5	116	105·0	91·1	512	521	306	301	111·0	122·0	100·2
43250	1138·4	174	131	125	129	110·5	93·6	91·8	486	493	298	291	100·1	104·0	98·0
43253	1197·7	179·5	128	134	135	109	100·2	91·2	498	512	292	290	108·0	111·5	89·1
43254	—	188	129	136	140	115	100·9	—	506	523	305	297	107·6	120·3	95·0
43256	1435·9	187·5	130	142	145·5	119·5	104·4	96·5	518	525	314	309	108·5	124·0	99·5
43258	1306·1	184	142	134	136	118	96·2	95·4	519	530	318	314	113·1	115·6	97·0
43292	1249·0	186·5	132	137	138·5	113	99·1	87·2	502	516	306	303	116·1	118·2	98·0
43293	1408·2	187	142·5	136	141	124	103·5	94·1	526	535	329	322	111·0	121·2	97·0
43295	1304·3	194	134	141	142	117	100·4	97·5	525	542	310	306	116·0	122·0	101·0

Bregmatic Q′	S_1'	S_2'	S_3'	S_1	S_2	S_3	S	*fml*	*fmb*	*EOW*	*IOW*	*G′H*	*J*	*ZB*	*GL*
299	108·1	111·0	95·1	122	124	111	357	33·2	26·9	107·0	98·0	64·1	135	98·3	104·0
305	110·1	120·6	91·0	128	140	110	378	35·6	27·9	106·0	102·6	67·4	133	100·0	97·1
309	108·0	120·4	96·0	124	137	120	381	34·8	27·9	107·0	99·1	67·0	—	90·1	104·0
302	111·5	114·2	97·5	126	127	117	370	34·7	29·0	104·2	97·7	65·0	132	94·4	105·0
287	107·3	107·3	95·0	121	120	114	355	34·9	31·0	104·0	97·8	65·6	128	92·4	99·3
314	112·3	117·5	102·7	127	130	124	381	33·3	29·4	111·0	105·4	67·0	141	102·2	102·3
326	107·0	132·5	100·4	121	147	124	392	32·5	24·8	—	—	66·0	—	—	106·0
312	111·0	114·0	100·1	128	125	122	375	36·7	30·9	105·0	98·0	65·5	133	95·0	102·0
295	103·0	114·0	93·2	116	128	114	356	37·0	31·2	106·3	102·1	65·2	130	94·0	100·0
302	102·6	117·2	109·2	112	131	141	384	29·0	25·5	108·1	103·2	66·0	131	92·2	103·0
321	110·0	126·5	103·1	126	142	125	393	41·4	31·1	116·6	110·3	71·0	—	101·2	107·2
301	110·2	115·0	99·0	125	128	121	374	36·9	30·1	109·4	105·6	69·5	136	85·3	104·3
306	113·0	117·2	95·6	130	134	120	384	35·0	28·0	111·1	101·9	70·9	138	96·0	99·3
294	108·0	118·0	97·1	123	133	114	370	32·5	28·9	108·0	101·1	63·3	134	95·0	109·7
296	100·0	111·5	103·0	115	121	132	368	35·3	33·0	112·1	105·0	72·0	152	99·8	109·5
303	115·0	106·2	100·0	132	120	119	371	35·9	30·9	—	—	75·1	—	—	109·0
304	106·4	121·2	91·0	122	140	114	376	35·2	31·0	116·0	108·0	66·1	144	90·5	104·0
315	116·2	131·1	105·6	133	147	129	409	30·7	28·5	112·5	101·0	—	139	104·0	—
297	109·3	123·0	89·3	126	138	109	373	36·5	31·2	107·2	101·0	73·1	129	100·9	105·1
305	109·0	115·0	100·0	124	134	118	376	35·2	25·9	—	—	66·3	—	—	99·2
305	110·0	117·5	96·4	123	133	115	371	33·9	27·0	112·1	104·3	65·2	141	104·7	103·0
309	111·0	121·0	98·0	125	134	122	381	36·0	32·0	—	—	64·0	—	—	99·5
307	111·4	115·2	87·0	130	128	107	365	35·2	27·1	108·0	101·1	63·5	—	96·0	112·0
306	107·0	115·0	95·2	122	131	116	369	36·0	28·4	106·3	99·6	66·3	143	99·1	100·0
299	114·0	119·2	94·3	130	135	111	376	34·9	29·3	109·1	104·0	62·0	136	97·0	102·5
292	106·0	111·0	91·0	121	127	111	359	34·1	29·0	109·3	104·4	63·0	136	100·5	101·0
298	106·3	108·2	97·6	120	122	112	354	36·0	29·0	108·1	100·7	67·8	133	95·9	101·0
310	110·0	118·1	91·6	126	135	108	369	32·0	28·0	104·1	100·2	63·6	140	96·4	107·3
320	112·0	115·3	100·0	128	130	122	380	34·0	28·0	110·4	103·6	70·0	146	98·5	112·3
297	115·0	123·1	92·1	128	136	117	381	34·2	26·1	108·0	101·0	79·1	132	99·1	100·0
314	106·2	126·3	99·0	123	143	123	389	34·1	29·3	111·1	104·0	72·0	137	99·0	102·1
308	105·2	120·1	94·0	118	135	112	365	37·1	31·5	112·3	104·0	76·6	—	96·1	111·0
301	109·0	111·7	99·6	122	125	122	369	35·3	29·6	106·6	101·3	63·0	138	95·0	103·2
300	111·0	122·1	92·2	126	143	109	377	37·0	30·3	110·4	104·1	68·2	142	98·0	106·0
324	109·9	119·1	99·0	126	135	123	384	32·9	29·1	109·2	101·1	77·2	139	93·4	103·0
308	101·8	120·3	102·2	117	136	120	373	36·0	28·5	113·4	108·5	66·3	138	97·9	111·0
304	106·0	113·0	100·3	119	127	119	365	34·0	27·0	105·8	98·3	65·1	132	98·0	109·0
310	112·0	121·1	91·3	130	142	108	380	34·1	27·0	104·3	98·2	70·3	131	98·6	102·2
288	103·5	109·0	96·0	121	120	116	356	36·2	31·2	100·9	93·2	65·0	128	91·2	106·1
290	107·1	107·5	93·0	123	120	110	353	32·4	26·9	102·2	97·1	61·2	125	91·3	99·0
305	109·1	123·0	95·2	124	137	116	377	33·9	26·0	112·2	107·1	66·6	145	97·1	107·2
307	110·3	114·0	89·0	128	130	108	366	34·2	26·6	108·0	101·2	64·1	138·5	99·2	99·0
317	113·2	118·0	105·0	127	132	131	390	36·0	28·5	115·0	107·1	63·0	142	99·2	96·0
298	110·0	114·1	95·0	125	131	116	372	36·6	27·0	107·6	101·0	68·3	137	94·2	100·0
312	110·3	117·0	94·2	127	132	114	373	36·5	27·1	111·8	105·0	63·3	144	100·4	102·0
306	112·0	118·0	97·6	127	135	120	382	38·0	30·0	108·2	103·6	66·0	—	94·0	105·0
309	108·0	122·0	100·0	125	137	122	384	34·5	28·9	105·5	100·0	71·0	140·5	102·3	105·0
307	108·0	107·0	105·2	125	119	132	376	36·1	26·5	107·1	101·0	62·1	128	92·0	97·0
300	113·4	119·0	88·0	127	132	106	365	38·0	31·0	108·8	101·0	70·5	134	99·0	100·0
308	112·0	120·4	92·1	129	138	117	384	32·0	27·2	115·3	107·2	—	141·5	103·3	—
300	108·0	115·2	95·4	124	131	115	370	34·0	29·8	105·4	99·1	71·0	—	97·0	98·0
312	111·8	120·1	95·8	126	137	116	379	34·8	27·5	111·9	107·0	66·2	137	99·0	108·0
305	109·4	117·6	97·0	122	134	118	374	35·1	28·1	109·0	103·2	65·2	131·5	101·0	106·0
309	106·6	115·0	89·3	122	132	106	360	37·7	31·8	110·0	103·5	71·2	145	101·4	110·0
301	111·0	122·0	100·2	125	136	116	377	32·1	26·9	105·0	99·0	63·0	130	98·0	105·0
291	100·1	104·0	98·0	116	117	123	356	33·0	29·0	109·8	103·1	66·1	135	97·4	98·0
290	108·0	111·5	89·1	124	124	109	357	34·0	27·0	108·0	101·0	68·6	137	98·4	99·0
297	107·6	120·3	95·0	125	139	118	382	31·7	27·5	108·1	102·7	63·3	131·5	94·0	107·0
309	108·5	124·0	99·5	122	142	116	378	39·0	29·1	104·0	97·1	66·0	134	93·1	96·0
314	113·1	115·6	97·0	129	133	125	387	31·9	26·5	113·2	106·1	70·0	139·5	100·0	104·0
303	116·1	118·2	98·0	131	132	120	383	34·0	28·0	98·8	95·2	68·8	133·5	101·6	98·0
322	111·0	121·2	97·0	127	142	126	395	31·7	26·0	110·4	103·0	71·5	138	102·0	110·0
306	116·0	122·0	101·0	136	137	125	398	34·0	25·0	109·3	105·5	68·0	141	99·4	104·0

* A number

	GB	*GL*	*PH*	*NHR*	*NHL*	*NH'*	*NB*	O_1, *R*	O_1', *R*	*Lacr.* O_1, *R*	O_2, *R*	O_1, *L*	O_1', *L*	*Lacr.* O_1, *L*	O_2, *L*	G_1	G_1'	G_2	*EH*	*Alveolar* $P\angle$	*Prosthion* $P\angle$
	98·3	104·0	—*	47·2	47·1	44·3	28·0	42·9	39·3	38·0	33·0	42·1	39·0	36·2	32·3	57·0	51·9	—	—	78°·0	75°·8
	100·0	97·1	—	47·3	48·1	43·0	25·4	44·8	41·7	41·0	33·6	47·3	44·0	—	32·5	50·9	48·0	39·9	11·2	77°·6	74°·2
	90·1	104·6	—	51·1	49·1	46·5	27·0	42·5	40·1	39·3	33·0	44·0	40·3	39·5	32·9	53·9	48·0	36·7	10·3	77°·0	74°·0
	94·4	105·6	—	50·5	50·5	45·1	27·0	43·2	39·5	38·9	32·5	43·1	39·5	38·8	33·0	56·1	52·0	40·0	10·1	81°·1	79°·0
	92·4	99·3	—	50·0	49·6	47·3	22·0	43·0	40·0	39·0	33·4	42·6	40·6	39·7	33·1	51·1	49·0	40·7	8·2	75°·8	74°·2
	102·2	102·1	—	50·4	50·4	50·0	28·2	43·9	41·0	40·1	34·0	45·2	42·0	40·9	33·9	54·0	49·1	40·3	12·0	80°·3	78°·0
	—	106·0	—	49·0	49·0	46·0	30·0	42·8	40·0	39·0	32·9	—	—	—	—	—	—	—	—	77°·6	75°·5
	95·0	102·0	—	51·1	51·5	50·4	26·1	42·4	40·0	—	33·9	42·9	39·5	38·1	32·9	51·9	48·9	42·9	12·5	82°·0	80°·3
	94·0	100·0	—	47·0	46·0	45·1	26·0	42·1	39·8	38·2	30·9	43·5	41·0	38·9	31·0	50·9	50·5	48·3	12·6	78°·0	76°·1
	92·2	103·0	—	48·8	48·5	44·8	27·6	45·1	42·0	41·2	32·1	43·0	40·2	40·0	32·2	54·1	49·0	43·3	10·0	80°·0	78°·2
	101·2	107·4	—	51·1	51·0	49·1	28·2	47·2	43·0	43·0	34·7	46·5	43·0	41·9	33·9	59·7	53·0	44·1	12·2	76°·2	74°·1
	85·3	104·3	—	53·1	53·9	51·0	27·0	46·8	—	—	37·0	48·0	42·4	42·4	37·0	54·8	47·9	—	—	85°·1	83°·0
	96·0	99·3	—	52·0	52·6	46·9	27·1	43·9	41·0	40·0	35·0	43·6	40·5	39·9	35·1	52·3	47·0	39·4	13·2	78°·3	76°·1
	95·0	109·7	—	48·5	47·4	44·7	23·5	43·8	41·0	39·5	28·5	42·0	39·8	38·0	29·1	55·4	54·0	43·9	12·3	75°·0	71°·2
	99·8	109·5	—	53·4	54·5	47·7	25·5	45·1	42·1	42·1	32·1	44·1	42·4	42·4	34·0	62·9	55·5	45·1	14·0	72°·3	70°·1
	—	109·0	—	58·1	58·3	55·1	27·5	—	—	—	—	46·0	42·9	42·0	36·0	53·2	50·1	43·0	11·6	82°·1	—
	90·5	104·0	—	51·0	51·0	48·9	28·8	45·1	42·5	41·0	32·0	44·3	42·4	40·1	31·8	59·2	51·2	39·1	10·0	78°·7	75°·3
	104·0	—	—	53·0	53·2	46·9	29·0	43·5	41·9	40·2	34·1	43·0	40·1	39·0	33·8	—	—	—	—	—	—
	100·9	105·1	—	55·5	55·5	54·0	24·1	45·0	42·0	42·0	34·2	44·0	40·0	—	35·0	55·2	50·9	38·9	13·0	78°·2	77°·1
	—	99·2	—	—	—	—	—	41·0	—	—	30·3	—	—	—	—	—	—	—	—	—	—
	104·7	103·6	—	48·2	48·2	46·0	27·0	43·7	41·2	40·0	31·9	42·3	40·2	39·0	32·1	54·1	49·5	40·9	10·0	81°·0	78°·2
	—	99·5	—	48·8	48·7	47·5	28·0	43·9	—	—	31·1	—	—	—	—	—	—	—	—	—	—
	96·0	112·0	—	45·9	45·3	43·0	27·2	43·2	41·0	38·9	30·1	44·0	40·3	38·9	30·0	—	—	—	—	74°·3	73°·0
	99·1	100·0	—	52·0	52·0	50·5	27·1	43·8	40·5	40·5	35·1	44·3	41·3	41·3	33·4	55·1	47·9	37·5	9·0	82°·2	79°·8
	97·0	102·5	—	50·5	52·1	49·0	28·9	43·5	41·1	40·1	32·0	45·9	43·0	41·9	31·8	59·0	51·3	42·2	9·5	77°·2	75°·0
	100·5	101·8	—	—	—	46·0	27·0	44·9	41·0	40·0	34·0	45·2	41·9	41·0	33·5	53·1	49·0	38·0	8·2	80°·1	—
	95·9	101·0	—	53·3	53·3	50·5	29·9	44·0	40·9	39·1	33·0	43·5	41·0	39·8	33·5	54·9	48·1	43·2	10·8	82°·3	80°·3
	96·4	107·3	—	50·8	51·0	48·1	28·6	41·6	40·0	39·0	29·2	42·9	40·0	38·8	30·4	55·2	51·0	45·5	9·0	77°·0	75°·0
	98·5	112·1	—	57·0	57·0	51·5	28·9	46·0	42·0	40·9	34·0	45·0	41·8	40·7	33·2	60·0	53·8	40·8	—	78°·0	75°·8
	99·1	100·0	—	62·0	62·0	55·2	28·5	46·0	42·0	—	34·4	45·0	41·6	40·1	35·0	54·0	48·9	42·2	13·2	85°·2	83°·1
	99·0	102·2	—	55·1	55·2	50·9	27·6	45·5	42·0	40·0	30·3	42·9	41·9	39·0	30·9	61·0	55·2	—	—	81°·0	78°·1
	96·1	111·0	—	55·1	55·1	47·0	28·9	45·3	42·0	—	35·5	44·9	42·7	41·5	34·1	57·9	55·1	—	—	78°·1	75°·1
	95·0	103·2	—	49·8	49·8	46·5	25·1	43·0	41·5	38·2	31·3	43·1	41·0	38·2	31·3	55·2	48·5	39·7	10·0	81°·8	80°·5
	98·0	106·0	—	52·5	52·8	48·3	28·2	46·0	43·0	41·2	32·5	45·0	41·0	41·0	32·8	54·9	52·0	42·9	11·7	77°·0	73°·7
	93·4	103·0	—	58·2	58·4	50·7	26·7	44·2	42·0	40·0	35·6	46·2	43·0	41·1	35·1	58·0	53·0	40·0	14·1	79°·6	78°·0
	97·9	111·0	—	47·5	47·5	45·4	30·9	46·2	41·8	41·8	32·3	44·0	40·0	40·0	32·2	58·0	55·0	44·6	7·3	77°·3	75°·0
	98·0	109·7	14·0	54·1	54·1	50·7	26·8	41·9	—	37·3	32·0	42·0	—	38·9	31·0	54·1	50·6	44·0	10·5	77°·0	75°·1
	98·6	102·2	20·5	53·5	53·6	48·4	26·0	43·3	—	—	34·0	44·0	—	—	33·9	57·9	52·1	—	—	78°·0	75°·3
	91·2	106·1	18·0	49·0	49·0	46·6	24·0	40·5	—	—	33·0	42·0	39·5	38·2	33·1	56·0	53·0	41·3	9·5	75°·1	72°·8
	91·3	99·1	22·2	45·0	42·4	41·9	25·9	43·0	41·0	38·2	28·4	44·2	40·3	40·0	28·5	55·0	50·0	36·4	8·3	77°·2	75°·1
·5	97·1	107·3	19·1	51·0	51·0	48·0	30·8	48·1	44·5	43·—	34·6	47·0	43·0	41·2	34·0	64·0	56·1	43·1	9·0	78°·0	76°·4
	99·2	99·3	15·2	48·9	49·2	49·0	27·6	40·7	38·1	38·1	32·2	42·0	37·9	37·9	30·5	56·2	50·4	—	—	81°·9	80°·0
	99·2	96·5	17·0	49·3	48·7	46·5	31·0	47·0	43·0	43·0	32·0	47·0	42·1	40·9	32·0	53·0	48·0	38·5	10·2	82°·1	80°·2
	94·2	100·2	19·9	50·0	52·0	50·0	28·8	43·3	40·2	38·9	31·1	42·0	39·2	38·1	31·2	58·2	52·9	41·5	11·0	81°·3	78°·0
	100·4	102·3	18·2	46·1	46·8	45·0	27·3	45·3	42·1	41·0	33·0	46·1	42·1	40·4	32·0	54·5	50·2	—	—	87°·0	83°·0
	94·0	105·4	12·2	52·5	51·5	51·5	29·0	46·1	—	42·0	34·2	46·1	—	41·5	36·1	55·6	50·2	44·1	11·2	76°·7	76°·0
·5	102·3	105·5	22·0	55·3	55·3	51·0	28·5	43·4	—	—	31·7	44·1	—	—	32·6	57·1	52·0	40·2	14·5	80°·2	77°·9
	92·0	97·4	17·5	45·5	46·0	45·0	28·0	44·2	38·9	—	29·8	43·9	40·9	38·1	29·1	53·5	50·1	38·9	8·0	81°·0	79°·5
	99·0	100·0	19·0	54·0	54·0	52·2	25·5	46·2	42·7	41·0	32·1	46·0	41·0	40·0	32·9	53·1	51·4	38·0	10·6	81°·4	80°·0
·5	103·3	—	—	47·1	47·0	45·9	29·0	45·2	42·0	41·0	32·5	46·0	42·0	41·0	33·2	—	—	—	—	—	—
	97·0	98·2	18·0	53·9	54·1	52·2	27·9	43·0	40·0	40·0	33·0	43·3	40·5	39·1	33·0	53·9	49·1	42·1	12·1	80°·3	78°·0
	99·0	108·9	18·0	51·2	51·0	48·4	26·5	46·1	43·0	41·2	34·9	47·0	43·7	41·4	34·1	55·3	50·0	44·0	14·2	78°·5	76°·3
·5	101·0	106·3	24·1	43·7	42·9	40·5	28·9	45·1	—	—	34·2	44·0	—	—	33·4	56·9	52·1	38·0	9·0	76°·4	73°·9
	101·4	110·0	16·0	55·7	55·7	54·6	28·0	46·8	43·8	42·1	32·8	47·3	45·0	43·0	34·9	58·8	54·2	45·9	14·0	74°·8	73°·3
	98·0	105·3	16·5	49·1	49·1	45·9	27·2	44·9	—	—	36·0	44·1	—	—	36·5	53·5	49·4	41·0	12·6	—	—
	97·4	98·2	20·5	47·8	47·6	44·6	28·0	45·8	42·2	41·1	33·4	46·0	43·0	42·1	33·3	52·5	49·8	44·6	14·3	80°·3	77°·0
	98·4	99·5	19·8	51·2	51·1	48·0	25·1	44·8	41·3	39·9	32·8	44·2	41·4	40·0	31·9	49·0	46·4	43·0	11·0	82°·0	80°·3
·5	94·0	107·0	21·8	46·8	46·6	42·0	26·9	45·3	41·9	40·3	29·9	44·1	41·3	39·7	29·1	56·0	52·1	36·1	11·9	75°·1	72°·7
	93·1	96·2	—	51·0	52·1	49·1	25·0	44·1	—	—	34·0	43·9	—	—	32·1	—	—	—	—	83°·3	81°·0
·5	100·0	104·0	21·0	50·9	50·8	48·0	28·0	45·1	42·6	41·0	31·9	45·5	42·9	40·1	31·1	56·4	52·1	49·0	12·0	80°·2	78°·0
·5	101·6	98·1	18·0	52·1	52·0	49·9	26·8	42·0	39·8	38·0	33·6	41·0	39·7	37·2	32·0	52·1	49·8	43·9	12·8	80°·4	79°·0
	102·0	110·5	23·5	53·9	53·8	47·8	30·9	45·1	43·0	40·5	34·1	45·8	42·0	40·0	33·0	63·5	55·0	48·0	15·7	80°·0	78°·6
	99·4	104·8	18·0	47·9	48·0	47·4	20·3	46·0	—	42·0	31·1	45·8	—	—	31·3	55·9	51·8	38·8	13·0	74°·0	72°·7

* A number of *PH*'s were not recorded.

NHL	*NH′*	*NB*	O_1, R	O_1', R	*Lacr.* O_1, R	O_2, R	O_1, L	O_1', L	*Lacr.* O_1, L	O_2, L	G_1	G_1'	G_2	*EH*	*Alveolar* $P \angle$	*Prosthion* $P \angle$	$N \angle$	$A \angle$	$B \angle$	*Oc. I.*	$100 \frac{B}{L}$
47·1	44·3	28·0	42·9	39·3	38·0	33·0	42·1	39·0	36·2	32·3	57·0	51·9	—	—	78°·0	75°·8	75°·7	67°·7	36°·6	63·0	73·3
48·1	43·0	25·4	44·8	41·7	41·0	33·6	47·3	44·0	—	32·5	50·9	48·0	39·9	11·2	77°·6	74°·2	72°·1	66°·9	41°·0	59·4	73·6
49·1	46·5	27·0	42·5	40·1	39·3	33·0	44·0	40·3	39·5	32·9	53·9	48·0	36·7	10·3	77°·0	74°·0	74°·4	67°·4	38°·2	57·1	71·3
50·5	45·1	27·0	43·2	39·5	38·9	32·5	43·1	39·5	38·8	33·0	56·1	52·0	40·0	10·1	81°·1	79°·0	73°·5	70°·2	36°·3	60·0	73·5
49·6	47·3	22·0	43·0	40·0	39·0	33·4	42·6	40·6	39·7	33·1	51·1	49·0	40·7	8·2	75°·8	74°·2	73°·9	67°·0	39°·1	60·0	69·9
50·4	50·0	28·2	43·9	41·0	40·1	34·0	45·2	42·0	40·9	33·9	54·0	49·1	40·3	12·0	80°·3	78°·0	71°·6	69°·9	38°·5	59·5	73·2
49·0	46·0	30·0	42·8	40·0	39·0	32·9	—	—	—	—	—	—	—	—	77°·6	75°·5	76°·4	66°·3	37°·3	57·8	73·7
51·5	50·4	26·1	42·4	40·0	—	33·9	42·9	39·5	38·1	32·9	51·9	48·9	42·9	12·5	82°·0	80°·3	72°·7	69°·3	38°·0	58·7	72·4
46·0	45·1	26·0	42·1	39·8	38·2	30·9	43·5	41·0	38·9	31·0	56·9	50·5	48·3	12·6	78°·0	76°·1	74°·6	66°·1	39°·3	58·5	73·2
48·5	44·8	27·6	45·1	42·0	41·2	32·1	43·0	40·2	40·0	32·2	54·1	49·0	43·3	10·0	80°·0	78°·2	75°·8	65°·8	38°·4	55·5	71·7
51·0	49·1	28·2	47·2	43·0	43·0	34·7	46·5	43·0	41·9	33·9	59·7	53·0	44·1	12·2	76°·2	74°·1	73°·0	68°·0	39°·0	59·2	70·6
53·9	51·0	27·0	46·8	—	—	37·0	48·0	42·4	42·4	37·0	54·8	47·9	—	—	85°·1	83°·0	67°·1	74°·9	38°·0	58·5	65·2
52·6	46·9	27·1	43·9	41·0	40·0	35·0	43·6	40·5	39·9	35·1	52·3	47·0	39·4	13·2	78°·3	76°·1	71°·0	66°·4	42°·6	56·8	70·7
47·4	44·7	23·5	43·8	41·0	39·5	28·5	42·0	39·8	38·0	29·1	55·4	54·0	43·9	12·3	75°·0	71°·2	79°·3	66°·3	34°·4	62·2	67·4
54·5	47·7	25·5	45·1	42·1	42·1	32·1	44·1	42·4	42·4	34·0	62·9	55·5	45·1	14·0	72°·3	70°·1	77°·8	62°·4	39°·8	55·8	71·7
58·3	55·1	27·5	—	—	—	—	46·0	42·9	42·0	36·0	53·2	50·1	43·0	11·6	82°·1	—	68°·0	72°·2	39°·8	60·8	69·6
51·0	48·9	28·8	45·1	42·5	41·0	32·0	44·3	42·4	40·1	31·8	59·2	51·2	39·1	10·0	78°·7	75°·3	71°·5	71°·5	37°·0	56·9	74·1
53·2	46·9	29·0	43·5	41·9	40·2	34·1	43·0	40·1	39·0	33·8	—	—	—	—	—	—	—	—	—	58·5	68·5
55·5	54·0	24·1	45·0	42·0	42·0	34·2	44·0	40·0	—	35·0	55·2	50·9	38·9	13·0	78°·2	77°·1	71°·1	67°·7	41°·2	58·6	67·0
—	—	—	41·0	—	—	30·3	—	—	—	—	—	—	—	—	—	—	73°·5	66°·4	40°·1	61·7	75·8
48·2	46·0	27·0	43·7	41·2	40·0	31·9	42·3	40·2	39·0	32·1	54·1	49·5	40·9	10·0	81°·0	78°·2	70°·9	72°·7	36°·4	60·5	70·5
48·7	47·5	28·0	43·9	—	—	31·1	—	—	—	—	—	—	—	—	—	—	67°·4	76°·4	36°·2	57·3	64·9
45·3	43·0	27·2	43·2	41·0	38·9	30·1	44·0	40·3	38·9	30·0	—	—	—	—	74°·3	73°·0	80°·3	65°·3	34°·4	58·1	70·8
52·0	50·5	27·1	43·8	40·5	40·5	35·1	44·3	41·3	41·3	33·4	55·1	47·9	37·5	9·0	82°·2	79°·8	69°·6	71°·9	38°·5	58·8	71·2
52·1	49·0	28·9	43·5	41·1	40·1	32·0	45·9	43·0	41·9	31·8	59·0	51·3	42·2	9·5	77°·2	75°·0	72°·9	71°·8	35°·3	62·0	71·2
—	46·0	27·0	44·9	41·0	40·0	34·0	45·2	41·9	41·0	33·5	53·1	49·0	38·0	8·2	80°·1	—	71°·9	71°·8	36°·3	58·6	71·3
53·3	50·5	29·9	44·0	40·9	39·1	33·0	43·5	41·0	39·8	33·5	54·9	48·1	43·2	10·8	82°·3	80°·3	68°·4	72°·9	38°·7	65·3	73·2
51·0	48·1	28·6	41·6	40·0	39·0	29·2	42·9	40·0	38·8	30·4	55·2	51·0	45·5	9·0	77°·0	75°·0	78°·5	65°·9	35°·6	61·8	75·4
57·0	51·5	28·9	46·0	42·0	40·9	34·0	45·0	41·8	40·7	33·2	60·0	53·8	40·8	—	78°·0	75°·8	76°·2	66°·7	37°·1	58·6	73·7
62·0	55·2	28·5	46·0	42·0	—	34·4	45·0	41·6	40·1	35·0	54·0	48·9	42·2	13·2	85°·2	83°·1	66°·8	66°·8	46°·4	56·2	65·8
55·2	50·9	27·6	45·5	42·0	40·0	30·3	42·9	41·9	39·0	30·9	61·0	55·2	—	—	81°·0	78°·1	71°·4	66°·8	41°·8	57·4	72·8
55·1	47·0	28·9	45·3	42·0	—	35·5	44·9	42·7	41·5	34·1	57·9	55·1	—	—	78°·1	75°·1	72°·5	66°·3	41°·2	60·7	71·0
49·8	46·5	25·1	43·0	41·5	38·2	31·3	43·1	41·0	38·2	31·3	55·2	48·5	39·7	10·0	81°·8	80°·5	73°·1	71°·1	35°·8	58·4	68·3
52·8	48·3	28·2	46·0	43·0	41·2	32·5	45·0	41·0	41·0	32·8	54·9	52·0	42·9	11·7	77°·0	73°·7	73°·6	67°·8	38°·6	61·5	69·7
58·4	50·7	26·7	44·2	42·0	40·0	35·6	46·2	43·0	41·1	35·1	58·0	53·0	40·0	14·1	79°·6	78°·0	70°·4	64°·8	44°·8	57·4	77·2
47·5	45·4	30·9	46·2	41·8	41·8	32·3	44·0	40·0	40·0	32·2	58·0	55·0	44·6	7·3	77°·3	75°·0	74°·6	70°·1	35°·3	62·2	71·3
54·1	50·7	26·8	41·9	—	37·3	32·0	42·0	—	38·9	31·0	54·1	50·6	44·0	10·5	77°·0	75°·1	78°·3	65°·3	36°·4	61·1	72·5
53·6	48·4	26·0	43·3	—	—	34·0	44·0	—	—	33·9	57·9	52·1	—	—	78°·0	75°·3	72°·7	66°·4	40°·9	61·4	72·1
49·0	46·6	24·0	40·5	—	—	33·0	42·0	39·5	38·2	33·1	56·0	53·0	41·3	9·5	75°·1	72°·8	77°·8	65°·4	36°·8	59·4	71·8
42·4	41·9	25·9	43·0	41·0	38·2	28·4	44·2	40·3	40·0	28·5	55·0	50·0	36·4	8·3	77°·2	75°·1	72°·4	71°·2	36°·4	61·4	72·8
51·0	48·0	30·8	48·1	44·5	43·0	34·6	47·0	43·0	41·2	34·0	64·0	56·1	43·1	9·0	78°·0	76°·4	74°·2	69°·1	36°·7	58·8	69·5
49·2	49·0	27·6	40·7	38·1	38·1	32·2	42·0	37·9	37·9	30·5	56·2	50·4	—	—	81°·9	80°·0	71°·4	71°·2	37°·4	59·1	75·0
48·7	46·5	31·0	47·0	43·0	43·0	32·0	47·0	42·1	40·9	32·0	53·0	48·0	38·5	10·2	82°·1	80°·2	69°·8	72°·2	38°·0	57·1	72·9
52·0	50·0	28·8	43·3	40·2	38·9	31·1	42·0	39·2	38·1	31·2	58·2	52·9	41·5	11·0	81°·3	78°·0	69°·7	70°·6	39°·7	58·6	71·1
46·8	45·0	27·3	45·3	42·1	41·0	33·0	46·1	42·1	40·4	32·0	54·5	50·2	—	—	87°·0	83°·0	71°·2	72°·5	36°·3	59·3	74·1
51·5	51·5	29·0	46·1	—	42·0	34·2	46·1	—	41·5	36·1	55·6	50·2	44·1	11·2	76°·7	76°·0	74°·8	68°·6	36°·6	58·1	69·6
55·3	51·0	28·5	43·4	—	—	31·7	44·1	—	—	32·6	57·1	52·0	40·2	14·5	80°·2	77°·9	72°·8	67°·2	40°·0	58·6	67·2
46·0	45·0	28·0	44·2	38·9	—	29·8	43·9	40·9	38·1	29·1	53·5	50·1	38·9	8·0	81°·0	79°·5	72°·8	69°·3	37°·9	56·9	74·7
54·0	52·2	25·5	46·2	42·7	41·0	32·1	46·0	41·0	40·0	32·9	53·1	51·4	38·0	10·6	81°·4	80°·0	72°·0	65°·7	42°·3	59·6	72·0
47·0	45·9	29·0	45·2	42·0	41·0	32·5	46·0	42·0	41·0	33·2	—	—	—	—	—	—	—	—	—	56·2	72·9
54·1	52·2	27·9	43·0	40·0	40·0	33·0	43·3	40·5	39·1	33·0	53·9	49·1	42·1	12·1	80°·3	78°·0	68°·9	68°·8	42°·3	59·6	74·0
51·0	48·4	26·5	46·1	43·0	41·2	34·9	47·0	43·7	41·4	34·1	55·3	50·0	44·0	14·2	78°·5	76°·3	76°·5	67°·3	36°·2	59·2	72·4
42·9	40·5	28·9	45·1	—	—	34·2	44·0	—	—	33·4	56·9	52·1	38·0	9·0	76°·4	73°·9	76°·8	66°·2	37°·0	58·9	74·2
55·7	54·6	28·0	46·8	43·8	42·1	32·8	47·3	45·0	43·0	34·9	58·8	54·2	45·9	14·0	74°·8	73°·3	74°·4	66°·8	38°·8	61·0	79·1
49·1	45·9	27·2	44·9	—	—	36·0	44·1	—	—	36·5	53·5	49·4	41·0	12·6	—	—	72°·9	71°·9	35°·2	64·0	72·6
47·6	44·6	28·0	45·8	42·2	41·1	33·4	46·0	43·0	42·1	33·3	52·5	49·8	44·6	14·3	80°·3	77°·0	73°·7	66°·0	40°·3	56·8	75·3
51·1	48·0	25·1	44·8	41·3	39·9	32·8	44·2	41·4	40·0	31·9	49·0	46·4	43·0	11·0	82°·0	80°·3	69°·0	70°·6	40°·4	58·5	71·3
46·6	42·0	26·9	45·3	41·9	40·3	29·9	44·1	41·3	39·7	29·1	56·0	52·1	36·1	11·9	75°·1	72°·7	77°·9	66°·6	35°·5	57·4	68·6
52·1	49·1	25·0	44·1	—	—	34·0	43·9	—	—	32·1	—	—	—	—	83°·3	81°·0	64°·2	77°·5	38°·3	63·1	69·3
50·8	48·0	28·0	45·1	42·6	41·0	31·9	45·5	42·9	40·1	31·1	56·4	52·1	49·0	12·0	80°·2	78°·0	75°·5	63°·6	40°·9	55·6	77·2
52·0	49·9	26·8	42·0	39·8	38·0	33·6	41·0	39·7	37·2	32·0	52·1	49·8	43·9	12·8	80°·4	79°·0	68°·7	70°·5	40°·8	58·4	70·8
53·8	47·8	30·9	45·1	43·0	40·5	34·1	45·8	42·0	40·0	33·0	63·5	55·0	48·0	15·7	80°·0	78°·6	76°·5	64°·8	38°·7	55·3	76·2
48·0	47·4	20·3	46·0	—	42·0	31·1	45·8	—	—	31·3	55·9	51·8	38·8	13·0	74°·0	72°·7	74°·2	67°·0	38°·8	57·6	69·1

recorded.

n Collection of Male New Britain Crania at Chicago (Field Museum).

∠	*Oc. I.*	100 $\frac{B}{L}$	100 $\frac{H'}{L}$	100 $\frac{B}{H'}$	100 $\frac{(B-H')}{L}$	100 $\frac{G'H}{GB}$	100 $\frac{NB}{NH'}$	100 $\frac{NB}{NH'}$, *R*	100 $\frac{O_2}{O_1}$, *R*	100 $\frac{O_1}{O_2}$, *L*	100 $\frac{O_2}{O_1'}$, *R*	100 $\frac{O_2}{Lacr.O_1}$, *R*	100 $\frac{G_1}{G_1}$	100 $\frac{G_2}{G_1'}$	100 $\frac{EH}{G_2}$	100 $\frac{fmb}{fml}$	*Frontal,* F_2 R	*Frontal,* F_2 L	*Parietal,* P_2 R	*Parietal,* P_2 L
·6	63·0	73·3	71·1	103·1	+ 2·2	65·2	63·2	59·3	76·9	76·7	84·0	86·8	—	—	—	81·0	82	81	158	146
·0	59·4	73·6	76·7	96·0	− 3·1	67·4	59·1	53·7	75·0	68·7	80·6	82·0	78·4	83·1	28·1	78·4	89	81	164	162
·2	57·1	71·3	73·2	97·7	− 1·9	74·4	58·1	52·8	77·6	74·8	82·3	84·0	68·1	76·5	28·1	80·2	84	83	167	169
·3	60·0	73·5	71·9	102·3	+ 1·6	68·9	59·9	53·5	75·2	76·6	82·3	83·5	71·3	76·9	25·2	83·6	81	82	166	160
·1	60·0	69·9	73·3	95·3	− 3·4	71·0	46·5	44·0	77·7	77·7	83·5	85·6	79·6	83·1	20·1	88·8	77	77	156	150
·5	59·5	73·2	74·1	98·9	− 0·8	65·6	56·4	56·0	77·4	75·0	82·9	84·8	74·6	82·1	29·8	88·3	88	88	163	161
·3	57·8	73·7	75·8	97·2	− 2·2	—	65·2	61·2	76·9	—	82·2	84·4	—	—	—	76·3	93	90	175	174
·0	58·7	72·4	73·2	98·9	− 0·8	68·9	51·8	51·1	80·0	76·7	84·8	—	82·7	87·7	29·1	84·2	88	87	164	159
·3	58·5	73·2	71·8	102·0	+ 1·4	69·4	57·6	55·3	73·4	71·3	77·6	80·9	84·9	95·6	26·1	84·3	77	76	156	156
·4	55·5	71·7	71·4	100·4	+ 0·3	71·6	61·6	56·6	71·2	74·9	76·4	77·9	80·0	88·4	23·1	87·9	86	86	162	159
·0	59·2	70·6	75·6	93·4	− 4·9	70·2	57·4	55·2	73·5	72·9	80·7	80·7	73·9	83·2	27·7	75·1	88	92	—	—
·0	58·5	65·2	67·7	96·3	− 2·5	81·5	52·9	50·9	79·1	77·1	—	—	—	—	—	81·6	76	74	156	160
·6	56·8	70·7	72·8	97·0	− 2·2	73·9	57·8	52·1	79·7	80·5	85·4	87·4	75·3	83·8	33·5	80·0	—	—	163	161
·4	62·2	67·4	74·0	91·1	− 6·6	66·6	52·6	48·5	65·1	69·3	69·5	72·2	79·2	81·3	28·0	88·9	87	86	153	155
·8	55·8	71·7	71·7	100·0	0·0	72·1	53·5	47·8	71·2	77·1	76·2	76·2	71·7	81·3	31·0	93·5	71	71	163	156
·8	60·8	69·6	69·6	100·0	0·0	—	49·9	47·3	—	78·3	—	—	80·8	85·8	27·0	86·1	81	83	158	159
·0	56·9	74·1	73·5	100·7	+ 0·5	73·0	58·9	56·5	71·0	71·8	75·3	78·0	66·0	76·4	25·6	88·1	91	83	170	163
–	58·5	68·5	71·0	96·4	− 2·6	—	61·8	54·7	78·4	78·6	81·4	84·8	—	—	—	92·8	85	83	178	171
·2	58·6	67·0	69·9	95·8	− 2·9	72·4	44·6	43·4	76·0	79·5	81·4	81·4	70·5	76·4	33·4	85·5	79	77	158	155
·1	61·7	75·8	76·4	99·3	− 0·6	—	—	—	73·9	—	—	—	—	—	—	73·6	—	—	—	—
·4	60·5	70·5	76·5	92·1	− 6·0	62·3	58·7	56·0	73·0	75·9	77·4	79·8	75·6	82·6	24·4	79·7	78	78	165	164
·2	57·3	64·9	75·4	86·1	−10·5	—	59·0	57·4	70·8	—	—	—	—	—	—	88·9	—	—	—	—
·4	58·1	70·8	75·0	94·4	− 4·2	66·1	63·3	59·3	69·7	68·2	73·4	77·4	—	—	—	77·2	91	91	157	159
·5	58·8	71·2	74·0	96·3	− 2·7	66·9	53·7	52·1	80·1	75·4	86·7	86·7	68·1	78·3	24·0	78·9	81	75	160	161
·3	62·0	71·2	72·8	97·8	− 1·6	63·9	59·0	57·2	73·6	69·3	77·9	79·8	71·5	82·3	22·5	84·0	79	81	157	163
·3	58·6	71·3	72·4	98·5	− 1·1	62·7	58·7	—	75·7	74·1	82·9	85·0	71·6	77·6	21·6	85·1	78	81	161	161
·7	65·3	73·2	76·9	95·2	− 3·7	70·7	59·2	56·1	75·0	77·0	80·7	84·4	78·7	89·8	25·0	80·6	83	81	155	155
·6	61·8	75·4	74·3	101·5	+ 1·1	66·0	59·5	56·3	70·2	70·9	73·0	74·9	82·4	89·2	19·8	87·5	84	82	158	152
·1	58·6	73·7	74·7	98·6	− 1·1	71·1	56·1	50·7	73·9	73·8	81·0	83·1	68·0	75·8	—	82·4	90	89	175	170
·4	56·2	65·8	69·7	94·4	− 3·9	79·8	51·6	46·0	74·8	77·8	81·9	—	78·1	86·3	31·3	76·3	87	79	156	157
·8	57·4	72·8	72·6	100·4	+ 0·3	72·7	54·2	50·1	66·6	72·0	72·1	75·8	—	—	—	85·9	—	—	—	—
·2	60·7	71·0	75·8	93·6	− 4·8	79·7	61·5	52·5	78·4	75·9	84·5	—	—	—	—	84·9	74	70	167	163
·8	58·4	68·3	71·5	95·5	− 3·2	66·3	54·0	50·4	72·8	72·6	75·4	81·9	71·9	81·9	25·2	83·9	81	82	154	158
·6	61·5	69·7	75·7	92·1	− 5·9	69·6	58·4	53·7	70·7	72·9	75·6	78·9	78·1	82·5	27·3	81·9	88	89	157	155
·8	57·4	77·2	76·4	101·1	+ 0·8	82·7	52·7	45·9	80·5	76·0	84·8	89·0	69·0	75·5	35·2	88·5	85	83	171	163
·3	62·2	71·3	71·8	99·3	− 0·5	67·7	68·1	65·1	69·9	73·2	77·3	77·3	76·9	81·1	16·4	79·2	86	89	166	161
·4	61·1	72·5	73·1	99·2	− 0·6	66·4	52·9	49·5	76·4	73·8	—	85·8	81·3	87·0	23·9	79·4	82	84	161	155
·9	61·4	72·1	75·4	95·7	− 3·3	71·3	53·7	48·6	78·5	77·0	—	—	—	—	—	79·2	—	—	166	166
·8	59·4	71·8	69·9	102·8	+ 1·9	71·3	51·5	49·0	81·5	78·8	—	—	73·8	77·9	23·0	86·2	75	73	148	153
·4	61·4	72·8	76·9	94·7	− 4·0	67·0	61·8	57·6	66·0	64·5	69·3	74·3	66·2	72·8	22·8	83·0	81	82	161	151
·7	58·8	69·5	73·7	94·3	− 4·2	68·6	64·2	60·4	71·9	72·3	77·8	80·5	67·3	76·8	20·9	76·7	—	—	166	168
·4	59·1	75·0	76·4	98·2	− 1·4	64·6	56·3	56·4	79·1	72·6	84·5	84·5	—	—	—	77·8	85	86	159	163
·0	57·1	72·9	75·0	97·2	− 2·1	63·5	66·7	62·9	68·1	68·1	74·4	74·4	72·6	80·2	26·5	79·2	83	83	166	165
·7	58·6	71·1	69·8	101·9	+ 1·3	72·5	57·6	57·6	71·8	74·3	77·4	79·9	71·3	78·4	26·5	73·8	84	82	162	152
·3	59·3	74·1	74·6	99·3	− 0·5	63·0	60·7	59·2	72·8	69·4	78·4	80·5	—	—	—	74·3	80	81	169	168
·6	58·1	69·6	71·2	97·8	− 1·6	70·2	56·3	55·2	74·2	78·3	—	81·4	79·3	87·8	25·4	78·9	86	85	165	162
·0	58·6	67·2	71·9	93·5	− 4·7	69·4	55·9	51·5	73·0	73·9	—	—	70·4	77·3	36·1	83·8	87	86	158	161
·9	56·9	74·7	70·9	105·4	+ 3·8	67·5	62·2	61·5	67·4	66·3	76·6	—	72·7	77·6	20·6	73·4	83	82	169	156
·3	59·6	72·0	74·2	97·0	− 2·2	71·2	48·9	47·2	69·5	71·5	75·2	78·3	71·6	73·9	27·9	81·6	—	—	154	153
—	56·2	72·9	71·3	102·2	+ 1·6	—	63·2	61·6	71·9	72·2	77·4	79·3	—	—	—	85·0	90	86	159	158
·3	59·6	74·0	77·3	95·7	− 3·4	73·2	53·4	51·8	76·7	76·2	82·5	82·5	78·1	85·7	28·7	87·6	84	82	163	156
·2	59·2	72·4	74·6	97·1	− 2·1	66·9	54·8	51·8	75·7	72·6	81·2	84·7	79·6	88·0	32·3	79·0	81	79	167	165
·0	58·9	74·2	77·8	95·4	− 3·6	64·6	71·4	66·1	75·8	75·9	—	—	66·8	72·9	23·7	80·1	77	75	163	159
·8	61·0	79·1	77·4	102·2	+ 1·7	70·2	51·3	50·3	70·1	73·8	74·9	77·9	78·1	84·7	30·5	84·4	88	87	158	158
·2	64·0	72·6	72·6	100·0	0·0	64·3	59·3	55·4	80·2	82·8	—	—	76·6	83·0	30·7	83·8	78	77	163	158
·3	56·8	75·3	71·8	104·8	+ 3·4	67·9	62·8	58·6	72·9	72·4	79·1	81·3	85·0	89·6	32·1	87·9	80	75	160	156
·4	58·5	71·3	74·6	95·5	− 3·3	69·7	52·3	49·0	73·2	72·2	79·4	82·2	87·8	92·7	25·6	79·4	84	82	161	156
·5	57·4	68·6	72·3	94·9	− 3·7	67·3	64·0	57·5	66·0	66·0	71·4	74·2	64·5	69·3	[illegible]	[illegible]	—	—	156	160
·3	63·1	69·3	75·7	91·5	− 6·4	70·9	50·9	49·0	77·1	73·1	—	—	—	—	—	[illegible]	86	87	164	166
·9	55·6	77·2	72·8	106·0	+ 4·3	70·0	58·3	55·0	70·7	68·4	74·9	77·8	86·9	94·0	24·5	83·1	85	83	167	173
·8	58·4	70·8	73·4	96·4	− 2·7	67·7	53·7	51·4	80·0	78·0	84·4	88·4	84·3	88·2	29·2	82·4	76	76	163	156
·7	55·3	76·2	72·7	104·8	+ 3·5	70·1	64·6	57·3	75·6	72·1	79·3	84·2	75·6	87·3	32·7	82·0	86	87	170	169
·8	57·6	69·1	72·7	95·0	− 3·6	68·4	42·8	42·4	67·6	68·3	—	74·0	69·4	74·9	33·5	73·5	86	86	169	167

Frontal, F_2		*Parietal*, P_2		*Occipital*, O_1		*Malar*, Ml_1		*Maxilla*, Mx_1		*Temporal*, T_1		*Temporal*, T_2		*Sphenoid*, S_3		Jugular foramina	Supra-orbital ridges	Pyriform aperture		Glenoid fossa
R	L	R	L	R	L	R	L	R	L	R	L	R	L	R	L					
82	81	158	146	86	82	61	64	62·2	62·2	87·7	85·0	44·0	46·2	37·1	36·9	JR	1	r	a	t
89	81	164	162	94	93	58	54	67·2	66·0	88·0	87·0	47·0	42·0	37·0	36·4	JR	2	r	i	sp. ang.
84	83	167	169	92	98	—	—	64·0	64·0	87·1	88·0	44·2	40·5	37·0	34·0	J=	3	r	a	sp. ang.
81	82	166	160	92	91	61	61	61·3	62·9	95·0	91·2	42·0	50·0	33·1	33·1	JR	2	r	a	sp. ang.
77	77	156	150	87	88	57	57	62·9	62·1	85·0	84·1	47·2	50·0	34·8	35·3	JR	2	sh	i	sp. ang.
88	88	163	161	92	94	66	66	64·0	63·0	90·0	88·4	55·1	52·0	37·0	37·0	JR	2	sh	i	sp. ang.
93	90	175	174	98	98	—	—	63·1	63·3	98·0	98·0	45·0	49·2	38·0	39·0	JR	2	r	i	sp. ang.
88	87	164	159	89	91	57	57	62·2	62·1	89·0	88·2	55·0	53·0	35·3	35·3	JR	2	sh	i	sp. ang. L, t R
77	76	156	156	86	94	56	56	62·3	61·7	95·0	92·1	44·1	44·0	32·3	32·3	JR	3	r	i	sp. ang. R, t L
86	86	162	159	98	102	68	68	63·0	63·0	87·5	89·6	52·0	53·0	32·6	32·5	JL	2	r	a	sp. ang.
88	92	—	—	—	—	—	—	—	—	—	—	—	—	—	—	JL	2	r	i	sp. ang.
76	74	156	160	98	96	65	65	68·0	67·2	89·5	89·0	45·7	51·2	35·0	36·0	JR	3	r	a	t
—	—	163	161	95	92	67	65	70·0	69·8	88·8	88·2	48·7	48·0	34·2	34·2	JR	3	r	a	sp. ang. R, t L
87	86	153	155	87	86	72	68	61·4	62·0	84·1	85·0	45·0	50·0	38·1	37·3	JR	2	r	a	sp. ang.
71	71	163	156	95	93	68	66	70·3	70·4	95·1	92·1	45·0	48·0	37·0	37·3	JR	3	r	p	t
81	83	158	159	91	89	—	—	71·1	70·6	102·0	99·0	51·2	51·5	39·2	39·2	JR	2	sh	i	t
91	83	170	163	89	92	63	63	63·6	63·6	95·0	94·3	44·8	46·2	36·7	39·0	JR	3	r	a	sp. ang.
85	83	178	171	96	97	66	64	—	—	96·2	94·3	52·0	50·5	38·1	36·0	JR	3	sh	a	t
79	77	158	155	90	88	61	67	69·6	70·0	89·0	84·5	50·0	49·2	35·2	37·0	JL	3	sh	i	t
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	JR	2–3	r	i	t
78	78	165	164	91	89	73	74	61·7	62·0	89·0	85·8	44·5	45·5	40·0	37·0	JR	2–3	sh	i	sp. ang.
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	JL	1	r	i	sp. ang. L, t R
91	91	157	159	78	83	—	—	—	—	—	—	—	—	—	—	JR	1–2	r	i	sp. ang.
81	75	160	161	95	103	62	62	63·2	63·2	91·0	91·0	39·0	45·0	36·3	35·2	JL	2–3	r	a	sp. ang.
79	81	157	163	89	88	60	58	—	—	99·5	96·2	49·0	48·6	33·0	34·3	JR	3	sh	a	sp. ang. L, t R
78	81	161	161	86	89	65	65	—	—	92·2	92·5	49·1	50·2	38·3	38·3	JR	2–3	r	i	t
83	81	155	155	91	87	66	66	65·4	65·0	96·0	96·2	41·0	46·0	36·0	38·0	J=	3	sh	a	t
84	82	158	152	84	81	59	59	62·0	62·2	84·6	89·5	45·4	45·0	35·0	34·7	JR	2	sh	i	sp. ang. R, t L
90	89	175	170	92	91	67	68	67·9	67·9	98·2	95·0	48·0	47·0	35·0	41·0	JR	2	r	a	sp. ang.
87	79	156	157	85	87	67	67	76·5	76·2	97·1.	98·1	44·2	44·0	36·2	37·0	JR	2	r	a	sp. ang.
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	JR	2–3	r	a	sp. ang.
74	70	167	163	88	81	—	—	74·0	74·0	99·0	95·0	52·1	54·5	36·2	39·0	JR	3	r	a	t
81	82	154	158	90	95	61	61	—	—	99·0	98·0	46·3	48·1	34·3	34·3	J=	2–3	sh	a	t
88	89	157	155	85	96	62	61	67·9	67·9	90·2	91·0	49·7	50·0	32·0	33·2	JR	3	sh	a	t
85	83	171	163	89	91	66	64	76·1	76·1	93·0	93·0	50·0	49·5	36·0	38·0	JR	2–3	r	a	sp. ang.
86	89	166	161	88	87	63	63	63·0	63·0	91·0	93·2	49·0	50·0	38·3	38·3	JR	2·	sh	i	sp. ang. R, t L
82	84	161	155	98	92	—	—	63·0	62·2	100·0	90·0	—	—	36·0	36·0	JR	2–3	r	a	sp. ang. L, t R
—	—	166	166	86	81	60	61	69·6	69·4	93·0	92·3	45·0	49·0	34·0	35·3	JL	2	r	a	sp. ang.
75	73	148	153	92	89	61	60	64·5	63·2	86·0	88·0	43·3	43·0	—	—	JR	2	r	a	t
81	82	161	151	85	90	58	59	59·8	60·9	87·0	85·0	46·0	48·3	34·1	34·1	JR	2	r	i	sp. ang.
—	—	166	168	88	88	65	64	62·8	63·4	90·5	93·0	55·0	53·2	36·8	36·2	JR	3	r	i	sp. ang.
85	86	159	163	84	82	62	62	61·0	61·0	88·0	90·6	45·0	45·0	32·6	34·9	JL	3	r	i	sp. ang.
83	83	166	165	91	100	64	67	59·6	60·2	97·0	97·1	48·5	48·0	38·2	35·2	JR	2–3	r	i	sp. ang. R, t L
84	82	162	152	89	86	72	71	64·1	64·0	94·0	95·0	49·0	49·6	35·2	37·0	JR	3	r	i	sp. ang.
80	81	169	168	91	92	53	58	62·0	62·0	94·2	91·0	49·0	52·2	36·2	35·0	JR	3	r	i	sp. ang.
86	85	165	162	91	94	—	—	64·0	63·0	108·2	105·2	44·1	44·1	32·2	35·0	JR	2–3	sh	i	sp. ang.
87	86	158	161	98	92	61	61	70·9	70·0	89·6	91·0	50·0	53·1	36·1	37·2	J=	2–3	r	a	t
83	82	169	156	102	100	62	62	59·4	59·6	85·2	85·7	41·3	45·0	36·0	36·0	JR	1	r	i	t
—	—	154	153	85	87	59	58	68·8	69·0	93·8	93·0	47·7	47·2	35·0	36·0	JL	2–3	sh	a	sp. ang.
90	86	159	158	87	82	70	68	—	—	97·1	96·2	49·3	55·3	35·3	37·2	JL	2	sh	p	t
84	82	163	156	92	89	—	—	68·0	68·0	86·2	86·0	43·6	45·7	34·6	34·6	JR	3	r	i	t
81	79	167	165	90	89	65	67	65·0	64·0	104·2	97·0	59·0	57·6	36·3	35·2	JR	2	sh	i	t
77	75	163	159	95	93	60	64	63·9	64·3	86·0	83·0	53·0	51·1	36·0	36·5	JR	2–3	r	i	sp. ang.
88	87	158	158	86	83	65	67	66·2	67·0	93·1	90·2	50·6	53·0	36·0	34·0	JR	2–3	sh	p	t
78	77	163	158	93	93	—	—	60·3	60·1	96·0	94·0	53·8	50·2	37·0	38·0	J=	2–3	r	a	sp. ang. L, t R
80	75	160	156	93	85	61	64	64·0	64·0	93·0	93·7	43·5	44·2	36·1	36·1	JL	2–3	sh	a	t
84	82	161	156	80	81	60	63	66·0	65·0	99·5	100·3	49·2	46·2	37·0	38·4	JR	3	r	i	t
—	—	156	160	89	86	68	71	63·8	62·5	83·0	87·0	43·2	47·0	36·0	37·0	JR	2	r	p	sp. ang.
86	87	164	166	94	100	63	63	62·2	62·2	87·1	88·0	49·4	51·2	—	—	JR	2	sh	i	sp. ang.
85	83	167	173	100	103	57	57	65·8	67·0	96·0	93·0	44·8	47·2	35·1	37·7	JR	2	sh	p	sp. ang.
76	76	163	156	79	82	70	67	66·7	66·7	97·6	94·2	47·0	49·0	40·0	38·5	JR	2–3	sh	i	sp. ang.
86	87	170	169	98	96	60	59	71·0	71·0	90·4	91·8	47·4	49·2	37·4	37·4	JR	3	r	a	sp. ang.
86	86	169	167	96	96	65	64	66·0	65·0	96·2	94·3	46·2	50·8	33·1	33·1	JR	3	sh	a	sp. ang.

Glenoid fossa	Sylvian fissure R	Sylvian fissure L	Third frontal gyre R	Third frontal gyre L	Tympanic perforations	Conformation at pterion
t	3	2	2	1	—	R and L os epipt.
sp. ang.	2	1	1	1	—	"
sp. ang.	3	2	1	0	R and L: large hole on medial side	"
sp. ang.	4	3	4	2	—	"
sp. ang.	4	4	4	3	—	"
sp. ang.	4	2	4	2	—	R sut. fronto-temp.
sp. ang.	2	3	1	2	—	"
sp. ang. L, t R	2	2	3	3	—	"
sp. ang. R, t L	1	1	1	2	L: on medial side	R and L sut. fronto-temp.
sp. ang.	2	3	2	1	R and L on medial side	"
sp. ang.	?	?	?	?	—	L two long ossa epipt.
t	4	3	3	3	—	"
sp. ang. R, t L	3	2	3	2	L small hole	"
sp. ang.	4	2	2	3	—	L large os epipt.
t	3	2	2	2	—	Stenokrotaphie
t	2	1	3	2	—	Stenokrotaphie
sp. ang.	2	3	2	1	—	R os epipt.
t	2	1	1	1	—	Sutures partly closed
t	3	1	1	1	—	"
t	2	1	1	1	L one small hole	"
sp. ang.	2	2	3	2	—	"
sp. ang. L, t R	1	1	2	1	—	"
sp. ang.	1	2	2	2	—	L os epipt.
sp. ang.	2	1	3	2	L one small hole	"
sp. ang. L, t R	2	1	2	2	—	R and L proc. front. ossis temp.
t	3	2	2	1	—	"
t	3	2	1	1	—	"
sp. ang. R, t L	2	1	2	1	R and L: small hole	"
sp. ang.	3	1	1	1	R and L: small holes	"
sp. ang.	2	1	0	0	R and L: small holes	L proc. front. ossis temp.
sp. ang.	2	2	2	2	—	Sutures closed
t	2	2	2	1	—	"
t	2	3	3	2	R small holes	R and L proc. front. ossis temp.
t	3	4	1	2	—	R proc. front. ossis temp. L Stenokrotaphie
sp. ang.	2	1	1	2	—	"
sp. ang. R, t L	2	1	2	1	—	R os epipt.
sp. ang. L, t R	2	1	2	2	—	R proc. front. ossis temp.
sp. ang.	1	2	1	2	—	"
t	2	2	1	1	—	Sutures closed
sp. ang.	3	2	2	1	—	Sutures closed
sp. ang.	3	2	1	1	—	"
sp. ang.	1	1	1	1	—	"
sp. ang. R, t L	3	3	1	1	—	Sutures closed
sp. ang.	2	1	2	3	—	"
sp. ang.	2	1	2	1	—	R os epipt.
sp. ang.	2	2	3	3	—	R and L proc. front. ossis temp.
t	1	1	1	1	—	"
t	1	1	1	1	—	"
sp. ang.	2	2	3	3	—	Stenokrotaphie
t	2	2	3	3	R one hole	R proc. front. ossis temp.
t	2	3	3	2	—	"
t	1	1	2	2	—	R proc. front. ossis temp.
sp. ang.	1	1	2	2	—	"
t	2	1	2	1	—	"
sp. ang. L, t R	1	1	2	2	—	"
t	1	1	1	1	Holes on both sides	R os epipt.
t	1	1	2	2	—	"
sp. ang.	2	1	1	0	—	"
sp. ang.	2	1	2	1	—	"
sp. ang.	2	1	1	3	—	"
sp. ang.	1	1	1	1	—	"
sp. ang.	3	3	2	1	—	"
sp. ang.	1	1	1	2	—	L proc. front. ossis temp. R os epipt.

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Serial No.	*L*	*B*	*H′*	*LB*	*B′*	*U*	*Bregmatic Q′*	S_1'	S_2'
3920	182	136	135	99	97	507	316	109	119
3921	185	124	133	100	90	502	300	110	124
3922	185	126	138	107·5	84	498	296	105	124
3925	185	137	136	101·5	98	521	315	106	125
3928	183	124	133·5	99	96·5	503	303	113	118
3929	194	129	140·5	111	101	533	315	115	124
3930	176·5	125	128	93	88	485	291	109	113
3931	186	132	140	101	95·5	511	314	115	122
3932	180	134	134	101	85·5	499	305	112	111
3934	181	132	131	100	88	495	300	106	114
3936	180	132	132	100·5	99	507	305	110	120
3938	186·5	132	137	99·5	91	508	295	108	124
3940	182	131	138	104	91·5	500	300	107	113
3942	186	127	141·5	102·5	95	510	305	113	118
3943	184	133	133	92	92	514	300	110	120
3944	183	128	139	101	94	505	297	114	119
3945	182	132	130	98	84·5	503	304	111	112
3947	188	135	139	104	99	519	310	113	120
3949	179	127	128	97	92	496	292	104	106
3959	183	127	—	—	86	506	295	109	120
3963	180	133	128	93	94	486	304	108	109
3969	190	134	138	105	105·5	534	308	110	119
3972	181	128	132	96·5	85	494	295	111	117
3978	182	133·5	134	96	92	512	308	109	121
3981	179·5	127	128	98	87	494	287	105	117
3985	185	131	136	97	96·5	517	303	109	116
3989	178·5	125	128	97	95	496	283	103	108
3990	178	125	127	95	92·5	496	293	107	115
3991	188	131·5	136	97·5	102	518	310	114	128
3993	195	129	134	98·5	102	533	299	112	132
3996	185	133·5	132	100·5	90	512	300	108	116
4000	191	132	125	108	89	515	290	111	112
4003	182	138	133·5	101	92	509	311	111	113
4005	187	134·5	134	98	94·5	525	310	107	118
4045	189	130	133	101	98·5	512	310	106	123
4046	186·5	132	132	100	97	514	300	112	119
4047	194	142	135	104	96	536	320	122	115
4048	187	134	143	106	101	522	317	113	121
4051	190	128·5	141	108·5	85	507	305	113	128
4054	185	138	132	104	96	510	305	111	112
4056	174	130	137	96	86	488	302	111	120
4057	185	130	135	100	93	502	305	107	124
4060	175	136	132	98	92	490	310	106	111
4061	190	134	130	98·5	97·5	524	302	108	121
4075	184	126	139	103·5	95	498	300	114	122
4087	184	136	—	—	100·5	513	317	110	120
4100	189	137	140	101	93	517	309	121	123
4102	176	126	134	96·5	92	485	300	108	119
4106	191	135	137	100·5	90·5	517	302	111	127
4112	190	131	142	100	92	520	307	121	130
4113	179	126	135	95·5	94	496	297	107	123
4114	181·5	127	136	99	90·5	502	302	112	110
4115	185	134·5	141·5	100	93	517	310	117	121
4116	180	127	126	95	95	499	290	106	108
4118	174·5	134·5	141·5	104	92	497	314	106	111
4122	184	127	132	96	90·5	498	295	110	117
4124	187	132	131	101	91·5	511	299	113	113
4127	182	137	129	99·5	90	504	301	108	115
4129	191	134	134	105	97	525	304	108	124
4133	187	136·5	139	103	95·5	522	310	116	107
4134	183·5	128	138	103	88	497	302	111	118
4137	181	137	136	98	101	512	306	109	112
4200	186	132	130	97·5	92	505	305	114	116
4201	186	127	134	105·5	99	512	311	111	120
Mean	184·2	131·3	134·5	100·2	93·4	508·0	303·4	110·4	118·1
Number	64	64	62	62	64	64	64	64	64

Appendix B. *Individual Measurements of Male New*

Bregmatic Q′	S_1'	S_2'	S_3'	S_1	S_2	S_3	S	*fml*	*fmb*	*G′H*	*J*	*GB*
316	109	119	96·1	125	131	118	375	32·5	25	63·5	127	98
300	110	124	89·6	124	142	108	374	38	27	62	—	98
296	105	124	97·2	116	142	111	369	38	30	71	131	99
315	106	125	99·5	119	142	120	381	32·5	31·5	71·5	143	107·5
303	113	118	97·0	127	128	118	373	36	27·5	61·5	128	95·5
315	115	124	103·0	130	137	121	388	35	28	69	133	95
291	109	113	100·0	121	130	123	374	32	24·5	60	129	94
314	115	122	96·5	128	134	117	379	35	28·5	73	135	104
305	112	111	94·5	128	120	115	363	31	27	64	131	93
300	106	114	93·0	117	128	114	360	36·5	26·5	66	138	103·5
305	110	120	90·5	126	134	108	367	33	28	69	—	98·5
295	108	124	99·2	121	139	120	380	39	34	66·5	134	96
300	107	113	105·0	120	128	125	374	32	28	59·5	137	97
305	113	118	92·0	126	135	113	375	37	31	71	136	102
300	110	120	103·0	125	135	124	384	37	28	70	136	95
297	114	119	94·0	131	134	112	377	33	27	68	137	101
304	111	112	99·0	126	124	117	368	34	27·5	69·5	139	96
310	113	120	103·0	127	134	126	386	34	28·5	69	139	107
291	104	106	101·2	115	117	126	359	35	28·5	65	132	98
295	109	120	—	127	137	—	—	—	—	68	125	92·5
304	108	109	92·0	125	120	113	358	35	25	65	131	98
308	110	119	98·0	123	133	121	377	38·5	29	—	148	99·5
295	111	117	96·0	124	131	111	366	34	28	68	132	88
308	109	121	97·0	127	135	117	379	36	31	70	137	99
287	105	117	89·3	117	131	108	356	34	32	66·5	123	89
303	109	116	98·0	127	128	126	381	33	27	68	139	100·5
283	103	108	101·0	118	120	124	362	34	30	62·5	130	99
293	107	115	88·9	121	127	110	358	35	25·5	70	133	97
310	114	128	86·8	138	142	106	386	33	23·5	60	130	93
299	112	132	97·5	126	149	121	396	36	28	71	139	98
300	108	116	96·7	122	131	117	370	34	29	67	131	91
290	111	112	100·2	124	123	122	369	32·5	27	66	—	—
311	111	113	99·0	122	127	115	364	40	32	73	136	98
310	107	118	105·3	120	132	131	383	35	27	72	135	99
310	106	123	101·8	119	137	120	376	37	29·5	68	136	98
300	112	119	96·5	126	134	114	374	37	29·5	73	138	96
320	122	115	110·5	136	124	136	396	34·5	30	68	146	100·5
317	113	121	100·0	128	137	121	386	36·5	33	70	138	101·5
305	113	128	93·0	128	147	105	380	36	30	65	131	94
305	111	112	102·0	122	127	123	372	33	31	73	136	105
302	111	120	90·5	124	137	110	371	29	26	64	—	99
305	107	124	94·5	118	139	116	373	31·5	26	66·5	137	100·5
310	106	111	96·3	121	126	115	362	32	29	—	133	94
302	108	121	99·5	120	135	123	378	41	32	70	—	102
300	114	122	96·2	127	135	112	374	34·5	27·5	77	—	100
317	110	120	96·5	123	137	110	370	—	—	—	—	98·5
309	121	123	94·0	140	140	108	388	35	28	75	142	100
300	108	119	91·2	117	137	109	363	35	28	65·5	132	94·5
302	111	127	95·0	128	145	116	389	35	29·5	62	138	91
307	121	130	87·0	144	147	106	397	35	27·5	68·5	140	96
297	107	123	94·0	122	141	112	375	34	29	—	132	102·5
302	112	110	110·0	125	121	135	381	31	29	64	138	91
310	117	121	93·4	132	138	112	382	35	29	68	142	101
290	106	108	94·2	122	121	117	360	33	28	76	138	99·5
314	106	111	94·8	119	126	112	357	38·5	29	62	139	96
295	110	117	91·7	125	133	116	374	36	27	64·5	131	95
299	113	113	91·5	129	126	113	368	36·5	26	65·5	135	91·5
301	108	115	98·2	123	130	120	373	29	29	75	138	95
304	108	124	98·0	120	141	119	380	36	29	66·5	142	103
310	110	107	102·6	130	118	129	377	35	28·5	66	138	102·5
302	111	118	98·2	125	131	116	372	33·5	26	61·5	132	94
306	109	112	97·1	125	125	121	371	32·5	28	—	138	96
305	114	116	92·2	130	130	115	375	34	27	68	131	94·5
311	111	120	93·0	124	134	118	376	31	30	67·5	142	102
303·4	110·4	118·1	96·6	124·8	132·5	117·1	374·3	34·6	28·4	67·6	135·4	97·7
64	64	64	63	64	64	63	63	62	62	59	57	63

Male New Britain Crania at Berlin (Kaiser Wilhelm Institut).

J	*GB*	*GL*	*NH'*	*NB*	O_1, R	O_2, R	G_1	G_2	$N\angle$	$A\angle$	$B\angle$	*Oc. I.*	$100\frac{B}{L}$	$100\frac{H'}{L}$	$100\frac{B}{H'}$
127	98	103	44·5	27·5	42	32	—	—	75°·7	68°·2	36°·1	58·2	74·7	74·2	100·7
—	98	101·5	47	28	45	32	—	—	73°·8	70°·6	35°·6	59·6	67·0	71·9	93·2
131	99	112	49	24	44·5	35	57	44·5	75°·2	67°·2	37°·6	66·1	68·1	74·6	91·3
143	107·5	115	51·5	26	45	32	62	45	81°·2	60°·7	38°·1	59·6	74·1	73·5	100·7
128	95·5	100·5	48·5	26	43·5	34	56	41	73°·7	70°·6	35°·7	58·9	67·8	73·0	92·9
133	95	112	48	27·5	44	33	59·5	42	73°·1	70°·9	36°·0	62·1	66·5	72·4	91·8
129	94	99	42	25	43	29	—	—	77°·4	66°·5	36°·1	58·1	70·8	72·5	97·7
135	104	108·5	48·5	28	45	36·5	62	42·5	75°·2	64°·2	40°·6	59·2	71·0	75·3	94·3
131	93	104	45	23	40	32·5	54	38	74°·1	69°·5	36°·4	58·8	74·4	74·4	100·0
138	103·5	107	46	27	43	33·5	56·5	41	77°·2	65°·7	37°·1	58·3	72·9	72·4	100·8
—	98·5	100·5	48	31	47	35	54	40·5	70°·0	70°·0	40°·0	60·5	73·3	73·3	100·0
134	96	104	48	28	44·5	34	55	32·5	74°·6	67°·2	38°·2	59·3	70·8	73·5	96·4
137	97	108	44	26	43	29	55·5	42	77°·7	69°·7	32°·6	60·8	72·0	75·8	94·9
136	102	104	49	29	44	34	55·5	41·5	71°·2	68°·4	40°·4	58·2	68·3	76·1	89·8
136	95	99	48	27	43	35	58	44·5	74°·0	63°·2	42°·8	59·7	72·3	72·3	100·0
137	101	108	45	31	43	33·5	58·5	—	76°·7	65°·3	38°·0	60·7	69·9	76·0	92·1
139	96	108·5	49	25	45	34	55	38	78°·4	62°·6	39°·0	61·5	72·5	71·4	101·5
139	107	107·5	45·5	31	42	32	60·5	45	73°·5	68°·1	38°·4	58·5	71·8	73·9	97·1
132	98	99	45	25	41	34·5	53	38	73°·0	68°·7	38°·4	57·3	70·9	71·5	99·2
125	92·5	—	44	27	39	34	—	—	—	—	—	—	69·4	—	—
131	98	95	48	25·5	42	35	51	37	71°·8	67°·8	40°·4	58·2	73·9	71·1	103·9
148	99·5	—	50	28	44	34	—	—	—	—	—	57·8	70·5	72·6	97·1
132	88	98	46	28	42	32	57·5	40	71°·1	68°·3	40°·6	64·2	70·7	72·9	97·0
137	99	102·5	46·5	25	42	35	—	—	74°·7	64°·2	41°·1	59·6	73·4	73·6	99·6
123	89	105	45	26	40	33	58	39	77°·1	65°·2	37°·7	59·4	70·8	71·3	99·2
139	100·5	95	44	25	46	34	49	—	67°·7	70°·8	41°·5	55·7	70·8	73·5	96·3
130	99	103	43·5	27	44·5	36	55	39	77°·2	66°·7	36°·1	58·2	70·0	71·7	97·7
133	97	104·5	48·5	26	41	33	54	42	77°·1	62°·6	40°·4	57·7	70·2	71·3	98·4
130	93	104	41	23	42	31	53	35	78°·5	67°·0	34°·5	58·6	69·9	72·3	96·7
139	98	103	44	27	46·5	35	62	36	73°·4	65°·7	40°·9	57·5	66·2	68·7	96·3
131	91	106	48	24	44	30·5	56	43·5	75°·9	66°·5	37°·6	55·5	72·2	71·4	101·1
—	—	105·5	52	28	46	33·5	56	42	70°·2	73°·8	36°·0	58·8	69·1	65·4	105·6
136	98	111	49	27	44	34	—	46	77°·6	62°·3	40°·1	63·5	75·8	73·4	103·4
135	99	107	49·5	24	46	34·5	53	39	76°·2	62°·7	41°·1	57·3	71·9	71·7	100·4
136	98	103	49	27	43	35	54	39·5	72°·2	68°·7	39°·1	61·8	68·8	70·4	97·7
138	96	106	51	28	43	38	60	38·5	74°·2	64°·7	41°·1	61·6	70·8	70·8	100·0
146	100·5	116	50	28	45	33	60	—	81°·8	62°·6	35°·6	58·0	73·2	69·6	105·2
138	101·5	112	52	29	44	34	—	45	76°·6	66°·7	36°·7	59·3	71·7	76·5	93·7
131	94	107	48	26	44	32	54	39	71°·7	73°·2	35°·1	58·1	67·6	74·2	91·1
136	105	112	48·5	29·5	45·5	31	62	39	76°·6	64°·7	38°·7	59·6	74·6	71·4	104·5
—	99	111	44	27	41	30·5	—	43	85°·4	59°·7	34°·9	59·0	74·7	78·7	94·9
137	100·5	111·5	44	27	40	31	62	38	81°·5	62°·4	36°·1	58·2	70·3	73·0	96·3
133	94	—	47	25	41	32	—	—	—	—	—	60·5	77·7	75·4	103·0
—	102	106	48	31	46	36	57	39	76°·2	64°·2	39°·6	57·7	70·5	68·4	103·1
—	100	106	55	27	43	38	—	41	70°·2	66°·2	43°·6	63·3	68·5	75·5	90·6
—	98·5	—	—	—	44·5	31	—	—	—	—	—	66·4	73·9	—	—
142	100	104	51·5	27	46	34	56·5	42	71°·1	66°·4	42°·5	65·1	72·5	74·1	97·9
132	94·5	103	47	27	43	30	58	42	76°·4	65°·7	37°·9	60·4	71·6	76·1	94·0
138	91	100·5	45·5	29	43·5	36·5	51·5	38	72°·2	72°·2	35°·6	58·6	70·7	71·7	98·5
140	96	100	46	27	45	32	52	38·5	70°·2	70°·2	39°·6	58·8	68·9	74·7	92·3
132	102·5	—	43	29·5	43	32	—	39	—	—	—	60·7	70·4	75·4	93·3
138	91	107	45·5	27	45	32	58	—	78°·9	65°·2	35°·9	58·2	70·0	74·9	93·4
142	101	107	45·5	24·5	45	34	54	—	76°·7	65°·2	38°·1	60·1	72·7	76·5	95·1
138	99·5	107·5	51	28	44	35	62	41·5	77°·3	59°·2	43°·5	57·4	70·6	70·0	100·8
139	96	107	45	25·5	42	31·5	59	44	75°·8	70°·2	34°·0	61·6	77·1	81·1	95·1
131	95	107·5	44	27	43	32	56	39	81°·7	61°·8	36°·5	56·4	69·0	71·7	96·2
135	91·5	101	45	26	43	36	53	—	71°·3	71°·3	37°·4	57·8	71·8	71·2	100·8
138	95	112	51	27·5	43	34·5	60	43	73°·7	60°·5	45°·8	58·5	75·3	70·9	106·2
142	103	113	44	29	46·5	32	—	47	79°·1	65°·9	35°·0	59·0	70·2	70·2	100·0
138	102·5	101·5	47	27	44	34	56·5	38·5	70°·1	72°·3	37°·6	56·75	73·0	74·3	98·2
132	94	103	44	27	42·5	33·5	52	39·5	72°·7	72°·7	34°·6	61·6	69·8	75·2	92·8
138	96	—	48	26	46	35	—	—	—	—	—	56·7	75·7	75·1	100·7
131	94·5	101	47·5	27·5	40·5	31	57·5	—	72°·8	67°·2	40°·0	57·2	71·0	69·9	101·5
142	102	109	48	26	46	34	57	40	74°·7	68°·7	36°·6	56·3	68·3	72·0	94·8
135·4	97·7	105·4	47·1	26·9	43·6	33·4	56·5	40·6	75°·1	66°·7	38°·2	59·4	71·3	73·0	97·7
57	63	58	63	63	64	64	49	48	58	58	58	63	64	62	62

dividual Measurements of Male New Britain Crania at Berlin (Kaiser Wilhelm Institut).

	S	*fml*	*fmb*	*G'H*	*J*	*GB*	*GL*	*NH'*	*NB*	O_1, R	O_2, R	G_1	G_2	*N* ∠	*A* ∠	*B* ∠	*Oc. I.*	
	375	32·5	25	63·5	127	98	103	44·5	27·5	42	32	—	—	75°·7	68°·2	36°·1	58·2	74
	374	38	27	62	—	98	101·5	47	28	45	32	—	—	73°·8	70°·6	35°·6	59·6	67
	369	38	30	71	131	99	112	49	24	44·5	35	57	44·5	75°·2	67°·2	37°·6	66·1	68
	381	32·5	31·5	71·5	143	107·5	115	51·5	26	45	32	62	45	81°·2	60°·7	38°·1	59·6	74
	373	36	27·5	61·5	128	95·5	100·5	48·5	26	43·5	34	56	41	73°·7	70°·6	35°·7	58·9	67
	388	35	28	69	133	95	112	48	27·5	44	33	59·5	42	73°·1	70°·9	36°·0	62·1	66
	374	32	24·5	60	129	94	99	42	25	43	29	—	—	77°·4	66°·5	36°·1	58·1	70
	379	35	28·5	73	135	104	108·5	48·5	28	45	36·5	62	42·5	75°·2	64°·2	40°·6	59·2	71
	363	31	27	64	131	93	104	45	23	40	32·5	54	38	74°·1	69°·5	36°·4	58·8	74
	360	36·5	26·5	66	138	103·5	107	46	27	43	33·5	56·5	41	77°·2	65°·7	37°·1	58·3	72
	367	33	28	69	—	98·5	100·5	48	31	47	35	54	40·5	70°·0	70°·0	40°·0	60·5	73
	380	39	34	66·5	134	96	104	48	28	44·5	34	55	32·5	74°·6	67°·2	38°·2	59·3	70
	374	32	28	59·5	137	97	108	44	26	43	29	55·5	42	77°·7	69°·7	32°·6	60·8	72
	375	37	31	71	136	102	104	49	29	44	34	55·5	41·5	71°·2	68°·4	40°·4	58·2	68
	384	37	28	70	136	95	9/	48	27	43	35	58	44·5	74°·0	63°·2	42°·8	59·7	72
	377	33	27	68	137	101	108	45	31	43	33·5	58·5	—	76°·7	65°·3	38°·0	60·7	69
	368	34	27·5	69·5	139	96	108·5	49	25	45	34	55	38	78°·4	62°·6	39°·0	61·5	73
	386	34	28·5	69	139	107	107·5	45·5	31	42	32	60·5	45	73°·5	68°·1	38°·4	58·5	71
	359	35	28·5	65	132	98	99	45	25	41	34·5	53	38	73°·0	68°·7	38°·4	57·3	70
	—	—	—	68	125	92·5	—	44	27	39	34	—	—	—	—	—	—	66
	358	35	25	65	131	98	95	48	25·5	42	35	51	37	71°·8	67°·8	40°·4	58·2	73
	377	38·5	29	—	148	99·5	—	50	28	44	34	—	—	—	—	—	57·8	70
	366	34	28	68	132	88	98	46	28	42	32	57·5	40	71°·1	68°·3	40°·6	64·2	70
	379	36	31	70	137	99	102·5	46·5	25	42	35	—	—	74°·7	64°·2	41°·1	59·6	73
	356	34	32	66·5	123	89	105	45	26	40	33	58	39	77°·1	65°·2	37°·7	59·4	70
	381	33	27	68	139	100·5	95	44	25	46	34	49	—	67°·7	70°·8	41°·5	55·7	70
	362	34	30	62·5	130	99	103	43·5	27	44·5	36	55	39	77°·2	66°·7	36°·1	58·2	70
	358	35	25·5	70	133	97	104·5	48·5	26	41	33	54	42	77°·1	62°·6	40°·4	57·7	70
	386	33	23·5	60	130	93	104	41	23	42	31	53	35	78°·5	67°·0	34°·5	58·6	66
	396	36	28	71	139	98	103	44	27	46·5	35	62	36	73°·4	65°·7	40°·9	57·5	66
	370	34	29	67	131	91	106	48	24	44	30·5	56	43·5	75°·9	66°·5	37°·6	55·5	71
	369	32·5	27	66	—	—	105·5	52	28	46	33·5	56	42	70°·2	73°·8	36°·0	58·8	69
	364	40	32	73	136	98	111	49	27	44	34	—	46	77°·6	62°·3	40°·1	63·5	73
	383	35	27	72	135	99	107	49·5	24	46	34·5	53	39	76°·2	62°·7	41°·1	57·3	71
	376	37	29·5	68	136	98	103	49	27	43	35	54	39·5	72°·2	68°·7	39°·1	61·8	69
	374	37	29·5	73	138	96	106	51	28	43	38	60	38·5	74°·2	64°·7	41°·1	61·6	71
	396	34·5	30	68	146	100·5	116	50	28	45	33	60	—	81°·8	62°·6	35°·6	58·0	71
	386	36·5	33	70	138	101·5	112	52	29	44	34	—	45	76°·6	66°·7	36°·7	59·3	71
	380	36	30	65	131	94	107	48	26	44	32	54	39	71°·7	73°·2	35°·1	58·1	69
	372	33	31	73	136	105	112	48·5	29·5	45·5	31	62	39	76°·6	64°·7	38°·7	59·6	71
	371	29	26	64	—	99	111	44	27	41	30·5	—	43	85°·4	59°·7	34°·9	59·0	71
	373	31·5	26	66·5	137	100·5	111·5	44	27	40	31	62	38	81°·5	62°·4	36°·1	58·2	71
	362	32	29	—	133	94	—	47	25	41	32	—	—	—	—	—	60·5	71
	378	41	32	70	—	102	106	48	31	46	36	57	39	76°·2	64°·2	39°·6	57·7	71
	374	34·5	27·5	77	—	100	106	55	27	43	38	—	41	70°·2	66°·2	43°·6	63·3	69
	370	—	—	—	—	98·5	—	—	—	44·5	31	—	—	—	—	—	66·4	71
	388	35	28	75	142	100	104	51·5	27	46	34	56·5	42	71°·1	66°·4	42°·5	65·1	71
	363	35	28	65·5	132	94·5	103	47	27	43	30	58	42	76°·4	65°·7	37°·9	60·4	71
	389	35	29·5	62	138	91	100·5	45·5	29	43·5	36·5	51·5	38	72°·2	72°·2	35°·6	58·6	71
	397	35	27·5	68·5	140	96	100	46	27	45	32	52	38·5	70°·2	70°·2	39°·6	58·8	69
	375	34	29	—	132	102·5	—	43	29·5	43	32	—	39	—	—	—	60·7	71
	361	31	29	64	138	91	107	45·5	27	45	32	58	—	78°·9	65°·2	35°·9	58·2	71
	382	35	29	68	142	101	107	45·5	24·5	45	34	54	—	76°·7	65°·2	38°·1	60·1	71
	360	33	28	76	138	99·5	107·5	51	28	44	35	62	41·5	77°·3	59°·2	43°·5	57·4	71
	357	38·5	29	62	139	96	107	45	25·5	42	31·5	59	44	75°·8	70°·2	34°·0	61·6	71
	374	36	27	64·5	131	95	107·5	44	27	43	32	56	39	81°·7	61°·8	36°·5	56·4	69
	368	36·5	26	65·5	135	91·5	101	45	26	43	36	53	—	71°·3	71°·3	37°·4	57·8	71
	373	29	29	75	138	95	112	51	27·5	43	34·5	60	43	73°·7	60°·5	45°·8	58·5	71
	380	36	29	66·5	142	103	113	44	29	46·5	32	—	47	79°·1	65°·9	35°·0	59·0	71
	377	35	28·5	66	138	102·5	101·5	47	27	44	34	56·5	38·5	70°·1	72°·3	37°·6	56·75	71
	372	33·5	26	61·5	132	94	103	44	27	42·5	33·5	52	39·5	72°·7	72°·7	34°·6	61·6	69
	371	32·5	28	—	138	96	—	48	26	46	35	—	—	—	—	—	56·7	71
	375	34	27	68	131	94·5	101	47·5	27·5	40·5	31	57·5	—	72°·8	67°·2	40°·0	57·5	71
	376	31	30	67·5	142	102	109	48	26	46	34	57	40	74°·7	68°·7	36°·6	56·3	69
·1	374·3	34·6	28·4	67·6	135·4	97·7	105·4	47·1	26·9	43·6	33·4	56·5	40·6	75°·1	66°·7	38°·2	59·4	71
	63	62	62	59	57	63	58	63	63	64	64	49	48	58	58	58	63	69

	Oc. I.	100 $\frac{B}{L}$	100 $\frac{H'}{L}$	100 $\frac{B}{H'}$	100 $\frac{(B-H')}{L}$	100 $\frac{G'H}{GB}$	100 $\frac{NB}{NH'}$	100 $\frac{O_2}{O_1}$, R	100 $\frac{G_2}{G_1}$	100 $\frac{fmb}{fml}$
I	58·2	74·7	74·2	100·7	+0·5	64·8	61·8	76·2	—	76·9
5	59·6	67·0	71·9	93·2	−4·9	63·3	59·6	71·1	—	71·0
5	66·1	68·1	74·6	91·3	−6·5	71·7	49·0	78·7	78·1	78·9
I	59·6	74·1	73·5	100·7	+0·5	66·5	50·5	71·1	72·6	96·9
7	58·9	67·8	73·0	92·9	−5·2	64·4	53·6	78·2	73·2	76·4
0	62·1	66·5	72·4	91·8	−5·9	72·6	57·3	75·0	70·6	80·0
I	58·1	70·8	72·5	97·7	−1·7	63·8	59·5	67·4	—	76·6
5	59·2	71·0	75·3	94·3	−4·3	70·2	57·7	81·1	68·5	81·4
4	58·8	74·4	74·4	100·0	0·0	68·8	51·1	81·2	70·4	87·1
I	58·3	72·9	72·4	100·8	+0·6	63·8	58·7	77·9	72·6	72·6
0	60·5	73·3	73·3	100·0	0·0	70·1	64·6	74·5	75·0	84·8
2	59·3	70·8	73·5	96·4	−2·7	69·3	58·3	76·4	59·1	87·2
5	60·8	72·0	75·8	94·9	−3·8	61·3	59·1	67·4	75·7	87·5
4	58·2	68·3	76·1	89·8	−7·8	69·6	59·2	77·3	74·8	83·8
8	59·7	72·3	72·3	100·0	0·0	73·7	56·3	81·4	76·7	75·7
0	60·7	69·9	76·0	92·1	−6·0	67·3	68·9	77·9	—	81·8
0	61·5	72·5	71·4	101·5	+1·1	72·4	51·0	75·6	69·1	80·9
4	58·5	71·8	73·9	97·1	−2·1	64·5	68·1	76·2	74·4	83·8
4	57·3	70·9	71·5	99·2	−0·6	66·3	55·6	84·1	71·7	81·4
	—	69·4	—	—	—	73·5	61·4	87·2	—	—
4	58·2	73·9	71·1	103·9	+2·8	66·3	53·1	83·3	72·5	71·4
	57·8	70·5	72·6	97·1	−2·1	—	56·0	77·3	—	75·3
6	64·2	70·7	72·9	97·0	−2·2	77·3	60·9	76·2	69·6	82·4
I	59·6	73·4	73·6	99·6	−0·3	70·7	53·8	83·3	—	86·1
7	59·4	70·8	71·3	99·2	−0·6	74·7	57·8	82·5	67·2	94·1
5	55·7	70·8	73·5	96·3	−2·7	67·7	56·8	73·9	—	81·8
I	58·2	70·0	71·7	97·7	−1·7	63·1	62·1	80·9	71·0	88·2
4	57·7	70·2	71·3	98·4	−1·1	72·2	53·6	80·5	77·8	72·9
5	58·6	69·9	72·3	96·7	−2·4	64·5	56·1	73·8	66·0	71·2
9	57·5	66·2	68·7	96·3	−2·6	72·4	61·4	75·3	58·1	77·8
6	55·5	72·2	71·4	101·1	+0·8	73·6	50·0	69·3	77·7	85·3
0	58·8	69·1	65·4	105·6	+3·6	—	53·8	72·8	75·0	83·1
I	63·5	75·8	73·4	103·4	+2·5	74·5	55·1	77·3	—	80·0
I	57·3	71·9	71·7	100·4	+0·3	72·7	48·5	75·0	73·6	77·1
I	61·8	68·8	70·4	97·7	−1·6	69·4	55·1	81·4	73·1	79·7
I	61·6	70·8	70·8	100·0	0·0	76·0	54·9	88·4	64·2	79·7
6	58·0	73·2	69·6	105·2	+3·6	67·7	56·0	73·3	—	87·0
7	59·3	71·7	76·5	93·7	−4·8	69·0	55·8	77·3	—	90·4
I	58·1	67·6	74·2	91·1	−6·6	69·1	54·2	72·7	72·2	83·3
7	59·6	74·6	71·4	104·5	+3·2	69·5	60·8	68·1	62·9	93·9
9	59·0	74·7	78·7	94·9	−4·0	64·6	61·4	74·4	—	89·7
I	58·2	70·3	73·0	96·3	−2·7	66·2	61·4	77·5	61·3	82·5
	60·5	77·7	75·4	103·0	+2·3	—	53·2	78·0	—	90·6
6	57·7	70·5	68·4	103·1	+2·1	68·6	64·6	78·3	68·4	78·0
6	63·3	68·5	75·5	90·6	−7·1	77·0	49·1	88·4	—	79·7
	66·4	73·9	—	—	—	—	—	69·7	—	—
5	65·1	72·5	74·1	97·9	−1·6	75·0	52·4	73·9	74·3	80·0
9	60·4	71·6	76·1	94·0	−4·5	69·3	57·4	69·8	71·9	80·0
6	58·6	70·7	71·7	98·5	−1·0	68·1	63·7	83·9	73·8	84·3
6	58·8	68·9	74·7	92·3	−5·8	71·4	58·7	71·1	74·0	78·6
	60·7	70·4	75·4	93·3	−5·0	—	68·6	74·4	—	85·3
9	58·2	70·0	74·9	93·4	−5·0	70·3	59·3	71·1	—	93·6
1	60·1	72·7	76·5	95·1	−3·8	67·3	53·8	75·6	—	82·9
5	57·4	70·6	70·0	100·8	+0·6	76·4	54·9	79·5	66·9	84·8
0	61·6	77·1	81·1	95·1	−4·0	64·6	56·7	75·0	74·6	75·3
5	56·4	69·0	71·7	96·2	−2·7	67·9	61·4	74·4	69·6	75·0
4	57·8	71·8	71·2	100·8	+0·5	71·6	57·8	83·7	—	71·2
8	58·5	75·3	70·9	106·2	+4·4	78·9	53·9	80·2	71·7	100·0
0	59·0	70·2	70·2	100·0	0·0	64·6	65·9	68·8	—	80·6
6	56·75	73·0	74·3	98·2	−1·3	64·4	57·4	77·3	68·1	81·4
6	61·6	69·8	75·2	92·8	−5·4	65·4	61·4	78·8	76·0	77·6
-	56·7	75·7	75·1	100·7	+0·6	—	54·1	76·1	—	86·2
0	57·-	71·0	69·9	101·5	+1·1	72·0	57·9	76·5	—	79·4
6	56·3	68·3	72·0	94·8	−3·8	66·2	54·2	73·9	70·2	96·8
2	59·4	71·3	73·0	97·7	−1·7	69·3	57·4	76·7	71·1	82·2
	63	64	62	62	62	58	63	64	42	62

[Turn over

Serial No.	*C*	*L*	*F*	*B*	*H′*	*LB*	*B′*	*U*	*Glabella U*	*Bregmatic Q′*	S_1'	S_2'	S_3'	S_1
43136	1213·0	177	174	133	133	95·0	86·0	485	494	299	108	116	94·5	123
43146	1147·4	169	170	127	129	92·6	81·9	471	475	290	101·5	113	96	114
43151	1166·0	173	173	136	123	93·0	95·3	487	492	295	101	115	85	113
43153	1198·2	173	175	126	130	96·0	85·5	488	495	284	109	108	97·5	124
43156	1427·3	177	176	137	131	92·7	97·3	500	508	311	109	121·5	92	127
43157	1325·6	179	176	135	130	96·0	91·5	497	502	293	—	—	—	—
43161	1090·7	171	169	132	125	93·0	86·4	498	505	298	105	117	92	117
43163	1318·5	180	178	133	132	101·8	94·3	496	505	296	103·5	111	100	117
43164	1273·5	175	174	130	126	92·0	85·5	490	493	283	101·5	111	93·5	116
43165	1136·4	170	169	122	122	90·1	78·5	466	471	278	94	97	103	109
43176	—	167	167	122	118	90·7	87·1	475	478	290	98	106	92	110
43178	1168·0	167	166	127	125	87·0	91·8	470	478	288	—	—	—	—
43180	1496·1	182	181	138	140	95·1	100·6	517	521	316	118	115·5	103	131
43187	—	172	171	127	128	93·8	83·0	475	483	294	96	110	91	110
43190	—	170	169	124	123	91·9	91·1	475	477	283	104	104	92·5	117
43193	1119·7	167	166	131	132	95·9	88·0	480	486	297	106	110	93	120
43194	1126·8	171	171	128	133	100·0	82·8	488	488	305	110	115	95·5	127
43195	—	171	172	128	134	97·3	96·6	485	490	303	—	—	—	—
43199	1532·2	190	190	138	139	105·6	99·6	529	534	320	121	115	100	139
43201	1131·9	173	173	127	130	100·0	93·5	479	483	288	104	116	94	119
43205	—	170	169	126	126	93·0	88·2	477	482	287	110·5	119·5	—	123
43206	1252·9	184	183	125	140	98·6	90·0	—	—	—	—	—	—	—
43215	—	175	171·5	128·5	127·5	92·0	84·6	486	496	298	113	114	92	129
43219	1173·7	171	171·5	132·5	133·5	95·2	94·5	485	491	297	105·5	114·5	91	123
43220	1123·6	175	172·5	132	120	95·3	88·0	484	495	280	102	104	95·5	115
43222	1089·4	173	170	128	127·5	98·0	93·0	478	487	285	104	110	89	117
43225	1310·2	178·5	179	131·5	133·5	97·7	88·6	503	507	306	109·5	111·5	98	125
43226	1074·0	164	162·5	120	127	90·0	83·0	460	464	286	103·5	110·5	93	116
43227	1137·1	175	174	126·5	133·5	100·5	87·1	493	501	296	104	109	97	119
43230	—	173	172	120	127	93·2	91·4	472	476	280	108	111	90·5	123
43231	1071·4	170	170	127	127·5	95·5	88·0	475	482	277	101	109·5	91	114
43233	1342·3	174·5	175·5	135	122·5	86·5	89·5	497	499	300	103	—	95	120
43234	1078·5	168·5	168·5	126	129	92·5	88·0	476	480	287	104	100	94	121
43237	—	170	168	123	127	91·4	85·5	470	479	275	97	107	92·5	111
43240	—	169	170	126	134	96·0	87·1	478	480	303	108	115	89·5	125
43242	1189·2	169	170·5	130	128	91·1	91·0	484	491	299	105·5	107	92	124
43243	1122·3	175	173·5	124	124	94·1	93·0	481	487	280	98	112	98	112
43246	—	180	177	131·5	—	101·0	88·2	493	505	296	109	112	96	121
43249	—	165	165	122	123	91·8	86·0	459	461	273	98·5	99	88·5	113
43251	—	178	177	131·5	140	102·3	88·4	493	503	302	109	114	98	123
43252	1202·7	173·5	171	132	131	98·0	94·5	484	490	291	107	108	93·5	121
43255	1193·0	177	176	132	—	—	88·7	496	503	297	104	107	97·5	120
43257	—	174	174	127	121·5	97·3	90·3	483	489	283	99	107	90·5	113
Mean	1207·7	173·4	172·6	128·8	128·9	95·0	89·4	484·7	490·6	292·6	104·9	110·6	94·1	119
Number	30	43	43	43	41	42	43	42	42	42	39	38	38	39

UNIV. OF MICH.

S_2'	S_1	S_2	S_3	S	*fml*	*fmb*	*EOW*	*IOW*	*G'H*	*J*	*GB*	*GL*	*PH*	*NH, R*	*NH, L*	*NH'*
94·5	123	133	111	365	35·0	27·8	99	92	66·5	128	90·3	95·3	19·0	48·0	48·0	47·0
96	114	127	116	357	30·2	26·0	99	95	59·7	—	86·0	99·0	13·4	47·2	47·3	45·5
85	113	129	103	345	35·0	25·5	101·5	95	70·0	125	88·6	99·0	23·0	52·1	52·1	46·0
97·5	124	119	122	363	36·4	28·3	102·5	94	66·6	125	92·1	100·0	19·0	52·0	51·1	47·0
92	127	138	110	375	33·0	28·0	105	96	67·2	134	92·2	94·5	16·1	52·1	52·1	50·0
—	—	—	—	365	30·4	26·0	103	98·5	63·0	134	90·0	92·7	14·7	50·8	50·8	47·1
92	117	131	113	361	34·1	30·0	100·5	93	58·0	126	92·0	96·1	14·8	46·5	46·2	43·0
100	117*	124	122	363	34·9	27·9	106·5	99	67·0	133	98·0	106·1	17·9	53·5	53·8	49·1
93·5	116	130	111	357	31·1	27·2	101	94	61·9	127	97·9	99·5	17·3	45·4	45·4	44·1
103	109	108	126	343	—	—	95·5	89·5	61·5	116	91·0	98·8	15·9	—	48·5	45·0
92	110	134	108	352	32·8	26·0	98	92·5	66·0	122	82·1	98·0	—	51·0	52·0	45·2
—	—	—	—	349	38·0	31·0	102	95	61·1	124	86·0	96·3	19·0	45·3	45·3	41·0
103	131	126	127	384	32·0	29·2	108	98	67·3	132	94·1	97·4	21·0	52·5	52·9	47·1
91	110	123	109	342	32·9	27·5	100	93·5	64·2	127	89·0	96·4	18·5	49·0	49·8	45·9
92·5	117	115	111	343	35·4	28·1	100	95·5	56·2	126	91·4	101·0	16·0	44·0	44·9	40·7
93	120	126	108	354	34·5	26·3	99	94	58·9	128	93·4	96·1	13·9	47·0	48·2	44·1
95·5	127	128	114	369	28·3	23·9	103	97	62·3	—	95·2	106·1	16·9	47·3	47·3	44·0
—	—	—	—	372	36·0	30·0	102	96	66·0	129	97·2	99·2	16·5	49·0	49·0	49·0
100	139	128	121	388	33·3	28·9	113	103	71·2	142	103·0	108·2	19·1	55·0	55·0	50·0
94	119	130	109	358	—	—	108	102	64·5	128	86·0	106·0	19·1	48·1	48·2	45·2
—	123	131	—	—	—	—	101	94	62·3	124	98·1	99·2	17·0	45·4	45·4	45·4
—	—	—	—	—	30·5	27·3	100·5	94	66·8	125	89·4	105·7	15·0	51·8	51·8	50·0
92	129	1[illegible]2	110	371	33·8	28·1	100·5	94	60·0	124	86·1	97·0	14·4	47·9	47·9	42·8
91	123	13[illegible]	105	359	31·1	27·9	105	99	—	125	90·0	—	—	49·0	49·0	44·1
95·5	115	117	120	351	31·2	27·0	100	95	63·8	123	92·0	97·4	18·9	45·0	45·0	45·0
89	117	123	107	347	33·5	27·0	104	98	61·6	129·5	93·0	102·3	15·7	49·1	49·1	45·1
98	125	124	118	367	28·0	24·5	103	98·5	68·4	131·5	97·8	102·3	20·1	51·0	52·2	47·0
93	116	125	110	351	33·0	28·7	98	94	62·0	120	91·5	95·2	17·1	46·3	46·3	44·9
97	119	121	117	357	31·7	24·3	104·5	97	—	127	98·0	—	17·8	49·0	49·9	44·9
90·5	123	126	109	358	30·0	27·8	101	95	61·2	122·5	91·1	98·9	18·2	47·0	47·0	41·8
91	114	122	109	345	30·3	26·4	102	95	65·7	126	93·0	100·2	19·0	48·2	49·1	44·1
95	120	121	128	369	33·1	27·2	98	94	58·0	118	88·3	87·5	14·0	48·0	48·3	43·2
94	121	112	112	345	31·8	26·0	101	98	64·6	129	92·7	100·0	20·4	45·0	45·0	42·9
92·5	111	125	113	349	30·2	25·9	101	95	—	124·5	87·0	—	—	—	—	43·0
89·5	125	134	103	362	32·3	27·0	95	88	65·0	124	85·2	101·0	20·0	50·8	50·8	44·9
92	124	120	112	356	32·2	26·0	101	93	62·8	126	87·1	101·5	16·8	49·0	49·0	46·2
98	112	127	114	353	34·1	28·0	102	97	62·6	126	87·0	99·0	19·8	49·2	49·0	41·7
96	121	128	115	364	—	—	103	98	66·5	128	97·0	102·1	20·0	51·9	51·9	48·0
88·5	113	113	109	335	31·1	25·0	98·5	94	65·1	120·5	91·2	95·4	19·9	45·2	45·2	45·0
98	123	127	116	366	—	—	—	—	—	—	—	—	—	—	—	—
93·5	121	127	109	357	32·0	27·3	103	98	68·3	—	94·1	103·0	—	50·0	50·0	47·8
97·5	120	123	118	361	—	—	103	95	66·0	—	96·0	—	22·0	46·5	48·1	45·9
90·5	113	120	112	345	30·0	27·0	100	94	61·3	121·5	92·0	98·1	14·4	49·0	49·2	46·0
94·1	119·5	125·1	113·3	357·9	32·5	27·2	101·7	95·5	63·9	126·3	91·7	99·25	17·7	48·8	49·0	45·4
38	39	39	38	41	37	37	42	42	39	38	42	38	38	40	41	42

APPENDIX C. *Individual Measurements of the Parkinson Collection of Female New Britain Cra*

EOW	*IOW*	*G'H*	*J*	*GB*	*GL*	*PH*	*NH, R*	*NH, L*	*NH'*	*NB*	O_1, *R*	O_1', *R*	*Lacr.* O_1, *R*	O_2, *R*	O_1, *L*	O_1
99	92	66·5	128	90·3	95·3	19·0	48·0	48·0	47·0	26·0	41·2	38·0	38·0	31·2	41·1	3
99	95	59·7	—	86·0	99·0	13·4	47·2	47·3	45·5	25·2	41·0	38·9	37·9	31·0	41·0	3
101·5	95	70·0	125	88·6	99·0	23·0	52·1	52·1	46·0	26·0	41·9	38·9	37·5	33·0	40·5	3
102·5	94	66·6	125	92·1	100·0	19·0	52·0	51·1	47·0	25·9	43·9	—	40·0	38·8	41·2	3
105	96	67·2	134	92·2	94·5	16·1	52·1	52·1	50·0	25·1	42·9	41·8	39·9	35·0	44·1	4
103	98·5	63·0	134	90·0	92·7	14·7	50·8	50·8	47·1	29·5	43·1	—	—	34·4	41·8	-
100·5	93	58·0	126	92·0	96·1	14·8	46·5	46·2	43·0	26·0	44·0	40·2	39·0	34·1	42·0	4
106·5	99	67·0	133	98·0	106·1	17·9	53·5	53·8	49·1	29·1	43·3	40·0	40·0	33·9	43·4	3
101	94	61·9	127	97·9	99·5	17·3	45·4	45·4	44·1	24·7	41·9	38·8	38·8	33·0	42·0	3
95·5	89·5	61·5	116	91·0	98·8	15·9	—	48·5	45·0	24·9	42·0	39·3	39·3	33·3	43·1	-
98	92·5	66·0	122	82·1	98·0	—	51·0	52·0	45·2	25·0	41·1	39·4	38·0	30·9	40·5	3
102	95	61·1	124	86·0	96·3	19·0	45·3	45·3	41·0	24·0	42·0	40·9	38·9	34·0	42·0	4
108	98	67·3	132	94·1	97·4	21·0	52·5	52·9	47·1	25·5	40·8	—	—	34·9	40·3	-
100	93·5	64·2	127	89·0	96·4	18·5	49·0	49·8	45·9	23·2	40·2	—	—	31·0	39·2	-
100	95·5	56·2	126	91·4	101·0	16·0	44·0	44·9	40·7	26·0	42·0	39·1	38·9	31·0	40·9	3
99	94	58·9	128	93·4	96·1	13·9	47·0	48·2	44·1	27·0	42·0	39·9	37·8	33·9	40·8	3
103	97	62·3	—	95·2	106·1	16·9	47·3	47·3	44·0	27·9	43·0	39·5	38·1	32·5	42·1	3
102	96	66·0	129	97·2	99·2	16·5	49·0	49·0	49·0	27·1	45·2	42·0	41·0	34·0	43·2	-
113	103	71·2	142	103·0	108·2	19·1	55·0	55·0	50·0	27·9	45·1	42·8	42·8	35·1	45·0	4
108	102	64·5	128	86·0	106·0	19·1	48·1	48·2	45·2	26·2	41·1	—	—	34·5	40·0	-
101	94	62·3	124	98·1	99·2	17·0	45·4	45·4	45·4	25·0	42·0	38·7	38·7	35·0	41·0	3
100·5	94	66·8	125	89·4	105·7	15·0	51·8	51·8	50·0	27·1	—	—	—	—	—	-
100·5	94	60·0	124	86·1	97·0	14·4	47·9	47·9	42·8	26·0	41·0	37·0	37·0	31·5	40·8	3
105	99	—	125	90·0	—	—	49·0	49·0	44·1	29·0	43·0	40·5	40·5	34·9	43·1	4
100	95	63·8	123	92·0	97·4	18·9	45·0	45·0	45·0	25·0	40·2	38·0	37·1	33·5	41·2	3
104	98	61·6	129·5	93·0	102·3	15·7	49·1	49·1	45·1	26·6	41·9	38·1	36·9	31·2	41·9	3
103	98·5	68·4	131·5	97·8	102·3	20·1	51·0	52·2	47·0	27·0	40·2	37·0	37·0	32·9	40·1	3
98	94	62·0	120	91·5	95·2	17·1	46·3	46·3	44·9	25·9	40·3	37·9	36·5	35·3	40·3	3
104·5	97	—	127	98·0	—	17·8	49·0	49·9	44·9	29·0	42·9	40·0	38·0	30·9	44·1	4
101	95	61·2	122·5	91·1	98·9	18·2	47·0	47·0	41·8	24·1	40·8	38·0	36·0	30·3	40·2	3
102	95	65·7	126	93·0	100·2	19·0	48·2	49·1	44·1	27·2	41·1	37·4	37·4	33·0	40·0	3
98	94	58·0	118	88·3	87·5	14·0	48·0	48·3	43·2	28·5	41·2	38·3	37·5	30·6	39·1	3
101	98	64·6	129	92·7	100·0	20·4	45·0	45·0	42·9	27·0	43·0	—	—	31·0	41·2	-
101	95	—	124·5	87·0	—	—	—	—	43·0	25·1	41·9	39·1	37·0	34·2	42·0	3
95	88	63·0	124	85·2	101·0	20·0	50·8	50·8	44·9	24·4	37·8	35·8	35·0	31·0	37·5	3
101	93	62·8	126	87·1	101·5	16·8	49·0	49·0	46·2	27·3	40·0	37·6	36·0	33·0	39·3	3
102	97	62·6	126	87·0	99·0	19·8	49·2	49·0	41·7	28·0	42·5	—	—	31·8	41·2	-
103	98	66·5	128	97·0	102·1	20·0	51·9	51·9	48·0	26·9	41·3	39·0	37·5	31·3	41·0	-
98·5	94	65·1	120·5	91·2	95·4	19·9	45·2	45·2	45·0	23·2	40·9	38·6	37·0	34·9	40·0	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
103	98	68·3	—	94·1	103·0	—	50·0	50·0	47·8	27·1	44·0	40·2	40·2	34·2	43·2	-
103	95	66·0	—	96·0	—	22·0	46·5	48·1	45·9	25·1	41·7	—	—	32·2	41·1	4
100	94	61·3	121·5	92·0	98·1	14·4	49·0	49·2	46·0	25·5	41·1	—	—	30·2	41·0	3
101·7	95·5	63·9	126·3	91·7	99·25	17·7	48·8	49·0	45·4	26·2	41·9	39·1	38·2	33·0	41·3	3
42	42	39	38	42	38	38	40	41	42	42	41	32	33	41	41	3

Britain Crania at Chicago (Field Museum).

R	O_1, L	O_1', L	*Lacr.* O_1, L	O_2, L	G_1	G_1'	G_2	*EH*	$N \angle$	$A \angle$	$B \angle$	*Oc. I.*	$100 \frac{B}{L}$	$100 \frac{H'}{L}$	$100 \frac{}{L}$
2	41·1	38·8	36·9	32·5	52·0	48·9	41·0	15·0	70°·4	68°·8	40°·8	62·1	75·1	75·1	10
0	41·0	38·2	38·0	31·0	54·0	50·8	39·3	10·2	77°·6	66°·0	36°·4	59·4	75·1	76·3	9
0	40·5	39·0	37·5	33·6	52·0	48·1	39·1	13·1	73°·3	64°·0	42°·7	59·2	78·6	71·1	11
8	41·2	38·9	38·2	34·7	51·0	46·3	35·9	13·1	73°·6	66°·6	39°·8	57·0	72·8	75·1	9
0	44·1	42·0	40·3	34·8	52·2	48·6	40·9	10·0	70°·5	67°·5	42°·0	60·3	77·4	74·0	10
4	41·8	—	—	34·5	49·9	45·2	38·0	8·1	67°·8	73°·3	38°·9	—	75·4	72·6	10
1	42·0	40·1	38·2	35·0	57·1	52·2	37·2	8·0	75°·0	69°·2	35°·8	58·2	77·2	73·1	10
9	43·4	39·5	39·5	33·5	52·3	49·0	37·0	9·1	74°·9	67°·3	37°·8	58·6	73·9	73·3	10
0	42·0	39·0	39·0	33·1	—	—	—	—	77°·8	64°·8	37°·4	61·0	74·3	72·0	10
3	43·1	—	—	33·5	52·5	48·3	37·0	10·5	79°·0	63°·5	37°·5	58·5	71·8	71·8	10
9	40·5	39·4	37·6	31·9	52·1	49·0	35·0	11·2	75°·7	63°·6	40°·7	62·2	73·1	70·7	10
0	42·0	40·3	39·7	34·0	52·2	48·1	41·3	10·5	79°·0	62°·5	38°·5	—	76·0	74·9	10
9	40·3	—	—	33·0	50·9	46·8	39·3	8·2	71°·5	67°·5	41°·0	57·9	75·8	76·9	9
0	39·2	—	—	31·0	52·0	49·0	33·0	—	72°·5	68°·0	39°·5	60·2	73·8	74·4	9
0	40·9	39·5	37·8	30·8	51·2	46·2	38·8	8·2	82°·1	64°·5	33°·4	60·0	72·9	72·4	10
9	40·8	39·8	37·2	33·0	—	—	41·8	10·0	72°·5	71°·5	36°·0	63·6	78·4	79·0	9
5	42·1	38·1	—	31·9	54·0	51·0	43·0	12·0	78°·2	66°·9	34°·9	60·5	74·9	77·8	9
0	43·2	—	—	33·1	56·5	50·9	43·9	—	71°·9	68°·8	39°·3	—	74·9	78·4	9
1	45·0	41·2	41·2	34·0	53·8	49·0	41·0	12·0	72°·4	68°·4	39°·2	59·3	72·6	73·2	9
5	40·0	—	—	33·5	49·1	45·4	42·0	11·2	77°·0	66°·5	36°·5	63·8	73·4	75·1	9
0	41·0	39·2	37·9	34·0	59·0	54·5	45·6	8·1	76°·5	66°·0	37°·5	—	74·1	74·1	10
.	—	—	—	—	—	—	—	—	77°·0	65°·0	38°·0	—	67·9	76·1	8
5	40·8	38·1	37·0	33·8	51·5	46·9	35·9	—	76°·1	67°·1	36°·8	60·3	73·4	72·9	10
9	43·1	40·2	40·2	34·2	48·0	45·3	—	—	—	—	—	64·5	77·3	78·1	9
5	41·2	39·1	37·3	32·9	53·0	48·8	31·1	—	72°·5	69°·0	38°·5	56·7	75·4	68·6	11
2	41·9	37·9	37·1	31·0	52·3	48·0	34·2	10·6	76°·2	68°·0	35°·8	59·9	74·0	73·7	10
9	40·1	37·1	37·1	32·0	53·5	50·3	40·5	11·9	73°·8	66°·3	39°·9	59·7	73·7	74·8	9
3	40·3	39·0	37·6	35·0	49·5	46·0	36·5	8·9	75°·0	66°·0	39°·0	61·4	73·2	77·4	9
9	44·1	40·1	38·9	31·2	53·1	50·9	41·2	13·0	—	—	—	59·6	72·3	76·3	9
3	40·2	38·4	36·3	30·0	52·0	47·2	37·9	10·0	76°·1	66°·9	37°·0	59·7	69·4	73·4	9
0	40·0	36·5	36·0	32·8	54·5	49·1	—	—	74°·8	66°·2	39°·0	60·2	74·7	75·0	9
6	39·1	37·4	36·5	31·0	46·0	43·0	39·0	12·1	71°·2	69°·8	39°·0	54·2	77·4	70·2	11
0	41·2	—	—	31·1	52·6	48·3	38·0	10·0	77°·0	64°·1	38°·9	60·7	74·8	76·6	9
2	42·0	39·0	37·1	33·8	—	—	—	—	—	—	—	58·5	72·4	74·7	9
0	37·5	34·0	33·5	30·5	—	49·0	38·5	9·1	74°·6	66°·9	38°·5	64·9	74·6	79·3	9
0	39·3	36·9	35·5	32·0	55·9	52·0	36·5	9·6	80°·2	62°·3	37°·5	58·8	76·9	75·7	10
8	41·2	—	38·1	31·0	55·9	52·0	36·5	8·1	75°·4	66°·8	37°·8	63·4	70·9	70·9	10
3	41·0	—	37·5	32·3	54·0	52·0	38·5	—	72°·1	69°·6	38°·3	60·2	73·1	—	—
9	40·0	38·2	—	31·7	51·9	50·1	40·5	10·7	73°·0	66°·3	40°·7	57·9	73·9	74·5	9
-	—	—	—	35·3	48·5	46·1	38·0	11·0	—	—	—	61·3	73·9	78·7	9
2	43·2	—	—	34·1	—	—	—	—	74°·3	66°·0	39°·7	63·1	76·1	75·5	10
2	41·1	40·3	39·8	32·5	49·0	46·1	38·7	12·2	—	—	—	59·3	74·6	—	—
2	41·0	39·0	37·0	30·2	50·0	46·0	41·0	—	73°·3	70°·1	36°·6	57·6	73·0	69·8	10
0	41·3	38·8	37·8	32·7	52·3	48·5	38·7	10·5	74°·8	66°·9	38°·3	60·1	74·3	74·5	9
	41	31	31	42	37	38	37	31	38	38	38	38	43	41	4

100 $\frac{B}{L}$	100 $\frac{H'}{L}$	100 $\frac{B}{H'}$	100 $\frac{(B-H')}{L}$	100 $\frac{G'H}{GB}$	100 $\frac{NB}{NH'}$	100 $\frac{NB}{NH}, R$	100 $\frac{O_2}{O_1}, R$	100 $\frac{O_2}{O_1}, L$	100 $\frac{O_2}{O_1''}, R$	100 $\frac{O_2}{Lacr.\,O_1}, R$	100 $\frac{G_2}{G_1}$	100 $\frac{G_2}{G_1'}$	100 $\frac{EH}{G_2}$	100 $\frac{fmb}{fml}$
75·1	75·1	100·0	0·0	73·6	55·3	54·2	75·7	79·1	82·1	82·1	78·8	83·9	36·6	79·4
75·1	76·3	98·4	−1·2	69·4	55·4	53·4	75·6	75·6	79·7	81·8	72·8	77·4	26·0	86·1
78·6	71·1	110·6	+7·5	79·0	56·5	49·9	78·8	83·0	84·8	88·0	75·2	81·3	33·5	72·9
72·8	75·1	96·9	−2·3	72·3	55·1	49·8	88·4	84·2	—	97·0	70·4	77·5	36·5	77·7
77·4	74·0	104·6	+3·4	72·9	50·2	48·2	81·6	78·9	83·7	87·7	78·4	84·2	24·5	84·8
75·4	72·6	103·8	+2·8	70·0	62·6	58·1	79·8	82·5	—	—	76·2	84·1	21·3	85·5
77·2	73·1	105·6	+4·1	63·0	60·5	55·9	77·5	83·3	84·8	87·4	65·1	71·3	21·5	88·0
73·9	73·3	100·8	+0·6	68·4	59·3	54·4	78·3	77·2	84·8	84·8	70·7	75·5	24·6	79·9
74·3	72·0	103·2	+2·3	63·2	56·0	54·4	78·8	78·8	85·1	85·1	—	—	—	87·5
71·8	71·8	100·0	0·0	67·6	55·3	—	79·3	77·7	84·7	84·7	70·5	76·6	28·4	—
73·1	70·7	103·4	+2·4	80·4	55·3	49·0	75·2	78·8	78·4	81·3	67·2	71·4	32·0	79·3
76·0	74·9	101·6	+1·2	71·0	58·5	53·0	81·0	81·0	83·1	87·4	79·1	85·9	25·4	81·6
75·8	76·9	98·6	−1·1	71·5	54·1	48·6	85·5	81·9	—	—	77·2	84·0	20·9	91·2
73·8	74·4	99·2	−0·6	72·1	50·6	47·4	77·1	79·1	—	—	63·5	67·3	—	83·6
72·9	72·4	100·8	+0·6	61·5	63·9	59·1	73·8	75·3	79·3	79·7	75·8	84·0	21·1	79·4
78·4	79·0	99·2	−0·6	63·1	61·2	57·5	80·7	80·9	85·0	89·7	—	—	23·9	76·2
74·9	77·8	96·2	−2·9	65·4	63·4	59·0	75·6	75·8	82·3	85·3	79·6	84·3	27·9	84·5
74·9	78·4	95·5	−3·5	67·9	55·3	55·3	75·2	76·6	81·0	82·9	77·7	86·2	—	83·3
72·6	73·2	99·3	−0·5	69·1	55·8	50·7	77·8	75·6	82·0	82·0	76·2	83·7	29·3	86·8
73·4	75·1	97·7	−1·7	75·0	58·0	54·5	83·9	83·8	—	—	85·5	92·5	26·7	—
74·1	74·1	100·0	0·0	63·5	55·1	55·1	83·3	82·9	90·4	90·4	77·3	83·7	17·8	—
67·9	76·1	89·3	−8·2	74·7	54·2	52·3	—	—	—	—	—	—	—	89·5
73·4	72·9	100·8	+0·6	69·7	60·7	54·3	76·8	82·8	85·1	85·1	69·7	76·5	—	83·1
77·3	78·1	99·2	−0·6	—	65·8	59·2	81·2	79·4	86·2	86·2	—	—	—	89·7
75·4	68·6	110·0	+6·9	69·3	55·6	55·6	83·3	79·9	88·2	90·3	58·7	63·7	—	86·6
74·0	73·7	100·4	+0·3	66·2	59·0	54·2	74·5	74·0	81·9	84·6	65·4	69·5	31·0	80·6
73·7	74·8	98·5	−1·1	70·0	57·5	52·9	81·9	79·8	88·9	88·9	75·7	80·5	29·4	87·5
73·2	77·4	94·5	−4·3	67·8	57·7	55·9	87·6	86·8	93·2	96·7	73·7	79·4	24·4	87·0
72·3	76·3	94·7	−4·0	—	64·6	59·2	72·0	70·7	77·2	81·3	77·6	81·0	31·6	76·7
69·4	73·4	94·5	−4·1	67·2	57·6	51·3	74·3	74·6	79·7	84·2	72·9	80·3	26·4	92·7
74·7	75·0	99·6	−0·3	70·6	61·7	56·4	80·3	82·0	88·2	88·2	—	—	—	87·1
77·4	70·2	110·2	+7·2	65·7	66·0	59·4	74·3	79·3	79·9	81·6	84·8	90·7	31·0	82·2
74·8	76·6	97·7	−1·8	69·7	62·9	60·0	72·1	75·5	—	—	72·2	78·7	20·3	81·8
72·4	74·7	96·8	−2·4	—	58·4	—	81·6	80·5	87·5	92·4	—	—	—	85·8
74·6	79·3	94·0	−4·7	76·3	54·3	48·0	82·0	81·3	86·6	88·6	—	78·6	23·6	83·6
76·9	75·7	101·6	+1·2	72·1	59·1	55·7	82·5	81·4	87·8	91·7	65·3	70·2	26·3	80·8
70·9	70·9	100·0	0·0	72·0	67·1	56·9	74·8	75·2	—	—	65·3	70·2	22·2	82·1
73·1	—	—	—	68·6	56·0	51·8	75·8	77·3	80·3	83·5	71·3	74·0	—	—
73·9	74·5	99·2	−0·6	71·4	51·6	51·3	85·3	79·2	90·4	94·3	78·0	80·8	26·4	80·4
73·9	78·7	93·9	−4·8	—	—	—	—	—	—	—	78·4	82·4	28·9	—
76·1	75·5	100·8	+0·6	72·6	56·7	54·2	77·7	78·9	85·1	85·1	—	—	—	85·3
74·6	—	—	—	68·8	54·7	54·0	77·2	79·1	—	—	79·0	83·9	31·5	—
73·0	69·8	104·5	+3·2	66·6	55·4	52·0	73·5	73·7	—	—	82·0	89·1	—	90·0
74·3	74·5	99·9	−0·2	69·7	58·0	54·1	78·8	79·1	84·3	86·7	73·9	79·6	26·9	83·8
43	41	41	41	39	42	40	41	41	32	33	35	36	31	37

A STUDY OF AN EGYPTIAN SERIES OF MANDIBLES, WITH SPECIAL REFERENCE TO MATHEMATICAL METHODS OF SEXING.

By E. S. MARTIN, Ph.D.

1. *Introduction.* This paper is concerned mainly with the mandibles of a series of Egyptian skulls of the 26th—30th Dynasties, which were excavated from a single cemetery near Gizeh*. The series of skulls, which will be referred to as the *E* series, consists of some 1800 crania and some 600 mandibles. Of the crania, it appears from the memoir by Professor Pearson and Miss A. G. Davin† that 1563 were sufficiently complete for an adequate number of measurements to be taken on each. They were divided anatomically into 935 ♂ and 628 ♀. Of the mandibles, all that is known is that they probably form a selection from the original 1800 mandibles belonging to the 1800 crania, but it is not known to which cranium any individual mandible belongs‡. It is also possible that there are a few mandibles of which the associated crania have not been preserved. The adult mandibles have been recently examined by Dr G. M. Morant, who has selected 432 of them as being sufficiently complete for the following six measurements at least to be taken on each, viz. the length of the condyle ($c_y l$), the length of the corpus ($c_p l$), the height of the coronoid process ($c_r h$), the bigonial breadth ($g_o g_o$), the mandibular angle ($M\angle$), and the angle between the condylar-coronoidal line and the ramus tangent ($R\angle$)§. The selected mandibles have been measured by the present writer in accordance with the technique described by Morant in the memoir cited, but the measurements are of little use if the sexes of the mandibles cannot be determined. The principal object of this paper is, therefore, to determine from the measurements the probable sexes of these mandibles and the constants of the distributions of the measured characters for the two sexes separately.

* [Professor Karl Pearson desiring that the Biometric Laboratory should possess at least one adequately long cranial series asked Professor Flinders Petrie if such could be obtained from Egypt. The latter expressed his willingness to excavate such a series if the costs of packing and transfer could be obtained. These were furnished by Professor Pearson's brother, the late Mr A. B. Pearson-Gee. Ed.]

† "On the Biometric Constants of the Human Skull," *Biometrika*, Vol. xvi. (1924), pp. 328—363.

‡ [My impression is that more mandibles than crania were brought from Gizeh, possibly many too fragmentary to be preserved. Dr Morant found only 600, but at my pressing request, he has recently further searched the stores and found another nearly complete 200. The numbering is peculiar, and seems to indicate that further mandibles must have been found originally. This somewhat weakens the argument that the mandibles belonged in bulk to the 1800 crania. Ed.]

§ The most recent definitions of these measurements are those given by Morant in "A Biometric Study of the Human Mandible." The six quoted were chosen because, judging from the material on which he worked, they appeared to distinguish the sexes more effectively than the other measurements: *Biometrika*, Vol. xxviii. see pp. 96—99.

In order to investigate a further problem, the series of mandibles has been divided into two groups which will be referred to as "Non-Aged" and "Aged" mandibles respectively. The criterion used was that a mandible was classed as "Aged" if it had lost two or more molars from at least one side before death, it being assumed that, where there was no socket for the third molar, it had erupted and had been lost before death. All other mandibles were classed as "Non-Aged." There were found to be 357 Non-Aged and 75 Aged mandibles. The object of this division is to discover if possible whether there is any significant change in any of the measurements, following the loss of molars. If this is found to be the case, the question will arise whether, in any series, the values of the measurements concerned taken from the "Aged" members of the series should be excluded.

2. *The Proportion of the Sexes in the Mandibles.* The only outside evidence that we possess in this connection is the fact that the crania are in the proportion 935 ♂ : 628 ♀, or roughly 3 : 2, this ratio having been obtained by anatomical appreciation. Assuming this to be correct, and that the selection of the whole series of mandibles under consideration is equivalent to a random selection, this is the most probable proportion of sexes in the mandibles. This would give us, for 432 mandibles, 258 ♂ and 174 ♀.

More generally, the chance of drawing m ♂ and f ♀ in a sample of n from a population N consisting of M ♂ and F ♀ is

$$\frac{n!}{m!\,f!}\,\frac{(N-n)!}{N!}\,\frac{M!\,F!}{(M-m)!\,(F-f)!}.$$

Taking $N = 1563$, $M = 935$, $F = 628$,

$n = 432$, $m = 258$, $f = 174$,

the chance

$$= \frac{432!}{258!\,174!}\,\frac{1131!\,935!\,628!}{1563!\,677!\,454!}$$

$$= \cdot 0459.$$

Hence the odds are roughly 20 to 1 against the proportion of sexes in the mandibles being exactly equal to that in the crania. If, however, we allow a 5 per cent. error in this assumption of equal proportions, i.e. 22 mandibles wrongly classified, then the chance of the proportion lying between 247 ♂ : 185 ♀ and 269 ♂ : 163 ♀ is the sum of 23 terms of the hypergeometric series, ·0459 being the middle term. We obtain the values for these terms shown in the table below.

Hence the odds are more than 4 to 1 in favour of our making an error of less than 5 per cent. in assuming the proportion of sexes in the mandibles to be equal to that in the crania. Again, the chance of 12 or fewer mandibles being misclassified is the sum of the 13 middle terms, i.e. ·5461. Now 12 mandibles = 2·8 per cent. of the total, hence we may say that the odds are in favour of our making an error of less than 3 per cent.

Considering now the two groups of "Non-Aged" and "Aged" mandibles, we can hardly expect that the sex-proportion will be the same in each. An examination

m	f	Chance	m	f	Chance
247	185	·0193	259	173	·0459
248	184	·0223	260	172	·0453
249	183	·0254	261	171	·0441
250	182	·0286	262	170	·0423
251	181	·0318	263	169	·0401
252	180	·0349	264	168	·0375
253	179	·0377	265	167	·0346
254	178	·0403	266	166	·0315
255	177	·0425	267	165	·0283
256	176	·0442	268	164	·0251
257	175	·0454	269	163	·0220
258	174	·0459			
				Total Chance	·8150

of the Registrar-General's Reports on the present population of England and Wales reveals that the proportion of deaths of aged women to deaths of aged men is greater than the corresponding proportion for younger people. We cannot, however, make an exact comparison between this population and that from which our Egyptian mandibles are drawn, for various reasons; thus the loss of molars probably occurs earlier in a modern population than in an ancient one, and the proportion of aged people in the total population is probably greater nowadays than at the period of our Egyptian series. Further, the names "Non-Aged" and "Aged" applied to our two groups are perhaps somewhat misleading; for while it is no doubt true that the mean age of the "Aged" group is greater than that of the "Non-Aged," yet the age-distributions of the two groups probably overlap considerably. Hence we cannot hope to obtain from consideration of a modern population any reasonably accurate idea of the relation between the sex-proportions of our two groups. We have therefore adopted the following method, which seems to us the best: assuming the sex-proportion of the whole series of 432 mandibles to be the same as that of the crania, we have sexed this series mathematically, leaving the sex-proportions in the two groups to emerge from this sexing. It will be seen that an anatomical sexing, carried out in a similar way, gives substantially the same proportions as those obtained from the mathematical sexing.

3. *Methods of Mathematical Sexing.* The distribution of values of any particular character should be a combination of the normal distributions of this character for males and females respectively. Professor Karl Pearson has given* the theoretical solution of the problem of resolving the combined distribution into its normal components. This solution, if carried out, would give the means and standard deviations of the character for the two sexes, and the actual numbers of males and females, but not the sexes of the individual bones. But unless these are determined, the work, which is laborious, will have to be performed for every

* "Contributions to the Mathematical Theory of Evolution," *Phil. Trans.*, Vol. 185 A (1894), pp. 71—110.

character, while the somewhat differing sex-proportions which may be expected to emerge for the various characters will have to be reconciled. Hence the sexing of the individual bones is a practical necessity, and in this connection Professor Pearson and Dr Julia Bell, dealing with the femur, have written* as follows:

"A dichotomy which gives, at that character value, only n % males, where n is a small number, may be taken to mark the bone for this character as ♀; one where only n % are females as ♂; dichotomies which give only n % to m % males, where m is a moderate number, and again only m % to n % females, may be used to form the classes ♀? and ♂? respectively; while between the dichotomies with m % males and m % females, we frankly record the bone as of unknown sex (♁). We have now five classes, and these categories should be made for, say, 4 to 6 suitable characters. We thus obtain for each bone, using six characters, a series like: ♀, ♀?, ♀, ♂?, ♀?, ♁. The balance of such a classification will almost invariably determine the sex of the bone as definitely as we know the sex of a cranium. In our actual work on these lines we gave a mark +2 for ♀, +1 for ♀?, 0 for ♁, −2 for ♂, −1 for ♂? The above series indicates femaleness by 5. Bones with zero may be marked ♁, or, better, settled by general anatomical appreciation as ♂? or ♀? Into which classes we may also put bones of 0 to −2 and of 0 to +2 respectively.... The actual choice of n and m is not of such importance as might at first appear, if all the characters are given approximately the same n and m. The following have been taken [for Dwight's femur data], based on the fitted frequency curves: $n = 8$ %, $m = 25$ %."

Now this method supposes that the previous work of resolving the distribution of each character into its components has been successfully carried out, for a knowledge of these components is necessary in order to determine the points of division corresponding to the numbers m and n. Assuming that the values of m and n are not of great importance, we might make arbitrary divisions in the distribution of each character, but we should have no means of ensuring that these divisions corresponded to approximately the same m and n for each distribution. Again, if by X is denoted the sum of the marks allotted to each bone for the various characters, there does not appear to be any reason for supposing that the number of bones for which X is zero will be small; in fact, unless the means of the component distributions are very widely separated, this group will be among the largest. Hence there will be a considerable number of bones of indeterminate sex which for purposes of subsequent calculation must be resolved into males and females. It is suggested that this be done anatomically, but a determination of sex by general anatomical appreciation will hardly be reliable in cases where the sex is mathematically doubtful, except possibly in cases of bones of abnormal form. Further, after considerable effort with certain characters, we have failed entirely to obtain any acceptable component normal distributions†. The method of marking,

* "A Study of the Long Bones of the English Skeleton. Text. Part I. The Femur." *Drapers' Company Research Memoirs, Biometric Series X* (1919). The passage quoted is taken from pp. 44—46.

† See Section 12 below.

however, suggests that if we can form a function X which will take a different value for each bone, a large value denoting, say, maleness, and a small value femaleness, then arranging these values in order of magnitude we may make a division at some point of the series and thus assign a sex to each bone. To make the division we must know the sex-proportion, and this in the case of our 432 mandibles we shall assume to be the same as the proportion in the crania.

If the six characters $g_o g_o$, $c_y l$, $c_p l$, $c_r h$, $M\angle$ and $R\angle$ be denoted respectively by x_1, x_2, x_3, x_4, x_5 and x_6, the simplest function is the linear function

$$X = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5 + a_6x_6.$$

We have tried three such functions, which will now be described.

(*a*) We know from other sexed series of mandibles that the male mean is significantly larger than the female mean in all of the six characters except the mandibular angle ($M\angle$), in which the position is reversed. Hence the simplest function possible is that in which the coefficients are 1, 1, 1, 1, −1, 1 respectively, i.e. $X = x_1 + x_2 + x_3 + x_4 - x_5 + x_6$.

(*b*) The objection to (*a*) may be made that quantities not comparable are being added and subtracted; that each x should be divided by its σ. But this means that when dealing with a male bone we should divide by the σ of the distribution of males, and with a female bone by the σ of the distribution of females. But we do not know which bones are male and which female, neither do we know the respective σ's. The best that can be done towards meeting this objection is to divide the value of each character by the σ of the combined distribution of that character. The function then becomes

$$X = \frac{x_1}{\sigma_1} + \frac{x_2}{\sigma_2} + \frac{x_3}{\sigma_3} + \frac{x_4}{\sigma_4} - \frac{x_5}{\sigma_5} + \frac{x_6}{\sigma_6}*.$$

This method is open to objection on the ground that the σ's used are dependent on other quantities besides the σ's of the component distributions, e.g. they are dependent on the separation of the means. Now if this separation is large in any character, then that character is probably an important one to use for the purpose of sexing. But, other things being equal, a larger separation of the means would give a larger σ for the combined distribution and hence a smaller $\frac{x}{\sigma}$, i.e. the importance of the character is decreased in the function X.

(*c*) The third function, which is the one we have finally adopted, has been suggested by Professor R. A. Fisher. The distribution of the function X will be a combination of the distributions of X for the two sexes separately; we shall thus make the least error in a dichotomic division if the function can be chosen so that the "overlap" of its components is a minimum. Professor Fisher has given the following method of obtaining this function.

* For calculation purposes it is more convenient to take the function

$$X = \frac{\sigma_6}{\sigma_1}x_1 + \frac{\sigma_6}{\sigma_2}x_2 + \frac{\sigma_6}{\sigma_3}x_3 + \frac{\sigma_6}{\sigma_4}x_4 - \frac{\sigma_6}{\sigma_5}x_5 + x_6.$$

Let $\bar{X}_{\delta}$ and $\bar{X}_{\varphi}$ be the means of the component distributions of X; then, if $d_s = \bar{x}_{s\delta} - \bar{x}_{s\varphi}$,

$$\bar{X}_{\delta} - \bar{X}_{\varphi} = a_1 d_1 + a_2 d_2 + a_3 d_3 + a_4 d_4 + a_5 d_5 + a_6 d_6.$$

Also, for either sex, of frequency n, if $\frac{V(X)}{n}$ be the variance,

$$\begin{aligned} V(X) &= \Sigma (X - \bar{X})^2 = \Sigma \{a_1 (x_1 - \bar{x}_1) + \ldots\}^2 \\ &= a_1^2 V(x_1) + \ldots + 2a_1 a_2 W(x_1 x_2) + \ldots, \end{aligned}$$

where

$$V(x_1) = \Sigma (x_1 - \bar{x}_1)^2 = \Sigma (x_1^2) - n\bar{x}_1^2,$$

$$W(x_1 x_2) = \Sigma (x_1 - \bar{x}_1)(x_2 - \bar{x}_2) = \Sigma (x_1 x_2) - n\bar{x}_1 \bar{x}_2.$$

Then to attain our object of "minimum overlap," we shall make the function

$$\frac{(\bar{X}_{\delta} - \bar{X}_{\varphi})^2}{V(X)}$$

a maximum. We have thus to choose the a's so as to make the expression

$$\frac{(a_1 d_1 + a_2 d_2 + a_3 d_3 + a_4 d_4 + a_5 d_5 + a_6 d_6)^2}{a_1^2 V(x_1) + \ldots + 2a_1 a_2 W(x_1 x_2) + \ldots}$$

a maximum. In the calculation the values of the V's and W's in the denominator of this expression will be obtained for the two sexes separately and then pooled.

Taking logarithms and differentiating with respect to a_1, a_2, a_3, a_4, a_5, a_6 in turn and equating to zero, we obtain six equations of which the first is

$$\frac{d_1}{\Sigma (ad)} = \frac{a_1 V(x_1) + a_2 W(x_1 x_2) + \ldots + a_6 W(a_1 a_6)}{V(X)},$$

or

$$a_1 V(x_1) + a_2 W(x_1 x_2) + \ldots + a_6 W(a_1 a_6) = \lambda d_1,$$

where

$$\lambda = \frac{V(X)}{\Sigma (ad)}.$$

Now there will be no loss of generality in our function if a_6 is taken to be unity. Hence putting $a_6 = 1$, we have the following six equations to solve for a_1, a_2, a_3, a_4, a_5 and λ:

$$\begin{aligned}
a_1 V(x_1) + a_2 W(x_1 x_2) + a_3 W(x_1 x_3) + a_4 W(x_1 x_4) + a_5 W(x_1 x_5) + W(x_1 x_6) &= \lambda d_1, \\
a_1 W(x_1 x_2) + a_2 V(x_2) \quad + a_3 W(x_2 x_3) + a_4 W(x_2 x_4) + a_5 W(x_2 x_5) + W(x_2 x_6) &= \lambda d_2, \\
a_1 W(x_1 x_3) + a_2 W(x_2 x_3) + a_3 V(x_3) \quad + a_4 W(x_3 x_4) + a_5 W(x_3 x_5) + W(x_3 x_6) &= \lambda d_3, \\
a_1 W(x_1 x_4) + a_2 W(x_2 x_4) + a_3 W(x_3 x_4) + a_4 V(x_4) \quad + a_5 W(x_4 x_5) + W(x_4 x_6) &= \lambda d_4, \\
a_1 W(x_1 x_5) + a_2 W(x_2 x_5) + a_3 W(x_3 x_5) + a_4 W(x_4 x_5) + a_5 V(x_5) \quad + W(x_5 x_6) &= \lambda d_5, \\
a_1 W(x_1 x_6) + a_2 W(x_2 x_6) + a_3 W(x_3 x_6) + a_4 W(x_4 x_6) + a_5 W(x_5 x_6) + V(x_6) \quad &= \lambda d_6.
\end{aligned}$$

Now the quantities in these equations are all unknown for the *E* series of mandibles. We have hence evaluated them for a sexed series of Egyptian mandibles fairly similar in type to the *E* series, hoping that the function thus obtained will be approximately the function of minimum overlap for the *E* series. The

series selected was the Kerma series, whose individual measurements have been published in this *Journal**. We found in this series 45 ♂ and 38 ♀ mandibles for which all six characters were given. Calculating the values of the coefficients in the left-hand sides of the above equations for males and females separately and pooling the results, we obtained the equations:

$$\begin{aligned}
3282{\cdot}41a_1 + 288{\cdot}19a_2 - 107{\cdot}30a_3 + 703{\cdot}19a_4 - 35{\cdot}99a_5 - 7{\cdot}1212\lambda &= 2{\cdot}30,\\
288{\cdot}19a_1 + 232{\cdot}96a_2 + 179{\cdot}78a_3 + 189{\cdot}07a_4 - 176{\cdot}70a_5 - 2{\cdot}0497\lambda &= -52{\cdot}32,\\
-107{\cdot}30a_1 + 179{\cdot}78a_2 + 1462{\cdot}12a_3 + 431{\cdot}58a_4 - 707{\cdot}07a_5 - 6{\cdot}0648\lambda &= -461{\cdot}41,\\
703{\cdot}19a_1 + 189{\cdot}07a_2 + 431{\cdot}58a_3 + 2092{\cdot}23a_4 - 1351{\cdot}44a_5 - 10{\cdot}7186\lambda &= -1230{\cdot}81,\\
-35{\cdot}99a_1 - 176{\cdot}70a_2 - 707{\cdot}07a_3 - 1351{\cdot}44a_4 + 3258{\cdot}97a_5 + 7{\cdot}3617\lambda &= 1145{\cdot}50,\\
-2{\cdot}30a_1 + 52{\cdot}32a_2 + 461{\cdot}41a_3 + 1230{\cdot}81a_4 - 1145{\cdot}50a_5 - 7{\cdot}3427\lambda &= -3655{\cdot}42.
\end{aligned}$$

The solution of these equations is

$$a_1 = 3{\cdot}201, \quad a_2 = 5{\cdot}035, \quad a_3 = 7{\cdot}164, \quad a_4 = 9{\cdot}665, \quad a_5 = 0{\cdot}524, \quad \lambda = 2522{\cdot}639.$$

Hence the function is

$$X = 3{\cdot}201\, g_o g_o + 5{\cdot}035\, c_y l + 7{\cdot}164\, c_p l + 9{\cdot}665\, c_r h + 0{\cdot}524\, M\angle + R\angle.$$

It is seen that the greatest weight is attached to the character $c_r h$, while the character $M\angle$ is comparatively insignificant. The latter character also appears with a positive weight, which is contrary to expectation.

If we denote $\sqrt{\frac{V(X)}{N}}$, where N is the total population (83)†, by s, then

$$\frac{\bar{X}_{\delta} - \bar{X}_{\varphi}}{s} = \Sigma(ad)\sqrt{\frac{N}{V(X)}} = \sqrt{\frac{N\Sigma(ad)}{\lambda}}.$$

Now $\Sigma(ad)$ works out to be 183·64785, hence

$$\frac{\bar{X}_{\delta} - \bar{X}_{\varphi}}{s} = 2{\cdot}4581.$$

We can get a rough estimate of the error we shall make by this method of sexing by supposing the distribution of X to consist of two equal normal distributions each of standard deviation s, and with the above separation of the means. The curves will intersect at a point of deviation $1{\cdot}229\sigma$ from the mean of either; from Sheppard's table we find that, for this deviation, $\frac{1}{2}(1+\alpha) = {\cdot}890$. Hence about 11 per cent. of each sex will be misclassified, i.e. about 11 per cent. of the total number of bones. This is the error we may expect on applying the method to the Kerma series (actually an error of 10 out of 83, i.e. 12 per cent. is obtained, see next section); on applying it to any other series the error may be greater.

The above work has also been carried out for only four characters, viz. $g_o g_o$, $c_p l$, $c_r h$ and $M\angle$. The weights work out to be 10·816, 24·068, 31·780 and 1 respectively.

* See footnote § to p. 149.

† Professor Fisher points out that it would be theoretically more correct to use $N-2$ or 81 here, as the number of degrees of freedom is 81.

Making the first weight 3·201 as for the case of six characters, and reducing the others proportionally, the function is

$$X = 3{\cdot}201\, g_o g_o + 7{\cdot}123\, c_p l + 9{\cdot}405\, c_r h + 0{\cdot}296\, M\angle.$$

The first three weights are substantially the same as those obtained above for six characters, but the weight attached to $M\angle$ is considerably reduced. The value of $(X_{\sigma^{\prime}} - X_{\varphi})/s$ is now 2·4103, and the estimated error is 11·4 per cent. Thus the use of four characters may give almost as accurate a result as the use of six, and probably the three characters $g_o g_o$, $c_p l$ and $c_r h$, representing a breadth, length and height measurement respectively, would suffice.

4. *Application to other Egyptian Series.* The above methods have been applied to three Egyptian series of mandibles which had previously been sexed, either by association with anatomically sexed crania or by graveyard evidence*. The following table gives the percentage agreement of this "accepted" sexing with the sexing by methods (*a*), (*b*) and (*c*) respectively, described above in Section 3.

Percentage Agreement of "Accepted" Sexing with Methods (*a*), (*b*), (*c*).

Method	Kerma (83)	Qau (99)	Sedment (30)
(*a*)	86	74	81
(*b*)	88	78	81
(*c*)	88	82	81

Thus the three methods give practically the same degree of agreement with the accepted sexing, except in the Qau series, where method (*c*) gives the highest agreement. In the case of method (*c*) the percentage agreement is highest for the Kerma series, as would have been expected since its measurements were used in determining the weights; but the Kerma series also shows the highest percentages for methods (*a*) and (*b*). The actual agreement between the methods is very high, as is seen in the following table:

Percentage Agreement in Sexing between Methods (*a*), (*b*), (*c*).

Methods	Kerma	Qau	Sedment
(*a*) and (*b*)	95	93	100
(*b*) and (*c*)	94	91	100
(*c*) and (*a*)	93	86	100
(*a*), (*b*) and (*c*)	90	85	100

* The Kerma and Qau series are dealt with in Morant's paper cited and the Sedment series in T. L. Woo's: "A Study of Seventy-one Ninth Dynasty Skulls from Sedment," *Biometrika*, Vol. XXII. (1930), pp. 65—92.

Thus the different weights given to the six characters in the function X, by the three methods, do not affect the sexing of the vast majority of mandibles, i.e. when a male bone has markedly male characteristics all three methods usually classify it as male, while if a male bone has female characteristics all the methods usually classify it incorrectly as female. (The anatomist no doubt does likewise.) Further, we do not gain, apparently, by taking the balance given by the three methods for each mandible, e.g. by classifying as male a bone sexed by (*a*) as ♂, by (*b*) as ♀, by (*c*) as ♂. For if this is done, the percentage agreements with the accepted sexing work out to be 87, 78 and 81 for the three series respectively, the first two being less than those given by (*c*) alone.

5. *Application to the E Series.* Similar results are obtained when the *E* series is sexed mathematically by the above three methods. The three functions are

(*a*) $X = g_o g_o + c_y l + c_p l + c_r h - M\angle + R\angle,$

(*b*)* $X = 1{\cdot}069\, g_o g_o + 4{\cdot}567\, c_y l + 1{\cdot}740\, c_p l + 1{\cdot}197\, c_r h - 1{\cdot}228\, M\angle + R\angle,$

(*c*) $X = 3{\cdot}201\, g_o g_o + 5{\cdot}035\, c_y l + 7{\cdot}164\, c_p l + 9{\cdot}665\, c_r h + 0{\cdot}524\, M\angle + R\angle.$

Their values have been calculated for each of the 432 mandibles and in each of the three cases a dichotomic division has been made so as to give 258 males and 174 females. Thus in (*c*) the values of X range from 1428 to 1961, and a division at the value 1632 produced the desired sex-proportion. The series has also been sexed anatomically by Mr F. H. Cleaver, under the direction of Dr Morant. Mr Cleaver assumed that there were 258 ♂ and 174 ♀ bones, but he did not know of our division of the series into "Non-Aged" and "Aged" groups, and the two groups were thoroughly mixed before sexing. On separating the two groups after sexing, the following numbers of males and females in the groups were found by the various methods:

Numbers of ♂ and ♀ Mandibles in "Non-Aged" and "Aged" Groups.

Method	Non-Aged		Aged	
	♂	♀	♂	♀
(*a*)	219	138	39	36
(*b*)	220	137	38	37
(*c*)	220	137	38	37
Anatomical	218	139	40	35

There is thus very close agreement in the sex-proportions of the two groups as given by the different methods. It is seen that the proportion of females is greater in the "Aged" series than in the "Non-Aged," a result which was expected from

* See footnote to p. 153.

other considerations. The percentage agreement between the methods is shown in the following table:

Percentage Agreement in Sexing between Various Methods.

Methods	Non-Aged	Aged	Together
(*a*) and Anatomical	82·9	74·7	81·5
(*b*) and Anatomical	84·6	78·7	83·6
(*c*) and Anatomical	86·6	81·3	85·6
(*a*) and (*b*)	93·3	96·0	93·7
(*b*) and (*c*)	93·3	92·0	93·1
(*c*) and (*a*)	92·4	88·0	91·7

These results are very similar to the results obtained for the Kerma, Qau and Sedment series. Of methods (*a*), (*b*) and (*c*), the last shows the closest agreement with the anatomical sexing; while the three mathematical methods show very high agreement in spite of the widely different weights given to the characters in the three functions. The "Non-Aged" series has also been sexed anatomically by Dr Evan Greene, Professor of Anatomy in Alberta University, but unfortunately we did not ask him to sex the "Aged" mandibles. Of the "Non-Aged" he designated 205 bones as ♂ and 152 as ♀, i.e. 15 fewer ♂ bones and 15 more ♀ than given by method (*c*); nevertheless, his agreement with method (*c*) is 88·0 per cent.

In the work which follows, the sexes as given by method (*c*) have been adopted.

6. *The Nature of the E Series of Mandibles.* An examination of the means given in Table I shows that, for the Non-Aged mandibles, the sex differences of 21 characters are significant, while those of 3 characters ($C'\angle$, $100\,c_r c_r/ml$ and $100\,g_o g_o/c_p l$) are insignificant. For the Aged mandibles the sex differences of 17 characters are significant, and those of 7 characters ($m_2 h$, $M\angle$, $C'\angle$, $100\,c_r c_r/ml$, $100\,g_o g_o/c_p l$, $100\,rb'/rl$, and $100\,g_o g_o/c_r c_r$) are insignificant. The criterion used is that a difference is or is not significant according as it is greater or less than 3·5 times its probable error.

Comparing the two series, we find that for no character does the male mean (Non-Aged) differ significantly from the male mean (Aged). For the females, however, the differences are significant for the characters

$$w_1\,(\Delta/\text{p.e.}\,\Delta = -4{\cdot}8),\quad g_o g_o\,(-4{\cdot}1),\quad c_r c_r\,(-4{\cdot}1),\quad m_2 p_1\,(+6{\cdot}1),\quad c_r h\,(-3{\cdot}6),$$
$$100\,g_o g_o/c_p l\,(-3{\cdot}6)\quad\text{and}\quad 100\,rb'/rl\,(+3{\cdot}6).$$

It is hard to deduce any material difference between the two series from these results. Comparing also the sex difference for each character in the two series, we note that for 13 characters the sex difference (Non-Aged) is greater than the corresponding sex difference (Aged), and *vice versa* for the other 11 characters. On investigation, i.e. dividing the difference between the sex differences by its probable error, we find that for no character is there any significant change in the sex difference on passing from the Non-Aged to the Aged series.

TABLE I.

Mean Measurements (with their Probable Errors) of the Egyptian E Series of Mandibles.*

Character	Non-Aged Mandibles		Aged Mandibles	
	Mean ♂	Mean ♀	Mean ♂	Mean ♀
w_1†	116·84 ± ·26 (209)	110·54 ± ·34 (128)	118·90 ± ·61 (33)	113·06 ± ·41 (31)
$g_o g_o$	93·67 ± ·29 (220)	85·57 ± ·33 (137)	95·61 ± ·72 (38)	88·26 ± ·56 (37)
$c_r c_r$	93·96 ± ·25 (202)	88·24 ± ·29 (130)	95·66 ± ·57 (29)	90·88 ± ·58 (33)
zz	44·46 ± ·10 (220)	42·86 ± ·12 (134)	44·29 ± ·30 (38)	42·68 ± ·30 (37)
$c_y l$	21·14 ± ·06 (220)	19·32 ± ·09 (137)	21·33 ± ·16 (38)	19·31 ± ·15 (37)
ml	103·63 ± ·23 (213)	98·79 ± ·25 (133)	103·87 ± ·50 (35)	98·61 ± ·56 (36)
$c_p l$	74·63 ± ·17 (220)	69·43 ± ·20 (137)	74·03 ± ·40 (38)	69·11 ± ·38 (37)
rb'	32·51 ± ·11 (220)	30·25 ± ·14 (137)	31·96 ± ·37 (38)	29·62 ± ·28 (37)
$c_y c_r$	33·10 ± ·14 (219)	31·92 ± ·16 (136)	34·25 ± ·39 (36)	32·21 ± ·38 (36)
$m_2 p_1$	27·93 ± ·07 (184)	26·84 ± ·09 (121)	27·37 ± ·33 (7)	25·44 ± ·21 (5)
h_1	32·63 ± ·14 (191)	29·15 ± ·19 (117)	33·64 ± ·38 (16)	29·28 ± ·59 (12)
$m_2 h$	25·79 ± ·11 (198)	23·12 ± ·15 (128)	25·87 ± ·40 (9)	23·90 ± ·51 (8)
$c_r h$	67·10 ± ·22 (220)	56·99 ± ·20 (137)	66·38 ± ·57 (38)	58·35 ± ·32 (37)
rl	61·42 ± ·19 (220)	54·14 ± ·21 (137)	60·86 ± ·50 (37)	55·22 ± ·47 (37)
ik'	13·17 ± ·08 (220)	11·50 ± ·08 (137)	13·28 ± ·18 (38)	11·18 ± ·17 (37)
$M\angle$	121°·70 ± ·27 (220)	126°·78 ± ·34 (137)	123°·66 ± ·60 (38)	125°·82 ± ·61 (37)
$R\angle$	77°·47 ± ·35 (220)	70°·53 ± ·34 (137)	76°·70 ± ·87 (38)	71°·07 ± ·67 (37)
$C'\angle$	70°·17 ± ·29 (172)	69°·76 ± ·32 (100)	71°·06 ± ·99 (16)	69°·64 ± 1·25 (14)
$100\, c_r h/ml$	64·97 ± ·26 (213)	57·81 ± ·24 (133)	64·16 ± ·58 (35)	59·37 ± ·44 (36)
$100\, c_r c_r/ml$	90·98 ± ·31 (198)	89·61 ± ·35 (127)	92·39 ± ·74 (29)	92·14 ± ·72 (33)
$100\, g_o g_o/c_p l$	125·85 ± ·51 (220)	123·64 ± ·60 (137)	129·52 ± 1·27 (38)	128·03 ± 1·06 (37)
$100\, rb'/rl$	53·17 ± ·24 (220)	56·10 ± ·32 (137)	52·63 ± ·60 (37)	53·32 ± ·70 (36)
$100\, ik'/c_y c_r$	40·04 ± ·27 (219)	36·32 ± ·31 (136)	39·11 ± ·59 (36)	35·01 ± ·67 (36)
$100\, g_o g_o/c_r c_r$	99·74 ± ·34 (202)	97·15 ± ·43 (130)	100·37 ± ·78 (29)	96·92 ± ·71 (33)

* The number of measurements on which each mean is based is given in brackets.

† The measurements are: w_1, bicondylar breadth; $g_o g_o$, bigonial breadth; $c_r c_r$, breadth between tips of coronoid processes; zz, minimum breadth between inner margins of *foramina mentalia*; $c_y l$, maximum length of left condyle; ml, total projective length on mandible board; $c_p l$, projective length of the corpus on mandible board; rb', minimum breadth of the left ramus; $c_y c_r$, chord from "apices" of condylar and coronoid processes on left side; $m_2 p_1$, distance between outer alveolar margin at middle of first premolar to middle of second molar on left side; h_1, symphyseal height from intradental to gnathion; $m_2 h$, height of outer alveolar margin at middle of left second molar; $c_r h$, height of left coronoid process; rl, projective length of left ramus (mandible board); ik', maximum depth of left incisura from chord $c_y c_r$; $M\angle$, mandibular angle; $R\angle$, angle between condylar-coronoidal line and ramus tangent on left side; $C'\angle$, mental angle between line joining infradental to pogonion and the standard horizontal plane.

Tables II and III give the standard deviations and the coefficients of variation for the characters of the two *E* series. Considering the coefficients of variation of the absolute measurements in the Non-Aged series, the male value exceeds the female value in 8 cases and in the remaining 7 cases the position is reversed. In only one case is the difference significant, viz. $c_r h$ (3·9, ♂ greater). Considering the standard deviations of the indices and angles, the male value is the greater in 6 cases and the female is the greater in the other 3 cases. Two of the differences are

TABLE II.

Standard Deviations for the Egyptian E Series of Mandibles.*

Character	Non-Aged Mandibles		Aged Mandibles	
	♂	♀	♂	♀
w_1	5·51 ± ·18	5·73 ± ·24	5·17 ± ·43	3·40 ± ·29
$g_o g_o$	6·37 ± ·20	5·72 ± ·23	6·57 ± ·51	5·03 ± ·39
$c_r c_r$	5·22 ± ·18	4·92 ± ·21	4·53 ± ·40	4·92 ± ·41
zz	2·23 ± ·07	2·09 ± ·09	2·76 ± ·21	2·75 ± ·22
$c_y l$	1·38 ± ·04	1·48 ± ·06	1·50 ± ·12	1·33 ± ·10
ml	4·91 ± ·16	4·27 ± ·18	4·39 ± ·35	4·97 ± ·40
$c_p l$	3·77 ± ·12	3·46 ± ·14	3·61 ± ·28	3·45 ± ·27
rb'	2·44 ± ·08	2·36 ± ·10	3·38 ± ·26	2·53 ± ·20
$c_y c_r$	3·08 ± ·10	2·82 ± ·12	3·47 ± ·28	3·39 ± ·27
$m_2 p_1$	1·44 ± ·05	1·52 ± ·07	1·31 ± ·24	0·69 ± ·15
h_1	2·91 ± ·10	3·02 ± ·13	2·25 ± ·27	3·01 ± ·41
$m_2 h$	2·28 ± ·08	2·44 ± ·10	1·77 ± ·28	2·14 ± ·36
$c_r h$	4·88 ± ·16	3·39 ± ·14	5·24 ± ·41	2·90 ± ·23
rl	4·28 ± ·14	3·68 ± ·15	4·50 ± ·35	4·25 ± ·33
ih'	1·68 ± ·05	1·42 ± ·06	1·69 ± ·13	1·53 ± ·12
$M\angle$	5°·88 ± ·19	5°·96 ± ·24	5°·50 ± ·43	5°·51 ± ·43
$R\angle$	7°·74 ± ·25	5°·96 ± ·24	7°·95 ± ·62	6°·08 ± ·48
$C'\angle$	5°·63 ± ·20	4°·68 ± ·22	5°·88 ± ·70	6°·95 ± ·89
$100\, c_r h/ml$	5·53 ± ·18	4·13 ± ·17	5·12 ± ·41	3·90 ± ·31
$100\, c_r c_r/ml$	6·51 ± ·22	5·82 ± ·25	5·87 ± ·52	6·09 ± ·51
$100\, g_o g_o/c_p l$	11·21 ± ·36	10·43 ± ·43	11·63 ± ·90	9·58 ± ·75
$100\, rb'/rl$	5·29 ± ·17	5·49 ± ·22	5·38 ± ·42	6·22 ± ·49
$100\, ih'/c_y c_r$	5·86 ± ·19	5·32 ± ·22	5·26 ± ·42	5·93 ± ·47
$100\, g_o g_o/c_r c_r$	7·26 ± ·24	7·28 ± ·30	6·25 ± ·55	6·04 ± ·50

TABLE III.

Coefficients of Variation for the Egyptian E Series of Mandibles.*

Character	Non-Aged Mandibles		Aged Mandibles	
	♂	♀	♂	♀
w_1	4·72 ± ·16	5·18 ± ·22	4·35 ± ·36	3·01 ± ·30
$g_o g_o$	6·80 ± ·22	6·68 ± ·27	6·87 ± ·53	5·70 ± ·45
$c_r c_r$	5·56 ± ·19	5·58 ± ·23	4·74 ± ·42	5·41 ± ·45
zz	5·02 ± ·16	4·88 ± ·20	6·23 ± ·48	6·44 ± ·51
$c_y l$	6·53 ± ·21	7·66 ± ·31	7·03 ± ·55	6·89 ± ·54
ml	4·74 ± ·16	4·32 ± ·18	4·23 ± ·34	5·04 ± ·40
$c_p l$	5·05 ± ·16	4·98 ± ·20	4·88 ± ·38	4·99 ± ·39
rb'	7·51 ± ·24	7·80 ± ·32	10·58 ± ·83	8·54 ± ·67
$c_y c_r$	9·31 ± ·30	8·83 ± ·36	10·13 ± ·81	10·52 ± ·85
$m_2 p_1$	5·16 ± ·18	5·66 ± ·25	4·79 ± ·87	2·71 ± ·58
h_1	8·92 ± ·31	10·36 ± ·46	6·69 ± ·80	10·28 ± 1·43
$m_2 h$	8·84 ± ·30	10·55 ± ·45	6·84 ± 1·09	8·95 ± 1·52
$c_r h$	7·27 ± ·24	5·95 ± ·24	7·89 ± ·61	4·97 ± ·39
rl	6·97 ± ·23	6·80 ± ·28	7·39 ± ·58	7·70 ± ·61
ih'	12·76 ± ·42	12·35 ± ·51	12·73 ± 1·00	13·69 ± 1·09

* The number of measurements on which each standard deviation and coefficient of variation is based is given in Table I of means.

significant, viz. those for $R\angle$ (5·1) and $100\,c_r h/ml$ (5·7), the male being the greater in each case.

For the absolute measurements of the Aged series, the male coefficient of variation is the greater in 6 cases and the female is the greater in the other 9 cases. Only one of the differences is significant, viz. that for $c_r h$ (4·1, ♂ greater). For the indices and angles the male standard deviation is the greater in 4 cases and the female in the other 5 cases, but no one of the differences is significant. There is thus some rather slight evidence that the variability is greater for males than for females in the Non-Aged series, but hardly any evidence of the same relation in the Aged series.

On comparing the variabilities of the Non-Aged and Aged series for each sex in a similar way, we find three significant differences, viz. that between the male coefficients of variation for rb' (3·6, Aged greater) and those between the female coefficients of variation for w_1 (5·9, Non-Aged greater) and m_2p_1 (4·7, Non-Aged greater).

The variability of the *E* series (Non-Aged) may now be compared with those of the Qau and Kerma Egyptian series discussed by Dr Morant*. Comparing the *E* and Qau males, we find two significant differences between the coefficients of variation, viz. $c_y l$ (4·8, Qau greater) and $c_y c_r$ (3·7, *E* greater). There are two significant differences between the standard deviations of indices and angles, viz. $100\,ih'/c_y c_r$ (3·6, *E* greater) and $100\,g_o g_o/c_r c_r$ (3·8, *E* greater). For the females we find only one significant difference between the coefficients of variation, viz. m_2p_1 (3·6, *E* greater) and none between the standard deviations of indices and angles. There is thus little difference in variability between the two series.

Comparing the *E* and Kerma males, there are two significant differences between the coefficients of variation, viz. rl (4·2, Kerma greater) and ih' (3·9, *E* greater), and one between the standard deviations of indices and angles, viz. $100\,ih'/c_y c_r$ (5·7, *E* greater). For the females there are significant differences between the coefficients of variation for $c_y c_r$ (3·9, *E* greater) and $c_r h$ (5·0, Kerma greater), and a significant difference in the standard deviations of $100\,c_r h/ml$ (3·5, Kerma greater). There is thus no indication of difference in variability between these two series.

Sex ratios (male means/female means) are given for the fifteen absolute measurements in Table IV. They may be compared with the sex ratios of cranial characters which are given for seven measurements of the *E* and other series, in a paper by Miss Elisabeth Kitson†. It is seen that the sex ratios for our mandibles are markedly larger than those for these series of crania; this suggests that the mandible may be a better bone than the cranium to use for mathematical sexing. The sex ratios of the *E* series of mandibles may also be compared with those of the mandibles of the Kerma, Qau and Anglo-Saxon series, some of which are given in Dr Morant's paper, the others being obtainable from the tables of

* *Loc. cit.*

† "A Study of the Negro Skull with Special Reference to the Crania from Kenya Colony," *Biometrika*, Vol. XXIII. (1931), pp. 271—314.

TABLE IV.

Sex Ratios for the Egyptian E Series of Mandibles.

Character	Non-Aged	Aged	Character	Non-Aged	Aged
w_1	1·057	1·052	$c_y c_r$	1·037	1·063
$g_o g_o$	1·095	1·083	$m_2 p_1$	1·041	1·076
$c_r c_r$	1·065	1·053	h_1	1·119	1·149
zz	1·037	1·038	$m_2 h$	1·115	1·082
$c_y l$	1·094	1·105	$c_r h$	1·177	1·138
ml	1·049	1·053	rl	1·134	1·102
$c_p l$	1·075	1·071	ih'	1·145	1·188
rb'	1·075	1·079			

means in that paper. It is found that for a majority of characters the sex ratios for the *E* series are greater (but not necessarily significantly) than the corresponding values for the other three series. This may be partly due to the method of sexing the *E* series, which may tend to increase sex ratios. It is seen from Table IV that the character $c_r h$ has the highest sex ratio in the case of the Non-Aged series, while the values for $g_o g_o$ and $c_y l$, also used for sexing, are also large.

7. *Coefficients of Racial Likeness between the Egyptian E and other Series.* These are given in Table V for the *E* Non-Aged series and all the others measured in the same way which are available at present*. A description of the method and references to the other sources giving measurements of the material are given in Dr Morant's paper, and he also provides the coefficients between every pair of the twelve male series and between every pair of the five female series. In carrying out this calculation he used the standard deviations of the longest series then available, viz. the Qau Egyptian, but the series described in the present paper is now by far the longest available. In order to estimate the extent to which the coefficients are influenced by the particular set of standard deviations used, all with the *E* series were calculated by using both the *E* (Non-Aged) and Qau constants. The reduced coefficients found in the two ways are compared in Table V. The two orders in which the series are arranged are very similar, and the only divergence between them which seems to be of any importance lies in the fact that the Qau standard deviations lead to appreciably lower coefficients with all the Asiatic series than those found when the *E* standard deviations are used. Some marked differences are found between corresponding male and female coefficients, but it must be remembered that several of the samples are obviously too small to provide reliable characterisations of racial types.

Our present purpose is to place these data on record and not to discuss their anthropological significance. It is satisfactory to find that the *E* has its lowest coefficients, indicating closest similarity, with the other three dynastic Egyptian

* With the exception of additional Anglo-Saxon Series published by J. C. Brash, Doris Layard and Matthew Young (*Biometrika*, Vol. XXVII. (1935), pp. 373—407) after the present paper had been completed.

TABLE V.

Coefficients of Racial Likeness between the Egyptian E and other Series.*

Egyptian *E* Series and	$\bar{n}$†		Crude C.R.L.'s using *E* σ's‡		Reduced C.R.L.'s			
					using *E* σ's		using Qau σ's§	
	♂	♀	♂	♀	♂	♀	♂	♀
Qau Egyptian ...	66·4	55·7	4·89	5·92	4·84 ± ·30	7·55 ± ·38	3·86	8·86
Sedment „	32·4	21·2	2·85	3·52	5·07 ± ·53	9·65 ± ·82	5·23	9·63
Kerma „	55·7	44·7	8·36	12·73	9·48 ± ·34	19·07 ± ·45	10·09	20·64
Anglo-Saxon ...	42·4	41·2	8·90	16·57	12·59 ± ·42	26·40 ± ·48	10·94	24·76
Dunstable ...	37·3	—	8·85	—	13·95 ± ·47	—	12·23	—
Tamil	33·0	—	10·69	—	18·72 ± ·53	—	13·60	—
Hylam Chinese	38·8	—	12·91	—	19·69 ± ·46	—	13·72	—
Nepalese ...	18·9	—	8·54	—	24·62 ± ·86	—	15·03	—
Badari Egyptian	33·5	18·9	15·14	7·66	26·18 ± ·52	23·19 ± ·91	24·84	22·23
Tibetan *A* ...	24·8	—	11·93	—	26·87 ± ·68	—	17·42	—
Fukien Chinese	37·5	—	19·50	—	30·60 ± ·47	—	23·92	—
Tibetan *B* ...	11·9	—	7·45	—	32·82 ± 1·32	—	27·22	—

* All the coefficients are based on the ten characters w_1, zz, $c_y l$, rb', $m_2 p_1$, h_1, ml, rl, $R\angle$ and $100\, g_o g_o / c_p l$.

† $\bar{n}$ in each case is the mean of the numbers of mandibles available for the ten characters used. For the male *E* series, $\bar{n} = 211{\cdot}7$, and for the female, $\bar{n} = 131{\cdot}8$.

‡ The probable error of each crude coefficient is ·30.

§ The probable errors of the reduced coefficients found by using the Qau standard deviations are the same as for the corresponding reduced coefficients found by using the *E* standard deviations.

series. Its wide separation from the Badari Egyptian appears to be anomalous, but there was a wide separation in time between the early predynastic and late dynastic populations.

8. *Comparison of Mandibles lacking Third Molars and Normal Mandibles.* The *E* series is sufficiently long to make profitable a statistical comparison between these two groups. All the Non-Aged bones were examined by Mr C. Bowdler Henry, M.R.C.S., L.D.S., and he picked out all those in which apparently neither third molar had erupted. This condition cannot be detected with certainty, since the absorbed alveolus of a wisdom tooth which had been lost before death may suggest in some cases that the tooth had never erupted; but it is probable that a high percentage of the bones selected, if not all, never had wisdom teeth. By holding each over an X-ray screen it was found that no one of them has an embedded third molar. There were 18 male and 27 female mandibles selected in this way, these numbers being 8·2 and 19·7 per cent. of the total numbers of Non-Aged male and female mandibles respectively. The percentage, it may be noted, is considerably higher for the female than for the male series, as is usually found to be the case.

It is of interest to enquire whether the measurements of these mandibles show any significant differences from those of the normal bones. Table VI gives the means of each character for the two series in the case of each sex. In constructing this table the means for the mandibles without third molars were first calculated from the individual measurements, and the means for the remainder were deduced

TABLE VI.

Mean Measurements for Mandibles without Third Molars and for the Remaining Mandibles of the E Non-Aged Series.

Character	Male Mandibles		Female Mandibles	
	Without third molars	Remainder	Without third molars	Remainder
w_1	115·23 (17)	116·98 (192)	110·18 (26)	110·63 (102)
$g_o g_o$	91·65 (18)	93·85 (202)	84·85 (27)	85·75 (110)
$c_r c_r$	93·49 (15)	94·00 (187)	87·09 (25)	88·51 (105)
zz	44·16 (18)	44·49 (202)	42·52 (26)	42·94 (108)
$c_y l$	20·79 (18)	21·17 (202)	19·15 (27)	19·36 (110)
ml	101·94 (17)	103·78 (196)	97·90 (26)	99·01 (107)
$c_p l$	73·68 (18)	74·71 (202)	68·11 (27)	69·75 (110)
rb'	32·13 (18)	32·54 (202)	29·89 (27)	30·34 (110)
$c_y c_r$	34·25 (18)	33·00 (201)	32·43 (26)	31·80 (110)
$m_2 p_1$	28·64 (17)	27·86 (167)	27·71 (23)	26·64 (98)
h_1	31·62 (17)	32·73 (174)	28·74 (22)	29·24 (95)
$m_2 h$	23·86 (17)	25·97 (181)	20·99 (26)	23·66 (102)
$c_r h$	67·03 (18)	67·11 (202)	55·73 (27)	57·30 (110)
rl	60·80 (18)	61·48 (202)	53·93 (27)	54·19 (110)
ih'	13·47 (18)	13·14 (202)	11·52 (27)	11·50 (110)
$M \angle$	121°·67 (18)	121°·70 (202)	126°·94 (27)	126°·74 (110)
$R \angle$	77°·64 (18)	77°·45 (202)	68°·54 (27)	71°·02 (110)
$C' \angle$	69°·50 (14)	70°·23 (158)	69°·26 (21)	69°·89 (79)
$100\, c_r h/ml$	66·05 (17)	64·88 (196)	57·10 (26)	57·98 (107)
$100\, c_r c_r/ml$	92·13 (15)	90·89 (183)	90·02 (24)	89·51 (103)
$100\, g_o g_o/c_p l$	124·79 (18)	125·94 (202)	124·90 (27)	123·33 (110)
$100\, rb'/rl$	53·23 (18)	53·16 (202)	55·61 (27)	56·22 (110)
$100\, ih'/c_y c_r$	39·81 (18)	40·06 (201)	35·85 (26)	36·43 (110)
$100\, g_o g_o/c_r c_r$	98·31 (15)	99·85 (187)	97·49 (25)	97·07 (105)

from these and the means for all mandibles given in Table I. Looking at the 15 absolute measurements we observe that for both sexes the means for 12 characters are less for mandibles without third molars than for the remaining mandibles, while for the other 3 characters ($c_y c_r$, $m_2 p_1$ and ih') the position is reversed. This suggests that mandibles in which third molars do not erupt are generally smaller than other mandibles. On examining the differences of the means we find that only one is significant in the case of the males, and four in the case of the females. The working for these characters is shown below.

In this table the suffix 1 refers to the mandibles without third molars and the suffix 2 refers to the remainder; σ_1 was calculated from the individual measure-

Character	σ_1	σ_2	p.e. M_1	p.e. M_2	p.e. $(M_1 - M_2)$	$\frac{M_1 - M_2}{\text{p.e.}(M_1 - M_2)}$
m_2p_1 ♂	1·40	1·38	0·23	0·07	0·24	+3·2
m_2h ♂	1·36	2·28	0·22	0·11	0·25	−8·4
m_2p_1 ♀	1·44	1·39	0·20	0·09	0·22	+4·9
m_2h ♀	2·79	2·04	0·37	0·14	0·40	−6·7
c_7h ♀	2·75	3·45	0·36	0·22	0·42	−3·7
$R\angle$ ♀	3°·65	6°·29	0°·47	0°·40	0°·62	−4·0

ments and σ_2 was deduced from σ_1 and from the standard deviation for all mandibles given in Table II. We have included in the table the difference of male means for the character m_2p_1, which is strictly not significant by the criterion

$$(M_1 - M_2)/\text{p.e.}(M_1 - M_2) \geqslant 3{\cdot}5,$$

but regarding it as such we have the interesting result that m_2p_1 is the only character whose mean is significantly larger for mandibles without third molars than for other mandibles, and this result is true for both sexes. Now the value of m_2p_1 has probably ceased to increase long before the bone has reached maturity*, and it has been found to be uncorrelated with other absolute measurements of the mature bone†. It appears from these results that the presence of a large m_2p_1 in a small bone tends to be concomitaneous with the absence of third molars.

Worthy of notice, also, is the highly significant nature of the mean difference for the character m_2h. A large value of m_2p_1 together with a small value of m_2h signifies that the first two molars are abnormally large compared with the region of the corpus which holds them. The index $100\,m_2h/m_2p_1$ may show a still more significant mean difference. This index has been calculated for each mandible, with the following results:

Constants of the Index $100\,m_2h/m_2p_1$.

	M_1	M_2	σ_1	σ_2	p.e. $(M_1 - M_2)$	$\frac{M_1 - M_2}{\text{p.e.}(M_1 - M_2)}$
♂	83·52 ± 1·07 (17)	93·87 ± ·55 (162)	6·57	10·36	1·20	−8·6
♀	78·17 ± 1·01 (23)	89·11 ± ·67 (96)	7·17	9·77	1·21	−9·0

These results show little advantage of $100\,m_2h/m_2p_1$ over m_2h for males, but a considerable advantage in the case of females. It is interesting to note, too, that the index shows significant sexual differences; for the series lacking third molars the difference of the male and female means is 3·6 times its probable error, and for the remainder the ratio is 5·5.

* This has been demonstrated for adequately long series of children of different ages in "Facial Growth of Children, with Special Reference to Dentition," by Corisande Smyth and Matthew Young, *Medical Research Council, Special Report Series*, No. 171.

† Morant, *loc. cit.*, p. 106.

9. *Analysis of Teeth lost before Death and Remarks on individual Mandibles.* All the mandibles measured have the dental arch complete and it is possible to determine from an inspection of the alveolar region which teeth have been lost before death, except in many cases where a third molar is absent. In these cases it has often been found impossible to determine with certainty from the appearance of the alveolar region whether the tooth had erupted and been lost before death or had never erupted. It is thus desirable to treat such cases separately, as in the following table:

	♂	♀
All teeth including third molars present at death	105 (40·7 °/₀)	77 (44·3 °/₀)
All teeth except possibly one or both third molars present at death	24 (9·3 °/₀)	31 (17·8 °/₀)
One or more teeth other than third molars lost before death ...	129 (50·0 °/₀)	66 (37·9 °/₀)
Totals	258	174

This table includes both "Non-Aged" and "Aged" mandibles; the latter all go into the third class by definition of an "Aged" mandible (see p. 150). If they are omitted the numbers in the first class represent 47·7 and 56·2 per cent., respectively, of the totals of Non-Aged mandibles.

An examination of the frequencies of the individual teeth lost gives the following results (for Non-Aged mandibles):

Analysis of Teeth lost before Death in Non-Aged Series.

Teeth	♂			♀		
	No. lost	Total No.	°/₀ lost	No. lost	Total No.	°/₀ lost
Central incisors	42	439*	9·6	13	273*	4·8
Lateral incisors	19	440	4·3	8	274	2·9
Canines ...	6	440	1·4	3	274	1·1
Premolars ...	28	880	3·2	5	548	0·9
First molars ...	75	440	17·0	16	274	5·8
Second molars	19	440	4·3	7	274	2·6
Third molars ...	62	440	14·1	67	274	24·5
All teeth ...	251	3519	7·1	119	2191	5·4

* One male and one female bone have sockets for only three incisors.

In this table all absent third molars have been included as lost before death even when we cannot be certain that they had ever erupted, so that the frequency

for this class of tooth is hardly comparable with the other frequencies. It is noteworthy, however, that the percentage of teeth lost by female bones is less for every class of tooth than the corresponding percentage lost by male bones, except in the case of the third molars. Excluding consideration of third molars, the tooth lost with greatest frequency by both sexes is the first molar, while the central incisors come next. The canines and premolars are lost with least frequency. There is a clear sex difference even for "All teeth" where the third molars have been included.

There are comparatively few anomalous dentitions in this series of mandibles. Serious disease of the alveolar margin has been noted in about a dozen cases and there are a few cases of slight overcrowding of the teeth in the front and premolar regions. Seven cases are noted of a divided socket of a canine. Five cases of impaction of one or both third molars are recorded, one of these (No. 308 ♂) being a peculiar coronoid impaction. In No. 109 ♂ the right central incisor is visible below the alveolar border but has not erupted, in No. 256 ♂ the left canine similarly appears, and in No. 393 ♂ both canines are below the alveolar border (see Plate I, A). There are sockets for only three incisors in Nos. 174 ♀ and 226 ♂. Two milk molars are retained in place of the second premolars in one bone (No. 180 ♀), but no other milk teeth have been found. There is a single edentulous specimen in the series (No. 407 ♀).

There is only one case of deformation of a condyle due to arthritis, sufficiently clear to be noted. No. 356 ♂ is the only bone on which an exostosis was observed, the excrescence in this case being blunt and not large. Two bones (Nos. 260 ♂ and 350 ♂: see Plate I, B) have anomalous cavities in the corpus, and one bone (No. 341 ♂) has a cavity resembling a crypt for a fourth molar. Two mandibles (not measured) were found to have been fractured and healed before death (Nos. 2202 ♀? and 2542 ♂: see Plate II).

10. *Verification of Normal Components: the Test of Goodness of Fit.* The components of the combined distributions for the various characters, specified by their frequencies, means and standard deviations given in Section 6, should (*a*) be normal, (*b*) provide a good fit to the combined distribution. To check up for all the 24 characters would be too arduous, but we have done the necessary work for the two characters $c_y l$ and $c_p l$. The former character was chosen because it is one of the smallest absolute measurements taken and its distribution appeared to be among the most erratic of those obtained, and because a certain bias was discovered in its measurement; the distribution of $c_p l$ was more regular and no bias could be discovered. The distributions are shown in Tables VII and VIII. In these tables are shown the total distribution of the 432 measurements, the Non-Aged and Aged distributions of males and females combined, and the components into which these distributions have been resolved by the mathematical sexing. Against each frequency in these components has been set (in brackets) the corresponding frequency of the normal distribution having the same total frequency, mean and standard deviation as the experimental distribution. These

TABLE VII.

Distribution of the Character $c_y l$.

Millimetres	Combined Distribution	Non-Aged			Aged		
		Combined	♂	♀	Combined	♂	♀
15·05—	1 (0·6)	1 (0·6)	—	1 (0·6)	—	—	—
15·45—	2 (0·8)	2 (0·7)	—	2 (0·7)	0 (0·1)	—	0 (0·1)
15·85—	2 (1·5)	2 (1·3)	—	2 (1·3)	0 (0·2)	—	0 (0·2)
16·25—	5 (2·8)	3 (2·4)	0 (0·1)	3 (2·3)	2 (0·4)	—	2 (0·4)
16·65—	4 (4·7)	3 (3·9)	1 (0·2)	2 (3·7)	1 (0·8)	—	1 (0·8)
17·05—	9 (7·5)	9 (6·1)	1 (0·5)	8 (5·6)	0 (1·4)	0 (0·1)	0 (1·3)
17·45—	8 (11·1)	6 (8·9)	0 (1·1)	6 (7·8)	2 (2·2)	0 (0·2)	2 (2·0)
17·85—	15 (15·5)	12 (12·3)	3 (2·1)	9 (10·2)	3 (3·2)	1 (0·4)	2 (2·8)
18·25—	14 (20·5)	9 (16·3)	3 (3·9)	6 (12·4)	5 (4·2)	1 (0·6)	4 (3·6)
18·65—	30 (25·7)	29 (20·5)	10 (6·5)	19 (14·0)	1 (5·2)	0 (1·0)	1 (4·2)
19·05—	21 (30·6)	14 (24·7)	4 (10·0)	10 (14·7)	7 (5·9)	0 (1·5)	7 (4·4)
19·45—	36 (35·1)	25 (28·6)	11 (14·2)	14 (14·4)	11 (6·5)	4 (2·2)	7 (4·3)
19·85—	47 (38·2)	41 (31·6)	21 (18·6)	20 (13·0)	6 (6·6)	4 (2·8)	2 (3·8)
20·25—	38 (39·9)	29 (33·4)	20 (22·4)	9 (11·0)	9 (6·5)	4 (3·4)	5 (3·1)
20·65—	47 (39·6)	41 (33·5)	33 (24·8)	8 (8·7)	6 (6·1)	5 (3·8)	1 (2·3)
21·05—	37 (37·1)	31 (31·6)	25 (25·3)	6 (6·3)	6 (5·5)	4 (4·0)	2 (1·5)
21·45—	36 (32·8)	34 (27·9)	27 (23·6)	7 (4·3)	2 (4·9)	2 (3·9)	0 (1·0)
21·85—	28 (27·2)	24 (23·1)	21 (20·4)	3 (2·7)	4 (4·1)	4 (3·6)	0 (0·5)
22·25—	13 (21·2)	10 (17·8)	10 (16·2)	0 (1·6)	3 (3·4)	2 (3·1)	1 (0·3)
22·65—	18 (15·2)	17 (12·7)	15 (11·8)	2 (0·9)	1 (2·5)	1 (2·4)	0 (0·1)
23·05—	5 (10·3)	3 (8·4)	3 (7·9)	0 (0·5)	2 (1·9)	2 (1·8)	0 (0·1)
23·45—	4 (6·3)	4 (5·1)	4 (4·9)	0 (0·2)	0 (1·2)	0 (1·2)	—
23·85—	11 (3·7)	7 (2·9)	7 (2·8)	0 (0·1)	4 (0·8)	4 (0·8)	—
24·25—	0 (2·0)	0 (1·5)	0 (1·5)	—	0 (0·5)	0 (0·5)	—
24·65—	0 (1·0)	0 (0·7)	0 (0·7)	—	0 (0·3)	0 (0·3)	—
25·05—	0 (0·4)	0 (0·3)	0 (0·3)	—	0 (0·1)	0 (0·1)	—
25·45—	0 (0·2)	0 (0·1)	0 (0·1)	—	0 (0·1)	0 (0·1)	—
25·85—26·25	1 (0·0)	1 (0·0)	1 (0·0)	—	—	—	—
Totals	432 (431·5)	357 (356·9)	220 (219·9)	137 (137·0)	75 (74·6)	38 (37·8)	37 (36·8)
Probability	P=·223	P=·034	P=·112	P=·334	P=·811	P=·884	P=·587

theoretical distributions have been added together to give the combined theoretical frequencies for Non-Aged, Aged, and all mandibles. It is seen that the components overlap too much for any "dip" to be produced in the combined theoretical distributions.

The $P\chi^2$ test has now been applied to all these distributions, with the results given in the last row of each table. The brackets at the extremities of each distribution indicate which frequencies have been clubbed together for the test. The values of P indicate that the component distributions are all well fitted by normal curves, the weakest agreement being that for $c_y l$ Non-Aged ♂, where P=·112. The combined distributions also show good fits except for $c_y l$ Non-Aged, and even here the value of P (·034) is not so small as to express a wild improbability.

TABLE VIII.

Distribution of the Character $c_p l$.

Millimetres	Combined Distribution	Non-Aged			Aged		
		Combined	♂	♀	Combined	♂	♀
61·05—	1 (2·9)	1 (2·2)	—	1 (2·2)	0 (0·7)	—	0 (0·7)
62·05—	2 (3·1)	2 (2·4)	0 (0·2)	2 (2·2)	0 (0·7)	—	0 (0·7)
63·05—	8 (5·4)	7 (4·1)	0 (0·3)	7 (3·8)	1 (1·3)	0 (0·1)	1 (1·2)
64·05—	11 (8·5)	8 (6·6)	1 (0·7)	7 (5·9)	3 (1·9)	0 (0·1)	3 (1·8)
65·05—	13 (12·5)	9 (9·7)	1 (1·3)	8 (8·4)	4 (2·8)	0 (0·3)	4 (2·5)
66·05—	11 (17·3)	8 (13·6)	3 (2·4)	5 (11·2)	3 (3·7)	0 (0·5)	3 (3·2)
67·05—	25 (22·3)	22 (17·6)	4 (4·0)	18 (13·6)	3 (4·7)	0 (0·8)	3 (3·9)
68·05—	28 (27·0)	20 (21·5)	4 (6·3)	16 (15·2)	8 (5·5)	2 (1·3)	6 (4·2)
69·05—	31 (31·2)	25 (25·1)	13 (9·4)	12 (15·7)	6 (6·1)	3 (1·9)	3 (4·2)
70·05—	39 (34·4)	28 (27·9)	13 (13·0)	15 (14·9)	11 (6·5)	5 (2·6)	6 (3·9)
71·05—	35 (36·4)	31 (29·8)	14 (16·7)	17 (13·1)	4 (6·6)	3 (3·3)	1 (3·3)
72·05—	37 (36·9)	31 (30·5)	20 (20·0)	11 (10·5)	6 (6·4)	3 (3·8)	3 (2·6)
73·05—	36 (36·1)	30 (30·1)	25 (22·3)	5 (7·8)	6 (6·0)	5 (4·1)	1 (1·9)
74·05—	34 (33·8)	31 (28·5)	24 (23·2)	7 (5·3)	3 (5·3)	3 (4·1)	0 (1·2)
75·05—	24 (30·4)	22 (25·8)	20 (22·5)	2 (3·3)	2 (4·6)	2 (3·8)	0 (0·8)
76·05—	29 (26·0)	24 (22·3)	23 (20·4)	1 (1·9)	5 (3·7)	4 (3·3)	1 (0·4)
77·05—	18 (21·0)	13 (18·2)	12 (17·2)	1 (1·0)	5 (2·8)	3 (2·6)	2 (0·2)
78·05—	15 (16·1)	13 (14·1)	12 (13·6)	1 (0·5)	2 (2·0)	2 (1·9)	0 (0·1)
79·05—	16 (11·4)	15 (10·1)	14 (9·9)	1 (0·2)	1 (1·3)	1 (1·3)	—
80·05—	6 (7·7)	5 (6·9)	5 (6·8)	0 (0·1)	1 (0·8)	1 (0·8)	—
81·05—	7 (4·9)	7 (4·4)	7 (4·4)	—	0 (0·5)	0 (0·5)	—
82·05—	2 (2·9)	2 (2·6)	2 (2·6)	—	0 (0·3)	0 (0·3)	—
83·05—	2 (1·6)	2 (1·5)	2 (1·5)	—	0 (0·1)	0 (0·1)	—
84·05—85·05	2 (1·5)	1 (1·4)	1 (1·4)	—	1 (0·1)	1 (0·1)	—
Totals	432 (431·3)	357 (356·9)	220 (220·1)	137 (136·8)	75 (74·4)	38 (37·6)	37 (36·8)
Probability	$P = ·909$	$P = ·926$	$P = ·957$	$P = ·631$	$P = ·741$	$P = ·957$	$P = ·811$

For the remaining characters, it has been thought sufficient to give only the Non-Aged combined distributions and their components by the mathematical sexing. These are given in an Appendix to this paper.

11. *The β's of the combined Distributions.* Professor Karl Pearson has given* the following equations from which to determine the moment-coefficients of a distribution consisting of two normal component distributions:

$$z_1 + z_2 = 1,$$

$$\gamma_1 z_1 + \gamma_2 z_2 = 0,$$

$$\mu_2 = \gamma_1^2 z_1 (1 + u_1^2) + \gamma_2^2 z_2 (1 + u_2^2),$$

$$\mu_3 = \gamma_1^3 z_1 (1 + 3u_1^2) + \gamma_2^3 z_2 (1 + 3u_2^2),$$

$$\mu_4 = \gamma_1^4 z_1 (1 + 6u_1^2 + 3u_1^4) + \gamma_2^4 z_2 (1 + 6u_2^2 + 3u_2^4),$$

$$\mu_5 = \gamma_1^5 z_1 (1 + 10u_1^2 + 15u_1^4) + \gamma_2^5 z_2 (1 + 10u_2^2 + 15u_2^4),$$

* Section 3, *loc. cit.*

where z_1 and z_2 are the component frequencies expressed as fractions of the total frequency, γ_1 and γ_2 are the component means referred to the mean of the combined distribution, and u_1 and u_2 are the respective standard deviations divided by these means. From these formulae we have evaluated the first three β's of the combined Non-Aged distributions for the characters $c_y l$ and $c_p l$, using the components specified in Section 6. They are given below, together with the values obtained from the experimental distributions.

β's of Combined Non-Aged Distributions.

	$c_y l$			$c_p l$		
	β_1	β_2	β_3	β_1	β_2	β_3
Experimental	·069 6746	3·289 1547	·451 1670	·002 1720	2·627 4856	·017 0131
Theoretical	·022 4290	2·908 4527	·183 0668	·000 0375	2·772 1825	·000 5538

It is seen that the experimental and theoretical values of β_2 are in substantial agreement for both characters, but that the values of β_1 and β_3 differ widely, even for $c_p l$, where the theoretical distribution fits excellently the observed distribution. In view of the large standard deviations of the β's, however, it is probable that none of the differences is significant; an examination of these standard deviations* appears to confirm this view for β_1 and β_2.

Comparing the two characters, we note that the values of β_1 and β_3 are much greater for $c_y l$ than for $c_p l$. From experience with other characters it appears that the smaller values as exhibited by $c_p l$ may usually be expected in mandible work, and that the values for $c_y l$ are exceptionally large; for example, the following values are obtained for the experimental Non-Aged distributions of the characters $g_o g_o$ and $M\angle$:

Character	β_1	β_2	β_3
$g_o g_o$	·000 1299	2·881 8970	·003 1031
$M\angle$	·000 3572	3·305 2191	·005 8354

We deduce from these results that the odd β's will probably be small for most characters, and in view of their large standard errors for small frequencies, may be unreliable measures on which to base calculations, in the case of our mandibles.

* Tables of the standard deviations of β_1 and β_2, for Pearson curves, are given in *Tables for Statisticians and Biometricians*, Part I, pp. 68—71. We have assumed that the values are roughly the same for our combined distributions.

12. *The Method of the Nonic.* The solution of the problem of resolving a compound distribution into its normal components, given by Professor Pearson, depends on the solution of a nonic equation in a variable which is the product of the means of the two components, measured from the mean of the compound distribution. We have carried out the calculation for the three characters $c_y l$, $c_p l$ and $M\angle$, for which the β's are given in the last section. In the case of $c_y l$ we found two negative roots of the nonic equation, one of which led to an imaginary solution of the problem. The other root gave the following solution:

	♂	♀
Mean	21·27	20·25
Standard Deviation	0·44	1·80
Frequency ...	64	293

This result is hardly acceptable, mainly on account of the very improbable sex-proportion.

Taking some perhaps more reliable characters, we found for $M\angle$ two negative solutions of the nonic; the first gave 95 per cent. males and the second almost 100 per cent. males. For the character $c_p l$ we found no real negative root of the nonic and hence no solution of the problem; a slight alteration in one or more of the coefficients of the equation, however, would evidently have produced a solution.

It appears to us that the method, based on complicated calculations with the first five moment-coefficients of a distribution, may be extremely sensitive to experimental errors in the distribution, and therefore, with small irregular frequencies, cannot be relied on to give reasonable results with any particular character, nor consistent results for two or more characters.

In a later memoir* Professor Pearson has given an approximate solution of the problem in the case when the numbers of males and females are approximately equal. In this case it is necessary (but not sufficient) that β_1 and β_2 shall be small. A more stringent necessary condition is that μ_3 and μ_5 shall be small, i.e. of the same order of magnitude as the expected deviations of the constants of the two components from equality. In the case of our mandibles we believe that the male and female frequencies are not approximately equal, and this belief is strengthened by the fact that in none of the characters we have investigated have μ_3 and μ_5 been small enough to satisfy the above condition. The method, therefore, cannot be applied to our case.

The method of obtaining the constants of each normal component of the compound distribution from its "tails"† has also been tried for several characters

* "On the Problem of Sexing Osteometric Material," *Biometrika*, Vol. x. (1915), pp. 479—487.

† Pearson and Lee, *Biometrika*, Vol. vi. (1908), pp. 65 and 68, and *Tables for Statisticians and Biometricians*, Part I, Introduction, pp. xxviii—xxxi.

without satisfactory results. This failure is apparently due to the irregular nature of the small frequencies in the "tails."

13. *Conclusions.* The method of obtaining the constants of mandibular characters for the sexes separately, by means of the nonic equation, is independent of any sexing of the corresponding series of crania. We have failed to obtain any reasonable results by this method, probably because our sample is too small. The mathematical methods (*a*), (*b*) and (*c*), described in Section 3, all depend on a previous sexing of the accompanying series of crania, in that they assume the same sex-proportion for the mandibles as that obtained for the crania. They provide sexes for the individual mandibles, and they differ only in the weights given to the several characters. We have found that for Egyptian series, method (*c*) gives results agreeing best with anatomical sexing. As, however, the formula of method (*c*) is based on the Egyptian Kerma series, its use would perhaps hardly be justified for a non-Egyptian series, and in this case method (*a*) or (*b*) would be used, method (*b*) possibly giving the better results.

Constants for the *E* series, sexed mathematically by method (*c*), are presented and discussed, and it is shown that the series is closely related to other Egyptian series. The differences between the constants of the Non-Aged and Aged series are small enough to suggest that in a shorter series it would be hardly necessary to treat the Non-Aged and Aged members separately. The section dealing with the mandibles with absent third molars reveals differences between them and normal bones which may possibly aid in attempting to predict for the living, whether or not third molars will appear, or whether, if they appear, they will cause trouble through having insufficient space to erupt.

I am indebted to Professor Karl Pearson for permission to use the *E* series of mandibles, and for several suggestions and criticisms; to Professor R. A. Fisher for his original method of mathematical sexing; and to Dr G. M. Morant for his general help and criticism (and in particular for the Plates at the end of this paper), which debts I gratefully acknowledge.

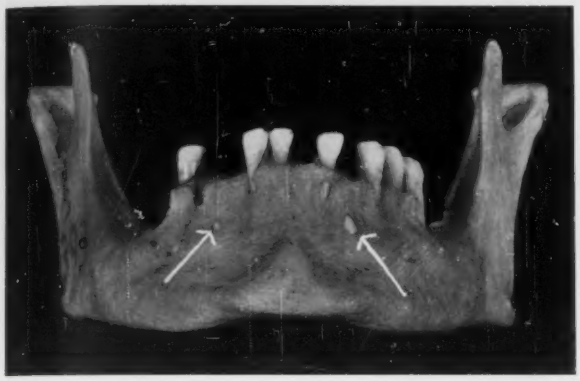
DESCRIPTION OF PLATES.

Plate I. A. A fully adult male mandible (No. 393) with embedded canines, the normal sites for these teeth being unoccupied: 0·7 natural size.

B. A male mandible (No. 350) with an anomalous depression below the left third molar: natural size.

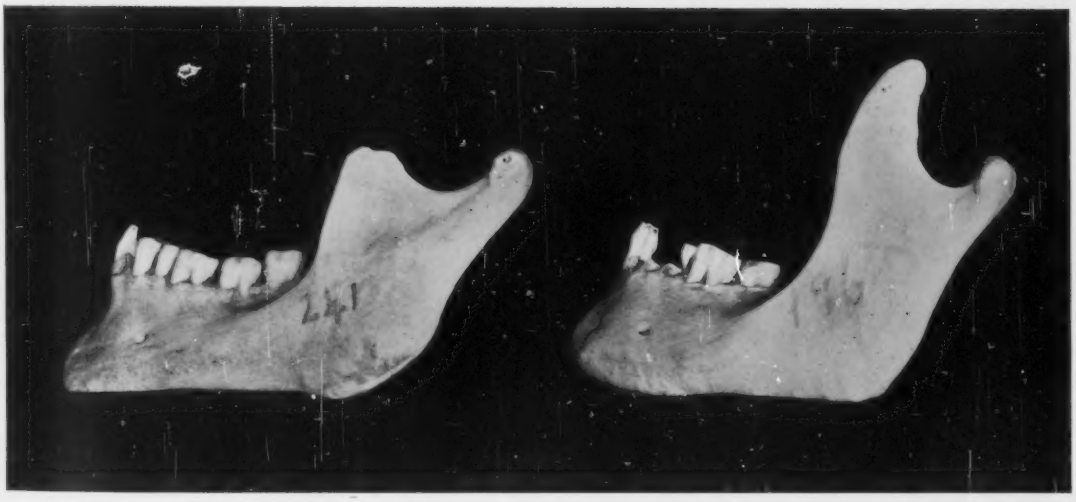
C. *Norma lateralis* views of two male mandibles (Nos. 241 and 196) with markedly dissimilar forms of the coronoid process: 0·7 natural size.

Plate II. Two mandibles (Nos. 2202 ♀? above and 2542 ♂) with healed fractures of their right rami: 1·1 natural size. These were not included in the measured series.



A

B



C

Anomalous Male Egyptian *E* Mandibles (see description of Plates p. 172).

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Egyptian *E* Mandibles with Fractured Rami.

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APPENDIX. *Distributions of Characters for the Non-Aged Series (those for $c_y l$ and $c_p l$ having been given in Tables VII and VIII).*

w_1				$g_o g_o$				$c_r c_r$				zz			
mm.	♂+♀	♂	♀	mm.	♂+♀	♂	♀	mm.	♂+♀	♂	♀	mm.	♂+♀	♂	♀
97·05—	1	—	1	67·45—	1	—	1	77·05—	1	—	1	36·55—	1	—	1
98·05—	4	—	4	68·85—	1	—	1	78·05—	0	—	0	37·05—	1	—	1
99·05—	3	1	2	70·25—	0	—	0	79·05—	6	2	4	37·55—	0	—	0
100·05—	1	0	1	71·65—	0	—	0	80·05—	5	0	5	38·05—	2	—	2
101·05—	3	0	3	73·05—	3	—	3	81·05—	2	2	0	38·55—	3	3	0
102·05—	4	0	4	74·45—	2	—	2	82·05—	9	1	8	39·05—	1	0	1
103·05—	4	2	2	75·85—	2	—	2	83·05—	12	4	8	39·55—	9	2	7
104·05—	9	4	5	77·25—	7	—	7	84·05—	11	2	9	40·05—	9	3	6
105·05—	14	3	11	78·65—	10	4	6	85·05—	10	4	6	40·55—	16	9	7
106·05—	7	2	5	80·05—	10	1	9	86·05—	18	3	15	41·05—	16	6	10
107·05—	7	3	4	81·45—	16	8	8	87·05—	16	6	10	41·55—	22	11	11
108·05—	10	3	7	82·85—	19	7	12	88·05—	19	6	13	42·05—	16	6	10
109·05—	11	4	7	84·25—	25	10	15	89·05—	28	17	11	42·55—	31	15	16
110·05—	15	8	7	85·65—	20	4	16	90·05—	16	10	6	43·05—	32	18	14
111·05—	20	11	9	87·05—	28	13	15	91·05—	22	14	8	43·55—	36	23	13
112·05—	13	7	6	88·45—	21	10	11	92·05—	21	17	4	44·05—	20	13	7
113·05—	21	13	8	89·85—	23	16	7	93·05—	14	9	5	44·55—	37	27	10
114·05—	17	7	10	91·25—	25	16	9	94·05—	23	20	3	45·05—	23	18	5
115·05—	35	21	14	92·65—	21	16	5	95·05—	18	14	4	45·55—	16	14	2
116·05—	25	19	6	94·05—	26	23	3	96·05—	14	11	3	46·05—	21	19	2
117·05—	17	14	3	95·45—	34	31	3	97·05—	10	8	2	46·55—	18	12	6
118·05—	21	19	2	96·85—	15	13	2	98·05—	20	18	2	47·05—	6	4	2
119·05—	12	10	2	98·25—	10	10	—	99·05—	10	9	1	47·55—	8	7	1
120·05—	14	12	2	99·65—	15	15	—	100·05—	10	9	1	48·05—	2	2	—
121·05—	10	9	1	101·05—	7	7	—	101·05—	4	4	0	48·55—	4	4	—
122·05—	7	6	1	102·45—	1	1	—	102·05—	6	6	0	49·05—	1	1	—
123·05—	8	8	0	103·85—	7	7	—	103·05—	3	2	1	49·55—	1	1	—
124·05—	10	9	1	105·25—	1	1	—	104·05—	2	2	—	50·05—	0	0	—
125·05—	8	8	—	106·65—	5	5	—	105·05—106·05	2	2	—	50·55—	0	0	—
126·05—	3	3	—	108·05—	0	0	—					51·05—	0	0	—
127·05—	2	2	—	109·45—110·85	2	2	—					51·55—52·05	2	2	—
128·05—	0	0	—												
129·05—130·05	1	1	—												
Totals	337	209	128	Totals	357	220	137	Totals	332	202	130	Totals	354	220	134

APPENDIX (*continued*).

ml				*rb'*				$c_y c_r$				$m_2 p_1$			
mm.	♂ + ♀	♂	♀	mm.	♂ + ♀	♂	♀	mm.	♂ + ♀	♂	♀	mm.	♂ + ♀	♂	♀
89·05—	2	—	2	25·05—	1	—	1	25·05—	5	4	1	23·05—	1	—	1
90·05—	3	2	1	25·45—	1	—	1	25·65—	0	0	0	23·35—	3	2	1
91·05—	3	0	3	25·85—	6	1	5	26·25—	3	2	1	23·65—	4	0	4
92·05—	3	1	2	26·25—	1	0	1	26·85—	10	4	6	23·95—	2	0	2
93·05—	5	1	4	26·65—	3	1	2	27·45—	5	3	2	24·25—	1	0	1
94·05—	19	5	14	27·05—	10	2	8	28·05—	7	4	3	24·55—	3	1	2
95·05—	14	4	10	27·45—	4	2	2	28·65—	17	6	11	24·85—	4	1	3
96·05—	23	8	15	27·85—	14	4	10	29·25—	15	6	9	25·15—	1	1	0
97·05—	21	5	16	28·25—	11	5	6	29·85—	20	10	10	25·45—	12	3	9
98·05—	11	6	5	28·65—	14	6	8	30·45—	23	12	11	25·75—	27	11	16
99·05—	26	14	12	29·05—	16	4	12	31·05—	25	14	11	26·05—	13	4	9
100·05—	28	18	10	29·45—	13	8	5	31·65—	37	21	16	26·35—	18	7	11
101·05—	21	14	7	29·85—	18	7	11	32·25—	28	19	9	26·65—	29	18	11
102·05—	25	15	10	30·25—	15	7	8	32·85—	26	17	9	26·95—	18	14	4
103·05—	30	20	10	30·65—	20	12	8	33·45—	17	12	5	27·25—	17	10	7
104·05—	27	21	6	31·05—	20	11	9	34·05—	33	21	12	27·55—	22	13	9
105·05—	17	17	0	31·45—	15	12	3	34·65—	23	19	4	27·85—	22	17	5
106·05—	14	12	2	31·85—	34	25	9	35·25—	9	5	4	28·15—	23	17	6
107·05—	15	14	1	32·25—	22	19	3	35·85—	15	10	5	28·45—	18	14	4
108·05—	11	9	2	32·65—	18	11	7	36·45—	9	7	2	28·75—	21	14	7
109·05—	7	7	0	33·05—	20	14	6	37·05—	10	8	2	29·05—	15	11	4
110·05—	7	7	0	33·45—	14	10	4	37·65—	4	4	0	29·35—	10	9	1
111·05—	4	4	0	33·85—	11	10	1	38·25—	5	5	0	29·65—	3	2	1
112·05—	1	1	0	34·25—	5	5	0	38·85—	2	1	1	29·95—	4	2	2
113·05—	0	0	0	34·65—	9	6	3	39·45—	2	1	1	30·25—	4	4	0
114·05—	3	3	0	35·05—	8	6	2	40·05—	2	2	0	30·55—	5	4	1
115·05—	2	2	0	35·45—	5	4	1	40·65—	1	1	0	30·85—	1	1	—
116·05—	2	2	0	35·85—	7	7	0	41·25—	1	1	0	31·15—	2	2	—
117·05—118·05	2	1	1	36·25—	1	1	0	41·85—42·45	1	—	1	31·45—	1	1	—
				36·65—	8	7	1					31·75—32·05	1	1	—
				37·05—	7	7	—								
				37·45—	5	5	—								
				37·85—38·25	1	1	—								
Totals	346	213	133	Totals	257	220	137	Totals	355	219	136	Totals	305	184	121

APPENDIX (*continued*).

h_1				m_2h				c_1h				rl			
mm.	♂+♀	♂	♀	mm.	♂+♀	♂	♀	mm.	♂+♀	♂	♀	mm.	♂+♀	♂	♀
22·65—	1	—	1	13·35—	1	—	1	47·85—	1	—	1	45·05—	1	—	1
23·25—	1	—	1	13·95—	1	—	1	49·05—	2	—	2	46·05—	2	—	2
23·85—	5	—	5	14·55—	0	—	0	50·25—	5	—	5	47·05—	2	—	2
24·45—	7	1	6	15·15—	0	—	0	51·45—	6	—	6	48·05—	8	—	8
25·05—	2	0	2	15·75—	0	—	0	52·65—	10	—	10	49·05—	5	—	5
25·65—	2	0	2	16·35—	0	—	0	53·85—	13	—	13	50·05—	11	—	11
26·25—	11	3	8	16·95—	1	—	1	55·05—	15	—	15	51·05—	12	2	10
26·85—	12	5	7	17·55—	0	—	0	56·25—	23	1	22	52·05—	19	4	15
27·45—	7	1	6	18·15—	1	—	1	57·45—	24	4	20	53·05—	17	3	14
28·05—	21	5	16	18·75—	2	—	2	58·65—	20	2	18	54·05—	19	1	18
28·65—	21	10	11	19·35—	3	—	3	59·85—	17	9	8	55·05—	17	8	9
29·25—	12	5	7	19·95—	6	—	6	61·05—	25	15	10	56·05—	28	13	15
29·85—	20	12	8	20·55—	11	3	8	62·25—	27	21	6	57·05—	25	20	5
30·45—	23	12	11	21·15—	8	1	7	63·45—	18	18	0	58·05—	30	21	9
31·05—	27	21	6	21·75—	25	9	16	64·65—	29	28	1	59·05—	20	15	5
31·65—	20	15	5	22·35—	17	4	13	65·85—	19	19	—	60·05—	19	17	2
32·25—	17	13	4	22·95—	28	13	15	67·05—	31	31	—	61·05—	23	20	3
32·85—	20	19	1	23·55—	32	19	13	68·25—	16	16	—	62·05—	28	26	2
33·45—	12	11	1	24·15—	35	24	11	69·45—	12	12	—	63·05—	13	13	0
34·05—	18	15	3	24·75—	33	22	11	70·65—	11	11	—	64·05—	17	16	1
34·65—	11	9	2	25·35—	16	12	4	71·85—	8	8	—	65·05—	11	11	—
35·25—	5	5	0	25·95—	30	22	8	73·05—	6	6	—	66·05—	5	5	—
35·85—	12	10	2	26·55—	17	15	2	74·25—	5	5	—	67·05—	8	8	—
36·45—	3	2	1	27·15—	10	10	0	75·45—	1	1	—	68·05—	4	4	—
37·05—	2	2	0	27·75—	18	15	3	76·65—	3	3	—	69·05—	6	6	—
37·65—	9	9	0	28·35—	10	9	1	77·85—	1	1	—	70·05—	2	2	—
38·25—	3	3	0	28·95—	10	9	1	79·05—	7	7	—	71·05—	3	3	—
38·85—	3	3	0	29·55—	3	3	—	80·25—	1	1	—	72·05—73·05	2	2	—
39·45—40·05	1	—	1	30·15—	3	3	—	81·45—	0	0	—				
				30·75—	4	4	—	82·65—83·85	1	1	—				
				31·35—31·95	1	1	—								
Totals	308	191	117	Totals	326	198	128	Totals	357	220	137	Totals	357	220	137

APPENDIX (*continued*).

ih'				*M* ∠				*R* ∠				*C'* ∠			
mm.	♂ + ♀	♂	♀	1°·0	♂ + ♀	♂	♀	1°·0	♂ + ♀	♂	♀	1°·0	♂ + ♀	♂	♀
7·85—	2	—	2	100·25—	1	1	—	55·75—	3	2	1	54·05—	1	1	—
8·25—	1	—	1	101·75—	0	0	—	57·25—	3	1	2	55·05—	0	0	—
8·65—	1	1	0	103·25—	0	0	—	58·75—	4	1	3	56·05—	2	2	—
9·05—	9	1	8	104·75—	0	0	—	60·25—	5	0	5	57·05—	2	1	1
9·45—	9	4	5	106·25—	1	1	—	61·75—	6	1	5	58·05—	5	3	2
9·85—	16	6	10	107·75—	1	1	—	63·25—	4	1	3	59·05—	4	4	0
10·25—	16	3	13	109·25—	6	6	—	64·75—	17	4	13	60·05—	3	3	0
10·65—	22	7	15	110·75—	5	5	—	66·25—	20	6	14	61·05—	6	3	3
11·05—	23	11	12	112·25—	6	6	—	67·75—	25	11	14	62·05—	5	3	2
11·45—	21	6	15	113·75—	11	9	2	69·25—	27	13	14	63·05—	5	3	2
11·85—	40	25	15	115·25—	17	13	4	70·75—	29	18	11	64·05—	10	5	5
12·25—	34	20	14	116·75—	19	16	3	72·25—	31	16	15	65·05—	16	9	7
12·65—	32	24	8	118·25—	34	24	10	73·75—	25	16	9	66·05—	21	9	12
13·05—	27	21	6	119·75—	30	20	10	75·25—	25	17	8	67·05—	21	16	5
13·45—	23	18	5	121·25—	25	19	6	76·75—	25	19	6	68·05—	22	15	7
13·85—	18	16	2	122·75—	34	23	11	78·25—	21	17	4	69·05—	14	8	6
14·25—	24	20	4	124·25—	34	18	16	79·75—	17	15	2	70·05—	23	14	9
14·65—	13	12	1	125·75—	27	21	6	81·25—	13	9	4	71·05—	17	12	5
15·05—	8	7	1	127·25—	26	12	14	82·75—	13	10	3	72·05—	18	10	8
15·45—	8	8	—	128·75—	26	13	13	84·25—	11	11	0	73·05—	19	9	10
15·85—	3	3	—	130·25—	22	5	17	85·75—	11	10	1	74·05—	17	11	6
16·25—	0	0	—	131·75—	12	1	11	87·25—	7	7	—	75·05—	10	5	5
16·65—	2	2	—	133·25—	6	4	2	88·75—	2	2	—	76·05—	13	11	2
17·05—	1	1	—	134·75—	7	1	6	90·25—	4	4	—	77·05—	5	4	1
17·45—	2	2	—	136·25—	1	0	1	91·75—	2	2	—	78·05—	3	3	0
17·85—	1	1	—	137·75—	2	1	1	93·25—	2	2	—	79·05—	2	2	0
18·25—18·65	1	1	—	139·25—	1	—	1	94·75—	2	2	—	80·05—	4	4	0
				140·75—	2	—	2	96·25—	1	1	—	81·05—	2	1	1
				142·25—	0	—	0	97·75—	1	1	—	82·05—83·05	2	1	1
				143·75—	0	—	0	99·25—	0	0	—				
				145·25—146·75	1	—	1	100·75—	0	0	—				
								102·25—	0	0	—				
								103·75—	0	0	—				
								105·25—106·75	1	1	—				
Totals	357	220	137	Totals	357	220	137	Totals	357	220	137	Totals	272	172	100

APPENDIX (*continued*).

APPENDIX (*continued*).

$100\,c_r h/ml$				$100\,c_r c_r/ml$				$100\,g_o g_o/c_p l$				$100\,rb'/rl$			
Index value	♂ + ♀	♂	♀	Index value	♂ + ♀	♂	♀	Index value	♂ + ♀	♂	♀	Index value	♂ + ♀	♂	♀
47·25—	2	—	2	72·45—	1	1	—	88·05—	1	—	1	41·05—	1	1	—
48·45—	1	—	1	73·75—	0	0	—	90·05—	0	—	0	42·05—	3	3	—
49·65—	2	—	2	75·05—	2	2	—	92·05—	0	—	0	43·05—	3	2	1
50·85—	5	—	5	76·35—	0	0	—	94·05—	0	—	0	44·05—	9	5	4
52·05—	16	4	12	77·65—	5	3	2	96·05—	0	—	0	45·05—	6	6	0
53·25—	9	1	8	78·95—	4	3	1	98·05—	3	1	2	46·05—	13	11	2
54·45—	13	4	9	80·25—	13	8	5	100·05—	2	1	1	47·05—	15	9	6
55·65—	19	3	16	81·55—	12	5	7	102·05—	5	5	0	48·05—	15	12	3
56·85—	21	7	14	82·85—	18	7	11	104·05—	5	1	4	49·05—	14	13	1
58·05—	25	11	14	84·15—	19	10	9	106·05—	6	3	3	50·05—	28	20	8
59·25—	30	15	15	85·45—	15	5	10	108·05—	11	8	3	51·05—	28	19	9
60·45—	21	10	11	86·75—	24	16	8	110·05—	13	7	6	52·05—	17	12	5
61·65—	27	22	5	88·05—	34	22	12	112·05—	11	7	4	53·05—	25	15	10
62·85—	30	22	8	89·35—	25	15	10	114·05—	16	12	4	54·05—	27	20	7
64·05—	29	20	9	90·65—	25	15	10	116·05—	20	14	6	55·05—	24	12	12
65·25—	22	21	1	91·95—	24	18	6	118·05—	21	10	11	56·05—	20	13	7
66·45—	11	11	0	93·25—	19	11	8	120·05—	25	13	12	57·05—	21	10	11
67·65—	21	20	1	94·55—	24	14	10	122·05—	21	12	9	58·05—	15	5	10
68·85—	8	8	—	95·85—	19	14	5	124·05—	31	13	18	59·05—	15	5	10
70·05—	8	8	—	97·15—	8	6	2	126·05—	30	20	10	60·05—	13	7	6
71·25—	6	6	—	98·45—	9	6	3	128·05—	21	13	8	61·05—	11	9	2
72·45—	8	8	—	99·75—	6	3	3	130·05—	29	18	11	62·05—	8	0	8
73·65—	1	1	—	101·05—	8	5	3	132·05—	9	7	2	63·05—	6	4	2
74·85—	3	3	—	102·35—	4	4	0	134·05—	18	13	5	64·05—	7	3	4
76·05—	1	1	—	103·65—	3	1	2	136·05—	12	9	3	65·05—	7	2	5
77·25—	1	1	—	104·95—	0	0	—	138·05—	15	10	5	66·05—	2	0	2
78·45—	4	4	—	106·25—	1	1	—	140·05—	10	6	4	67·05—	2	0	2
79·65—	0	0	—	107·55—	1	1	—	142·05—	9	9	0	68·05—69·05	2	2	—
80·85—	1	1	—	108·85—110·15	2	2	—	144·05—	8	3	5				
82·05—83·25	1	1	—					146·05—	2	2	—				
								148·05—	0	0	—				
								150·05—	1	1	—				
								152·05—	0	0	—				
								154·05—	0	0	—				
								156·05—	0	0	—				
								158·05—	1	1	—				
								160·05—162·05	1	1	—				
Totals	346	213	133	Totals	325	198	127	Totals	357	220	137	Totals	357	220	137

APPENDIX (*continued*).

$100\,ih'/c_y c_r$				$100\,g_o g_o/c_r c_r$			
Index value	♂ + ♀	♂	♀	Index value	♂ + ♀	♂	♀
23·35—	1	1	—	80·65—	2	—	2
24·55—	0	0	—	82·05—	0	—	0
25·75—	1	0	1	83·45—	8	4	4
26·95—	8	4	4	84·85—	5	3	2
28·15—	9	0	9	86·25—	7	3	4
29·35—	14	4	10	87·65—	11	7	4
30·55—	14	5	9	89·05—	17	7	10
31·75—	15	9	6	90·45—	12	7	5
32·95—	17	7	10	91·85—	16	9	7
34·15—	32	19	13	93·25—	23	12	11
35·35—	20	11	9	94·65—	21	12	9
36·55—	25	15	10	96·05—	18	8	10
37·75—	30	21	9	97·45—	29	16	13
38·95—	37	24	13	98·85—	22	13	9
40·15—	26	15	11	100·25—	24	19	5
41·35—	24	19	5	101·65—	24	19	5
42·55—	25	15	10	103·05—	22	12	10
43·75—	11	11	0	104·45—	15	11	4
44·95—	10	8	2	105·85—	18	12	6
46·15—	6	5	1	107·25—	10	8	2
47·35—	10	8	2	108·65—	10	7	3
48·55—	6	5	1	110·05—	2	2	0
49·75—	5	5	0	111·45—	5	4	1
50·95—	1	1	0	112·85—	4	2	2
52·15—	3	3	0	114·25—	3	3	0
53·35—	1	1	0	115·65—	1	0	1
54·55—	1	0	1	117·05—	1	0	1
55·75—	1	1	—	118·45—	1	1	—
56·95—	1	1	—	119·85—	0	0	—
58·15—	0	0	—	121·25—	0	0	—
59·35—60·55	1	1	—	122·65—124·05	1	1	—
Totals	355	219	136	Totals	332	202	130

MISCELLANEA.

(i) An Illustration of the Accuracy of the χ^2 Approximation.

By M. R. EL SHANAWANY, B.Sc.

1. *Introduction.*

If in an infinite population, the chance of an individual falling into the t^{th} of κ alternative categories is $p_t (t=1, 2, \dots \kappa)$, where $\Sigma(p_t)=1$, and a random sample of N is drawn, the chance that it will contain n_t individuals in the t^{th} category $(t=1, 2, \dots \kappa)$ is given by the term of the multinomial expansion; thus

$$c=\frac{N!}{n_1! \dots n_\kappa!} p_1^{n_1} \dots p_\kappa^{n_\kappa} \quad \dots\dots(1).$$

For any specified population and given size of sample, there will be a finite number of possible sets of sample frequencies $n_1, n_2, \dots n_\kappa$, that may be drawn, and with each is associated a chance c. The samples of this set may be arranged in order according to the magnitude of c, and if c_0 is the chance of drawing an observed sample, it is possible by summing the appropriate terms of the multinomial to find the probability, say $P\{c \leqslant c_0\}$, of drawing a sample with $c \leqslant c_0$. Following the common practice we might use this expression to test the hypothesis that a given sample has been drawn from a population with specified values of the p's*. For example, if $P\{c \leqslant c_0\} \leqslant \cdot 01$, we might decide to reject the hypothesis. Since however the calculation of the multinomial terms is extremely laborious, even when $\kappa=3$, the well-known χ^2 (or χ) approximation is almost invariably used†. This consists in calculating

$$\chi^2=\sum_{t=1}^{\kappa} \frac{(n_t-p_t N)^2}{p_t N} \quad \dots\dots(2),$$

and using as an approximation to $P\{c \leqslant c_0\}$ the integral

$$P(\chi \geqslant \chi_0)=\frac{\int_{\chi_0}^{\infty} \chi^{\kappa-2} e^{-\frac{1}{2}\chi^2} d\chi}{\int_0^{\infty} \chi^{\kappa-2} e^{-\frac{1}{2}\chi^2} d\chi} \quad \dots\dots(3).$$

While there is little doubt of the adequacy of this approximation when none of the expected frequencies Np_t are too small, there is less certainty of its value in the case of small samples. When $\kappa=2$, χ reduces to

$$\chi=\frac{n_1-Np_1}{\sqrt{Np_1(1-p_1)}} \quad \dots\dots(4),$$

and the approximation involved is that of representing a binomial $(p_2+p_1)^N$ by a normal curve with mean $=Np_1$, and standard deviation $=\sqrt{Np_1(1-p_1)}$. In this case, representing the binomial as a grouped frequency distribution or histogram, it is common to approximate to the sum of the tail terms by entering the tables of the Normal Probability Integral, not with χ of (4) but with an expression, say χ', in which the absolute value of the numerator is reduced by $\frac{1}{2}$‡, or

$$\chi'=\frac{|n_1-Np_1|-\frac{1}{2}}{\sqrt{Np_1(1-p_1)}} \quad \dots\dots(5).$$

* Such, for example, might be the case in testing a hypothesis regarding the factor constitution in Mendelian theory.

† For the relation between the multinomial and the χ^2 distribution see, for example, Neyman and Pearson, *Biometrika*, Vol. xx$^{\text{A}}$. p. 263.

‡ [It may be difficult to trace the origin of this correction for the binomial approximation. It has been used for many years in the Biometric Laboratory, University College, London, and I believed it due to De Moivre himself, but cannot now cite the source of my belief. Ed.]

In a recent paper* F. Yates has shown how an analogous correction for continuity applied to a 2×2 contingency table improves considerably the accuracy with which the Normal Integral represents the sum of the multinomial terms $P\{c\leqslant c_0\}$. He uses the expression

$$(\chi')^2=\sum_{t=1}^{\kappa}\frac{(|n_t-p_tN|-\frac{1}{2})^2}{p_tN} \qquad \text{.....(6).}$$

In the present paper, the situation will be considered when the individuals are classed into three categories ($\kappa=3$), and $P\{c\leqslant c_0\}$ will be compared with $P\{\chi\geqslant\chi_0\}$, in a particular case where

$$p_1=\cdot3,\quad p_2=\cdot5,\quad p_3=\cdot2 \qquad \text{.....(7),}$$

and $\qquad (a)\ N=10;\quad (b)\ N=20.$

It will be shown that the approximation is sometimes greater, sometimes less than the sum of the multinomial terms. As this is true at all levels of significance, a simple systematic correction of the type appropriate in Yates' problem is not here possible.

2. *Comparison of* $P\{c\leqslant c_0\}$ *and* $P\{\chi\geqslant\chi_0\}$.

Special case $\qquad \kappa=3,\quad N=10.$

In this case the integral (3) reduces to the simple value

$$P\{\chi\geqslant\chi_0\}=e^{-\frac{1}{2}\chi_0^2} \qquad \text{.....(8).}$$

For $N=10$, $\kappa=3$, there are 66 possible different sets of values n_1, n_2, n_3. The columns of Table I show (1) the series number of each set arranged in order of descending c value†, (2) the frequencies n_1, n_2, n_3, (3) c of Equation (1), (4) χ_0^2 of Equation (2), (5) $P_c=P\{c\leqslant c_0\}$, (6) $P_\chi=P\{\chi\geqslant\chi_0\}=e^{-\frac{1}{2}\chi_0^2}$, (7) $(P_\chi-P_c)/P_c$. It will be seen that the last quantity is sometimes positive, sometimes negative and that this is true throughout the region where greatest accuracy is required in applying the test, i.e. where

$$\cdot10>P_c>\cdot005,$$

that is to say for samples with series numbers 25—44.

The 66 possible types of sample can be represented as points in a three dimensioned space, in which they will all fall on the plane

$$n_1+n_2+n_3=N \qquad \text{.....(9).}$$

The projection of these points on the (n_1, n_2) plane is shown in Fig. 1; in the square surrounding each point is the series number from Table I, and the value of P_c. Samples for which $P_\chi>P_c$ are indicated by black circles, and those for which $P_\chi<P_c$ by open circles. The only sample for which P_c exactly equals P_χ is the "most probable" sample $n_1=3$, $n_2=5$, $n_3=2$ for which the values of the expressions are both unity. It will be seen that discrepancies of the same sign are roughly grouped together into the different regions in which the plane is divided by the lines

$$n_1=p_1N=3;\quad n_2=p_2N=5;\quad n_1+n_2=8 \qquad \text{.....(10),}$$

the last line being the projection of $n_3=p_3N=2$.

A correction in the form used by Yates in the case of the 2×2 contingency table would not here be applicable, for if we take

$$\chi'^2=\sum_{t=1}^{3}\frac{\{|n_t-p_tN|-\frac{1}{2}\}^2}{p_tN} \qquad \text{.....(11),}$$

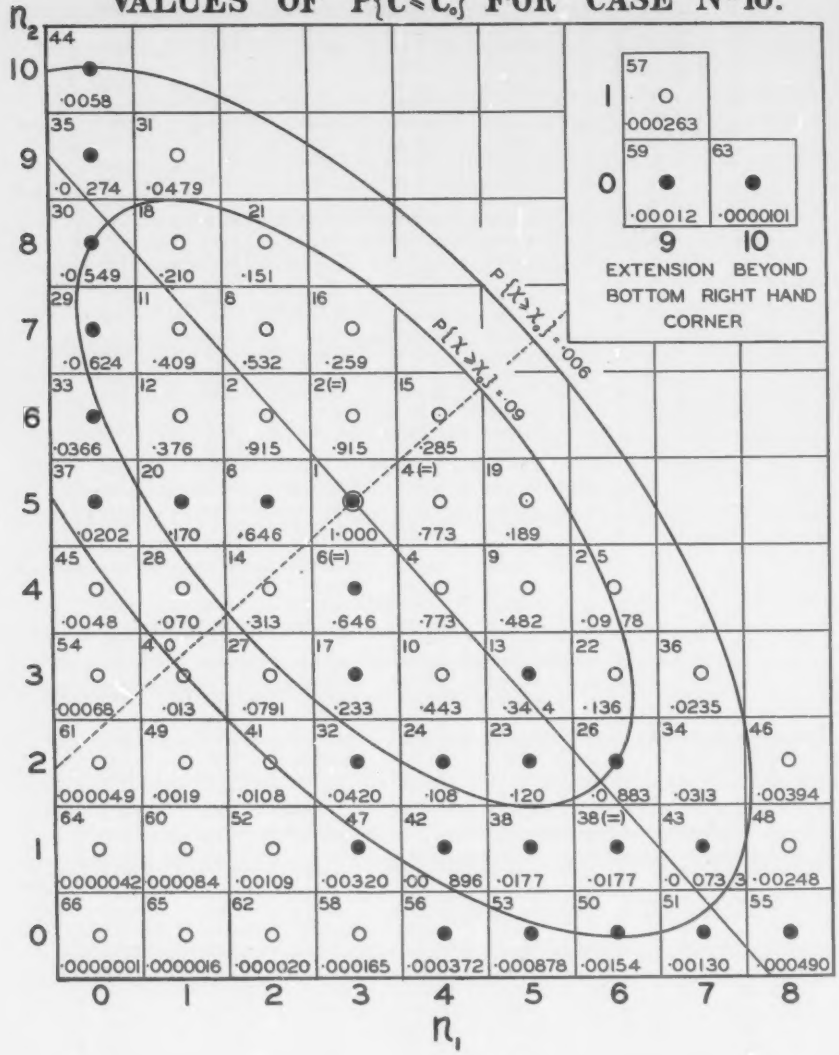
* Supplement to the *Journal of the Royal Statistical Society*, Vol. I. No. 2, 1934, p. 217, "Contingency Tables involving small numbers and χ^2 test."

† Where two values of c are identical, one has been given the order number and the other this number followed (in brackets) by the sign of equality. For example, the values of c for frequencies (2, 5, 3) and (3, 4, 3) are identical; the first has been given series No. 6, the second 6 (=). Other values of c, e.g. for Nos. 51 and 52, are only equal to 5 decimal places.

TABLE I. $N = 10$.

Series No. (1)	n_1, n_2, n_3 (2)	c (3)	χ^2 (4)	$P\{c \leqslant c_0\}$ (5)	$P\{\chi \geqslant \chi_0\}$ (6)	$\frac{(6)-(5)}{(5)}$ (7)
1	3, 5, 2	·08505	·0000	1·0000	1·0000	·000
2	2, 6, 2	·07087	·5333	·9149	·7859	−·141
2 (=)	3, 6, 1	·07087	·7000	·9149	·7047	−·230
4	4, 4, 2	·06379	·5333	·7732	·7659	−·009
4 (=)	4, 5, 1	·06379	·8333	·7732	·6592	−·147
6	2, 5, 3	·05670	·8333	·6456	·6592	·211
6 (=)	3, 4, 3	·05670	·7000	·6456	·7047	·092
8	2, 7, 1	·05062	1·6333	·5322	·4419	−·151
9	5, 4, 1	·03827	2·0333	·4816	·3618	−·249
10	4, 3, 3	·03402	1·6333	·4433	·4419	−·003
11	1, 7, 2	·03375	2·1333	·4093	·3442	−·159
12	1, 6, 3	·03150	2·0333	·3756	·3618	−·037
13	5, 3, 2	·03062	2·1333	·3441	·3442	·0003
14	2, 4, 4	·02835	2·5333	·3134	·2818	−·101
15	4, 6, 0	·02658	2·5333	·2851	·2818	−·012
16	3, 7, 0	·02531	2·8000	·2585	·2466	−·046
17	3, 3, 4	·02268	2·8000	·2332	·2466	·057
18	1, 8, 1	·02109	3·6333	·2105	·1626	−·228
19	5, 5, 0	·01914	3·3333	·1894	·1889	−·003
20	1, 5, 4	·01890	3·3333	·1703	·1889	·109
21	2, 8, 0	·01582	4·1333	·1514	·1266	−·164
22	6, 3, 1	·01531	4·3000	·1356	·1165	−·141
23	5, 2, 3	·01225	3·6333	·1203	·1626	·352
24	4, 2, 4	·01021	4·1333	·1080	·1266	·172
25	6, 4, 0	·00957	5·2000	·09781	·07427	−·241
26	6, 2, 2	·00919	4·8000	·08829	·09072	·028
27	2, 3, 5	·00907	5·6333	·07905	·05981	−·243
28	1, 4, 5	·00756	6·0333	·06998	·04897	−·300
29	0, 7, 3	·00750	4·3000	·06242	·1166	·867
30	0, 8, 2	·00703	4·8000	·05492	·09072	·652
31	1, 9, 0	·00586	6·5333	·04789	·03813	−·204
32	3, 2, 5	·00544	6·3000	·04203	·04285	·020
33	0, 6, 4	·00525	5·2000	·03658	·07427	1·030
34	7, 2, 1	·00394	7·6333	·03134	·02200	−·298
35	0, 9, 1	·00391	6·7000	·02740	·03508	·280
36	7, 3, 0	·00328	8·1333	·02349	·01713	−·271
37	0, 5, 5	·00252	7·5000	·02021	·02352	·164
38	5, 1, 4	·00245	6·5333	·01769	·03813	1·155
38 (=)	6, 1, 3	·00245	6·7000	·01769	·03508	·983
40	1, 3, 6	·00202	10·1333	·01280	·00630	−·508
41	2, 2, 6	·00181	10·1333	·01078	·00630	−·416
42	4, 1, 5	·00163	8·0333	·00896	·01801	1·009
43	7, 1, 2	·00157	8·5333	·00733	·01403	·914
44	0, 10, 0	·00098	10·0000	·00576	·00674	·170
45	0, 4, 6	·00084	11·2000	·00478	·00370	−·226
46	8, 2, 0	·00074	12·1333	·00394	·00232	−·411
47	3, 1, 6	·00073	11·2000	·00320	·00370	·155
48	8, 1, 1	·00059	12·0333	·00248	·00244	−·015
49	1, 2, 7	·00035	15·6333	·00189	·00040	−·362
50	6, 0, 4	·00024	10·0000	·00154	·00674	3·375
51	7, 0, 3	·00021	10·8333	·00130	·00444	2·430
52	2, 1, 7	·00021	16·0333	·00109	·00033	−·697
53	5, 0, 5	·00020	10·8333	·00088	·00444	4·059
54	0, 3, 7	·00019	16·3000	·00068	·00029	−·577
55	8, 0, 2	·00012	13·3333	·00049	·00127	1·598
56	4, 0, 6	·00011	13·3333	·00037	·00127	2·423
57	9, 1, 0	·00010	17·2000	·00026	·00018	−·300
58	3, 0, 7	·00004	17·5000	·00016	·00016	−·038
59	9, 0, 1	·00004	17·5000	·00012	·00016	·287
60	1, 1, 8	·00003	22·5333	·00008	·00001	−·847
61	0, 2, 8	·00003	22·8000	·00005	·00001	−·773
62	2, 0, 8	·00001	23·3333	·00002	·00001	−·581
63	10, 0, 0	·00001	23·3333	·00001	·00001	−·150
64	0, 1, 9	—	30·7000	—	—	−·610
65	1, 0, 9	—	30·8333	—	—	−·877
66	0, 0, 10	—	40·0000	—	—	−·979

FIG. I.
VALUES OF $P\{C \leqslant C_0\}$ FOR CASE N=10.



and calculate $P\{\chi' \geqslant \chi_0'\} = e^{-\frac{1}{2}(\chi_0')^2}$ .....(12),

we find that this probability is greater than P_c for every possible sample.

The diagram also shows the projection of two of the ellipses of equation (2). The inner ellipse which passes through the points with coordinates (0, 8) and (6, 2) corresponds very closely

to a probability level $P_\chi = \cdot 09$. The outer ellipse passes through the points (1, 3) and (2, 2) and corresponds very nearly to a probability level of $P_\chi = \cdot 006$. An examination of the true P_c values at points lying close to the ellipses shows that the χ-contours do provide a reasonable approximation for practical purposes even in this case where $N = 10$.

It is of interest to look at the problem from another point of view. We may regard a statistical test as providing us with a rule which will guide us in determining whether (*a*) to accept a hypothesis H, (*b*) remain in doubt, or (*c*) reject the hypothesis. In the present instance H is specified in equation (7) and we might adopt the following useful if arbitrary convention,

(*a*) Accept H if $P_c \geqslant \cdot 05$,

(*b*) Remain in doubt if $\cdot 05 > P_c \geqslant \cdot 01$,

(*c*) Reject H if $\cdot 01 > P_c$.

If we were to apply this rule whenever H were true, it is found from Table I that for the 66 types of samples, H would be accepted in 30 cases (series Nos. 1—30), left in doubt in 11 (series Nos. 31—41) and rejected in 25 cases (series Nos. 42—66). These numbers would be just the same if we were to use χ^2 as criterion, i.e. use P_χ instead of P_c. But for 6 out of the 66 samples we should reach different conclusions. The position is shown in Table II; if identical decisions were reached using either test, the frequencies would all lie in the three diagonal cells. The table also shows, in brackets, the chance associated with the frequency in each cell; for example, the cell "Rejection using P_c", "Doubt using P_χ" contains a frequency of 2; these correspond to the samples Nos. 42 and 43, for which Table I shows $C = \cdot 00163$ and $\cdot 00157$ giving a total of $\cdot 00320$.

TABLE II. $N = 10$.

Judging by P_c

Judging by χ^2 test	Acceptance	Doubt	Rejection	Total
Acceptance	29 (·9445)	1 (·0053)	—	30 (·9498)
Doubt	1 (·0076)	8 (·0298)	2 (·0032)	11 (·0406)
Rejection	—	2 (·0038)	23 (·0058)	25 (·0096)
Total	30 (·9521)	11 (·0389)	25 (·0090)	66 (1·0000)

It is clear from a study of the table that if we use the χ^2 approximation, the conclusions that we should reach regarding the hypothesis tested will be in practice almost exactly the same as those we should reach were we to calculate the sum of the multinomial terms, P_c. This result has of course been established only in the particular case of the illustration where the expected frequencies are 2, 5, 3. If any of the expectations were smaller the correspondence would no doubt be less.

3. *Comparison of P_c and P_χ, for case $\kappa = 3$, $N = 20$.*

Finally, the case with $N = 20$ and the same values of the p's was examined. Here there are 231 possible series of values for n_1, n_2, n_3; the 128 largest values of c were calculated from (1),

and are shown in Table III; since for these $\Sigma(c) = \cdot 999$, it was unnecessary to consider the remaining cases. The same convention as in Table I is adopted for samples having identical values of c (see footnote to p. 180).

In the range $\cdot 10 > P_c > \cdot 005$ (series Nos. 51—103) the differences $P_\chi - P_c$ are sometimes positive, sometimes negative. It is found that the mean of the absolute value of the ratio $(P_\chi - P_c)/P_c$ is now ·397, whereas in the case of $N = 10$, it was ·488. Thus the χ approximation is improved for the larger sample.

Fig. 2 shows the projection in the (n_1, n_2) plane of a part of the plane of equation (9), the scheme being as in Fig. 1 except that it has been necessary to omit the series numbers owing to

FIG. 2.

VALUES OF $P\{C \leqslant C_0\}$ FOR CASE N = 20.

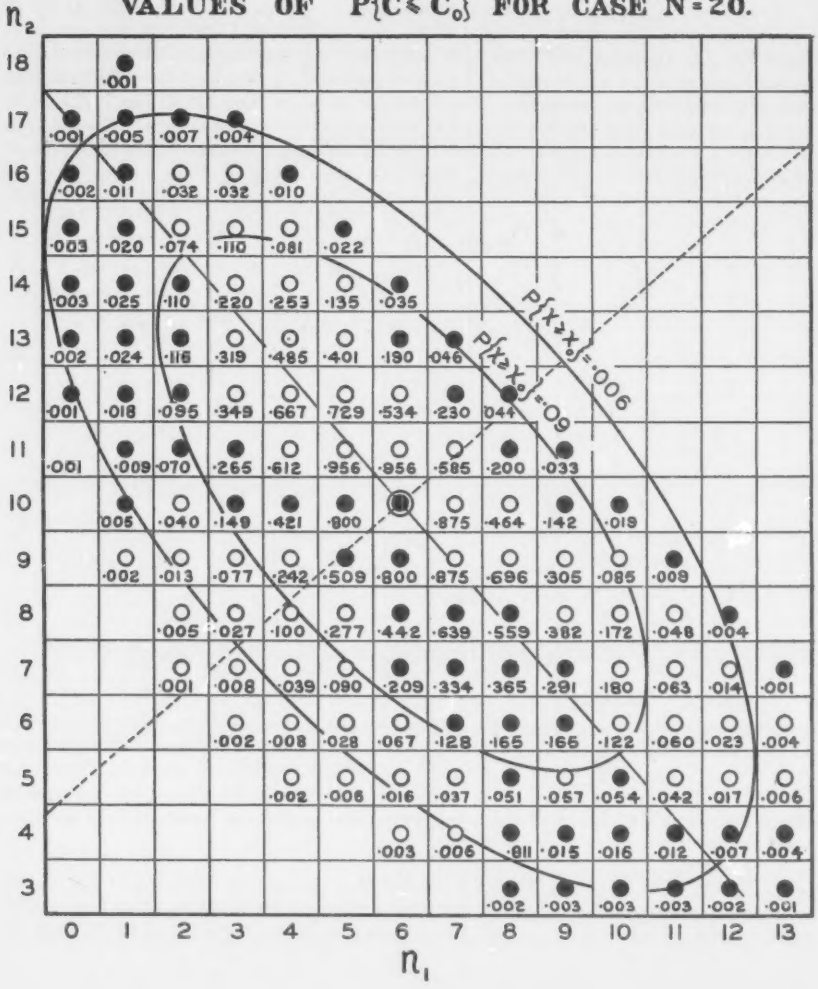


TABLE III. $N = 20$.

Series No. (1)	n_1, n_2, n_3 (2)	c (3)	χ^2 (4)	$P(c \leqslant c_0)$ (5)	$P(\chi \geqslant \chi_0)$ (6)	$\frac{(6)-(5)}{(5)}$ (7)
1	6, 10, 4	·04419	·000	1·0000	1·0000	·060
2	5, 11, 4	·04018	·266	·9558	·8752	−·090
2 (=)	6, 11, 3	·04018	·350	·9558	·8395	−·122
4	7, 9, 4	·03788	·266	·8755	·8752	−·000
4 (=)	7, 10, 3	·03788	·416	·8755	·8119	−·073
6	5, 10, 5	·03536	·416	·7997	·8119	·015
6 (=)	6, 9, 5	·03536	·350	·7997	·8395	·050
8	5, 12, 3	·03348	·816	·7290	·6648	−·088
9	8, 9, 3	·02841	1·016	·6955	·6015	−·135
10	4, 12, 4	·02790	1·016	·6671	·5867	−·121
11	7, 8, 5	·02727	·816	·6392	·6648	·040
12	4, 11, 5	·02678	1·066	·6119	·6015	−·017
13	7, 11, 2	·02583	1·266	·5851	·5308	−·093
14	8, 8, 4	·02557	1·066	·5593	·5867	·049
15	6, 12, 2	·02511	1·400	·5337	·4966	−·070
16	5, 9, 6	·02357	1·266	·5086	·5308	·044
17	4, 13, 3	·02146	1·816	·4851	·4032	−·169
18	8, 10, 2	·02131	1·666	·4636	·4346	−·063
19	6, 8, 6	·02121	1·400	·4423	·4966	·123
20	4, 10, 6	·01964	1·666	·4211	·4346	·032
21	5, 13, 2	·01932	2·066	·4014	·3558	−·114
22	9, 8, 3	·01705	2·150	·3821	·3413	−·107
23	8, 7, 5	·01637	1·816	·3651	·4032	·104
24	3, 12, 5	·01488	2·150	·3487	·3413	−·021
25	7, 7, 6	·01455	2·066	·3338	·3558	·066
26	3, 13, 4	·01431	2·400	·3193	·3012	−·057
27	9, 9, 2	·01421	2·600	·3050	·2724	−·107
28	9, 7, 4	·01364	2·400	·2908	·3012	·036
29	5, 8, 7	·01212	2·816	·2771	·2446	−·118
30	3, 11, 6	·01190	2·666	·2650	·2725	·028
31	4, 14, 2	·01150	3·266	·2531	·1953	−·228
32	4, 9, 7	·01122	3·016	·2416	·2213	−·084
33	7, 12, 1	·01076	2·816	·2304	·2446	·062
34	3, 14, 3	·01022	3·350	·2196	·1873	−·147
35	6, 7, 7	·00970	3·150	·2094	·2606	·245
36	8, 11, 1	·00969	3·016	·1997	·2213	·108
37	6, 13, 1	·00966	3·150	·1900	·2606	·372
38	10, 7, 3	·00818	3·816	·1804	·1483	−·178
39	10, 8, 2	·00767	4·066	·1722	·1309	·240
40	8, 6, 6	·00764	3·266	·1645	·1953	·187
40 (=)	9, 6, 5	·00764	3·350	·1645	·1873	·139
42	3, 10, 7	·00748	3·750	·1492	·1534	·028
43	9, 10, 1	·00710	3·750	·1417	·1534	·082
44	5, 14, 1	·00690	4·016	·1346	·1342	−·003
45	7, 6, 7	·00582	4·016	·1277	·1342	·051
46	10, 6, 4	·00573	4·266	·1219	·1184	−·029
47	2, 13, 5	·00572	3·816	·1162	·1483	·277
48	2, 14, 4	·00511	4·266	·1105	·1184	·072
48 (=)	3, 15, 2	·00511	5·000	·1105	·0821	−·257
50	4, 8, 8	·00505	5·066	·1003	·0794	−·208
51	2, 12, 6	·00496	4·066	·0952	·1309	·375
52	5, 7, 8	·00485	5·066	·0902	·0794	−·205
53	10, 9, 1	·00426	5·016	·0854	·0814	−·047
54	4, 15, 1	·00383	5·416	·0811	·0667	−·179
55	3, 9, 8	·00374	5·600	·0773	·0608	−·213
56	2, 15, 3	·00341	5·416	·0736	·0667	−·094
57	2, 11, 7	·00340	5·016	·0702	·0814	·160
58	6, 6, 8	·00339	5·600	·0668	·0608	−·089
59	11, 7, 2	·00335	6·066	·0634	·0482	−·240
60	11, 6, 3	·00312	6·016	·0600	·0494	−·177
61	9, 5, 6	·00306	5·000	·0569	·0821	−·443
62	10, 5, 5	·00275	5·416	·0538	·0667	·238
63	8, 5, 7	·00262	5·416	·0511	·0667	·305
64	11, 8, 1	·00209	6·816	·0485	·0331	−·317

TABLE III (*continued*).

Series No. (1)	n_1, n_2, n_3 (2)	c (3)	χ^2 (4)	$P(c \leqslant c_0)$ (5)	$P(\chi \geqslant \chi_0)$ (6)	$\frac{(6)-(5)}{(5)}$ (7)
65	7, 13, 0	·00207	5·066	·0464	·0794	·712
66	8, 12, 0	·00202	5·066	·0443	·0794	·792
67	11, 5, 4	·00188	6·666	·0423	·0357	−·156
68	2, 10, 8	·00187	6·666	·0404	·0357	−·117
69	4, 7, 9	·00180	7·816	·0385	·0201	−·479
70	7, 5, 8	·00175	6·666	·0367	·0357	−·029
71	6, 14, 0	·00173	5·600	·0350	·0608	·738
72	9, 11, 0	·00161	5·600	·0333	·0608	·828
73	2, 16, 2	·00160	7·266	·0317	·0264	−·165
73 (=)	3, 16, 1	·00160	7·350	·0317	·0254	−·199
75	5, 6, 9	·00151	8·016	·0285	·0182	−·362
76	3, 8, 9	·00150	8·150	·0270	·0170	−·370
77	1, 14, 5	·00136	6·016	·0255	·0494	·939
78	1, 13, 6	·00127	6·066	·0241	·0482	·999
79	12, 6, 2	·00117	8·600	·0228	·0136	−·405
80	5, 15, 0	·00115	6·666	·0216	·0357	·648
81	1, 15, 4	·00114	6·666	·0205	·0357	·740
82	10, 10, 0	·00107	6·666	·0194	·0357	·842
83	1, 12, 7	·00095	6·816	·0183	·0331	·809
84	12, 5, 3	·00094	8·750	·0174	·0126	−·274
85	10, 4, 6	·00092	7·266	·0164	·0264	·610
86	6, 5, 9	·00091	8·750	·0155	·0126	−·188
87	9, 4, 7	·00087	7·350	·0146	·0254	·737
88	12, 7, 1	·00084	9·150	·0137	·0103	−·249
89	2, 9, 9	·00083	9·016	·0129	·0110	−·145
90	11, 4, 5	·00075	8·016	·0121	·0182	·507
91	1, 16, 3	·00071	6·016	·0113	·0182	·607
92	8, 4, 8	·00066	8·266	·0106	·0160	·514
93	4, 16, 0	·00060	8·266	·0099	·0160	·613
94	11, 3, 0	·00058	8·266	·0093	·0160	·716
95	1, 11, 8	·00057	8·266	·0088	·0160	·830
96	4, 6, 10	·00050	11·266	·0082	·0036	−·563
97	3, 7, 10	·00048	11·400	·0077	·0033	−·564
98	2, 17, 1	·00048	9·616	·0072	·0082	·132
99	12, 4, 4	·00047	9·600	·0067	·0082	·221
100	7, 4, 9	·00039	13·016	·0063	·0015	−·762
101	5, 5, 10	·00036	11·666	·0059	·0029	−·501
102	13, 5, 2	·00032	11·666	·0055	·0029	−·469
103	1, 17, 2	·00031	9·866	·0052	·0072	·385
104	2, 8, 10	·00030	12·066	·0049	·0024	−·508
105	1, 10, 9	·00028	10·416	·0046	·0055	·192
106	13, 6, 1	·00027	12·016	·0043	·0025	−·429
107	12, 8, 0	·00026	10·400	·0040	·0055	·369
108	3, 17, 0	·00024	10·200	·0038	·0061	·614
109	13, 4, 3	·00022	12·016	·0035	·0055	·571
110	10, 3, 7	·00021	9·616	·0033	·0082	1·453
111	11, 3, 6	·00020	9·866	·0031	·0072	1·310
112	6, 4, 10	·00018	12·600	·0029	·0018	−·370
113	9, 3, 8	·00018	10·200	·0027	·0061	1·233
114	0, 14, 6	·00015	8·600	·0026	·0136	4·303
114 (=)	0, 15, 5	·00015	8·750	·0026	·0126	3·919
116	12, 3, 5	·00015	10·950	·0023	·0042	·855
117	3, 6, 11	·00012	15·350	·0021	·0005	−·762
118	0, 13, 7	·00012	9·150	·0020	·0103	4·150
119	0, 16, 4	·00012	9·600	·0019	·0083	3·368
120	8, 3, 9	·00012	11·616	·0017	·0030	·765
121	1, 9, 10	·00011	13·266	·0016	·0013	−·187
122	4, 5, 11	·00011	15·416	·0015	·0004	−·733
123	13, 7, 0	·00010	13·066	·0014	·0015	·071
124	2, 7, 11	·00009	15·816	·0013	·0004	−·692
125	1, 18, 1	·00009	12·816	·0012	·0016	·333
126	13, 3, 4	·00009	13·066	·0011	·0015	·364
127	0, 12, 8	·00008	10·400	·0011	·0055	4·000
128	0, 17, 3	·00007	10·950	·0010	·0042	3·200

lack of space. The two ellipses correspond to the same probability levels $P_\chi = \cdot09$ and $\cdot006$, and the values of P_c in the neighbourhood of the curve may be compared. The approximation appears very satisfactory.

Table IV has been prepared in the same way as Table II. It emphasises again how the conclusions regarding H obtained from using the χ^2 approximation would hardly differ from those obtained by the laborious process of calculating and summing the multinomial terms.

In conclusion, it must of course be remembered that the foregoing results have been established in two special cases only, where the expected group frequencies were (2, 5, 3) and (4, 10, 6). For larger expected frequencies better results might be anticipated; it is likely that the approximation will fail if any value of $p_s N$ is less than 2 and N much less than 10.

TABLE IV. $N = 20$.

Judging by P_c

Judging by χ^2 test	Acceptance	Doubt	Rejection	Total
Acceptance	61 (·9450)	4 (·0074)	—	65 (·9524)
Doubt	2 (·0065)	25 (·0311)	6 (·0021)	33 (·0397)
Rejection	—	—	133 (·0079)	133 (·0079)
Total	63 (·9515)	29 (·0385)	139 (·0100)	231 (1·000)

(ii) Note on the Application of a Theorem of Frau Pollaczek-Geiringer.

By J. PRZYBOROWSKI and H. WILEŃSKI.

Plant Breeding and Agricultural Department, University of Cracow.

In our previous paper "Statistical Principles of Routine Work in Testing Clover Seed for Dodder*" we have considered the upper limits of the probability of the first and second kind errors and the fiducial limits of the parameter $m = np$, when the sample is drawn from a population following the Poisson law

$$p(x) = e^{-m}\frac{m^x}{x!} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1),$$

where $x = a$.

Frau Pollaczek-Geiringer† has once given an approximation, which can be applied to our problem.

* *Biometrika*, XXVII. Parts III and IV.

† *Zeitschrift für angewandte Mathematik und Mechanik*, Bd. v. S. 493—501 (1925), and Mises, R. v., *Vorlesungen aus dem Gebiete der angewandten Mathematik*, I. Bd. *Wahrscheinlichkeitsrechnung*. S. 163—168, Leipzig und Wien, 1931.

Considering the Poisson law on the basis of Bayes' theorem of inverse probability Frau Pollaczek-Geiringer has obtained the following theorem:

The probability law of the parameter $m=np$, where $x=a$, is independent of the *a priori* unknown probability $v(p)=v\left(\frac{m}{n}\right)$ when n increases infinitely, and is expressed by the formula

$$p(m)\,dm=\frac{1}{a!}m^a e^{-m}dm \quad \text{.....(2).}$$

It can easily be seen from this that the mathematical expectation $\in(m)$ and the variance are both equal to $a+1$.

The probability that m does not exceed m_0 is given by

$$P\{m\leqslant m_0\}=\frac{1}{a!}\int_0^{m_0} m^a e^{-m}dm \quad \text{.....(3).}$$

Mises* says, that for $n=100$ and $a=1$ the approximation is satisfactory, but, as we have shown in our paper quoted above, in the cases where a is small the method of controlling the two kinds of errors that we have applied may give important information in answering such questions as treated in examples I, II and III (pp. 281—285).

We may add that following Frau Pollaczek-Geiringer's theorem we are entitled to consider the values of the upper limits of the probability of errors of the first kind (Table III) as practically the same as the values of the probability of m exceeding m_0.

The samples in testing clover for dodder are such that n is sufficiently large; moreover, if a (the number of dodder seeds found) is not too small, the approximation given by Frau Pollaczek-Geiringer's theorem may be practically applied; but, as we have mentioned above, if a is small the solution of the problem is found more precisely if the method of considering the two kinds of errors be used.

(iii) Note on the Method of Moments.

By V. Romanovsky.

In *Metron*, xi. N. 2 (1933), 103—136, E. Merzrath has published a paper on the theory of correlation in which he makes the following statements on the method of moments.

Let $\phi(x)$ be a law of distribution for which x remains between finite limits, a and b, and which we wish to approximate to by the aid of a function $f(x, c_0, c_1, \ldots, c_n)$ containing $n+1$ parameters c_i.

E. Merzrath considers f as a good approximation to ϕ if c_i are such that three conditions are satisfied:

$$\int_a^b [\phi-\sum_{i=0}^{n} a_i x^i]^2\,dx=\text{Min} \quad \text{.....(1),}$$

$$\int_a^b [f-\sum_{i=0}^{n} b_i x^i]^2\,dx=\text{Min} \quad \text{.....(2),}$$

$$\sum_i a_i x^i \equiv \sum_i b_i x^i \text{ or } a_i=b_i \text{ for } i=0, 1, 2 \ldots n \quad \text{.....(3).}$$

From (1) and (2) he obtains

$$\left.\begin{aligned}\int_a^b x^\kappa \phi\,dx&=\int_a^b \sum_i a_i x^{i+\kappa}\,dx\\ \int_a^b x^\kappa f\,dx&=\int_a^b \sum_i b_i x^{i+\kappa}\,dx\end{aligned}\right\}\ \kappa=0, 1, 2 \ldots n,$$

and therefore, by (3),

$$\int_a^b x^\kappa \phi\,dx=\int_a^b x^\kappa f\,dx,\ \kappa=0, 1, 2 \ldots n \quad \text{.....(4).}$$

* Mises, *loc. cit.* S. 166.

Thus, if the conditions (1)—(3) are satisfied, the functions ϕ and f have the same moments of order $\kappa = 0, 1, 2 \ldots n$ and the author concludes that the approximation of f to ϕ may be considered as good inasmuch as both functions have the same least square approximation $\Sigma a_i x^i$ of the order n.

But what we need in the proof of the moment method is rather the reversed conclusion: it is desirable from (4) to conclude that there is a good approximation of f to ϕ. This can be done very easily using the idea of E. Merzrath. Indeed we can demonstrate the following theorems.

I. If f is such that (4) *holds true, then f and ϕ have the same least square approximation*

$$P_n(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n.$$

To prove this, let $P_n(x)$ be the least square approximation to ϕ, so that

$$\int_a^b x^\kappa \phi \, dx = \int_a^b \sum_i a_i x^{i+\kappa} dx, \quad \kappa = 0, 1, 2 \ldots n.$$

Then, from (4), we have

$$\int_a^b x^\kappa f \, dx = \int_a^b \sum_i a_i x^{i+\kappa} dx, \quad \kappa = 0, 1, 2 \ldots n,$$

or

$$\frac{\partial}{\partial a_\kappa} \int_a^b [f - P_n(x)]^2 \, dx = 0, \quad \kappa = 0, 1, 2 \ldots n,$$

q.e.d.

II. Let

$$\int_a^b [\phi - P_n(x)]^2 \, dx \leqslant \epsilon^2,$$

$$\int_a^b [f - P_n(x)]^2 \, dx \leqslant \epsilon_1^2,$$

then

$$\int_a^b [f - \phi]^2 \, dx \leqslant (\epsilon + \epsilon_1)^2, \quad (\epsilon > 0, \epsilon_1 > 0).$$

In fact,

$$\int_a^b [f - \phi]^2 \, dx = \int_a^b [f - P_n]^2 \, dx + \int_a^b [\phi - P_n]^2 \, dx - 2 \int_a^b [f - P_n][\phi - P_n] \, dx$$

$$\leqslant \epsilon^2 + \epsilon_1^2 + 2 \left| \int_a^b [f - P_n][\phi - P_n] \, dx \right|.$$

But, by the inequality of Schwarz,

$$\left| \int_a^b [f - P_n][\phi - P_n] \, dx \right| \leqslant \sqrt{\int_a^b [f - P_n]^2 \, dx \int_a^b [\phi - P_n]^2 \, dx}$$

$$\leqslant \epsilon \epsilon_1,$$

and we have at once

$$\int_a^b [f - \phi]^2 \, dx \leqslant (\epsilon + \epsilon_1)^2.$$

Thus, if P_n is good approximation to f and ϕ, f will be also a good approximation to ϕ, if defined by the methods of moments.

Very often we have to deal with distributions of infinite range. For this case E. Merzrath demonstrates that if

$$\int_{-\infty}^{\infty} [\phi e^{h^2 x^2} - e^{-h^2 x^2} \sum_i a_i x^i]^2 \, dx = \text{Min},$$

$$\int_{-\infty}^{\infty} [f e^{h^2 x^2} - e^{-h^2 x^2} \sum_i b_i x^i]^2 \, dx = \text{Min},$$

$$a_i = b_i \text{ for } i = 0, 1, 2 \ldots n,$$

then

$$\int_{-\infty}^{\infty} x^\kappa \phi \, dx = \int_{-\infty}^{\infty} x^\kappa f \, dx, \quad \kappa = 0, 1, 2 \ldots n.$$

From this he concludes that f is a good approximation to ϕ inasmuch as $\phi e^{h^2x^2}$ and $fe^{h^2x^2}$ have the same least square approximation

$$e^{-h^2x^2}\sum_i a_i x^i = e^{-h^2x^2} P_n(x) \quad \ldots\ldots(5).$$

Here also we must reverse the problem and, besides, make some other alterations.

We shall consider least square approximations of the form (5) not to

$$\phi_1 = e^{h^2x^2}\phi, \quad f_1 = e^{h^2x^2} f,$$

as the author does, but to ϕ and f themselves defining a_i in such manner that, for example, we have

$$\int_{-\infty}^{\infty} e^{h^2x^2}\left[\phi - e^{-h^2x^2}\sum_i a_i x^i\right]^2 dx = \text{Min} \quad \ldots\ldots(6).$$

Then we can demonstrate:

III. If
$$\int_{-\infty}^{\infty} x^\kappa f\,dx = \int_{-\infty}^{\infty} x^\kappa \phi\,dx, \quad \kappa = 0, 1, 2 \ldots n,$$

then f and ϕ have the same least square approximation $e^{-h^2x^2} P_n(x)$ defined by (6).

IV. If
$$\int_{-\infty}^{\infty} e^{h^2x^2}\left[f - e^{-h^2x^2} P_n\right]^2 dx \leqslant \epsilon,$$
$$\int_{-\infty}^{\infty} e^{h^2x^2}\left[\phi - e^{-h^2x^2} P_n\right]^2 dx \leqslant \epsilon_1,$$

then
$$\int_{-\infty}^{\infty} [f - \phi]^2\, dx \leqslant (\epsilon + \epsilon_1)^2.$$

These theorems are proved in the same manner as I and II and lead to the same conclusion.

(iv) On the Validity of a Certain Pearson's Formula—a Rejoinder.

By JAN WIŚNIEWSKI.

In the first place I must confess that I have misunderstood Prof. Pearson's original memoir (*Biometrika*, Vol. IX. pp. 116—139) in that I have taken C_x as the mid-point of the class-interval (as clearly indicated by my formula (*b*); thus it cannot be said that I "never tell what I mean by a class-mark"—p. 368 of Prof. Pearson's note, *Biometrika*, Vol. XXVII). I thought that the developments, which are strictly true when applied to means of variates within given class-intervals, as formula (i), are transferred as approximations to class-marks. In so far as this was not Prof. Pearson's intention, I plead guilty.

But this does not exhaust the problem. I think that the most fundamental difference of opinion between Professor Pearson and me lies in the following proposition. He says: "The correlation of two constant quantities is zero" (p. 369 of the note and, somewhat differently worded, p. 118 of the original memoir). For me such a correlation is meaningless, as its mathematical expression is $\frac{0}{0}$. The same applies to the correlation of two quantities: one being constant and the other variable. This is why I indulged in a lengthy discussion of the possible interpretations of the partial correlation coefficient. I really sought such an interpretation which would give some definite meaning to such expressions as $_x\rho_{yC_x}$ or $_y\rho_{C_yC_x}$.

Once the above expressed point of view has been accepted, we readily see what are its consequences to the "broad categories" problem. Suppose now C_x means $\bar{x}_s$, the mean value of observed x in a given class-interval. Then the question, What is the value of $_x\rho_{C_yC_x}$?, persists still. In the former paper (*Biometrika*, Vol. XXVII. pp. 356—363) I attempted to give a discussion

of what the value of this coefficient will possibly turn out. This discussion applies equally well to the interpretation of C_x as identical with $\bar{x}_s$; indeed, I wrote on p. 361: "Our conclusions would be unchanged if instead of (e) we simply took $u_i = C_{yi} - y_i$." This is just the case now considered. So I believe the generality of $_x\rho_{C_x C_y} = 0$, and consequently $r_{C_y C_x} - r_{yC_y} r_{yC_x} = 0$ remains open to doubt with the interpretation of C_x now accepted.

Suppose $$r_{C_y C_x} - r_{yC_y} r_{yC_x} = \epsilon \neq 0,$$

then $$r_{yC_x} = \frac{r_{C_y C_x}}{r_{yC_y}} - \frac{\epsilon}{r_{yC_y}},$$

and $$r_{xy} = \frac{r_{C_y C_x}}{r_{yC_y} r_{xC_x}} - \frac{\epsilon}{r_{yC_y} r_{xC_x}} *.$$

What are the conditions for ϵ being sensibly different from zero? These are: (1) the grouping being performed into really broad categories; (2) the degree of correlation being high enough to make itself felt already within cells of the correlation table, and to produce a concentration of a great majority of observations in one cell only of each row. As the regressions are assumed linear, these cells will be crossed by the regression lines. If the above conditions are fulfilled, ϵ will be the sign of r_{xy} and will cause Prof. Pearson's formulae (viii) and (ix) to overcorrect. It was only these formulae that I proposed to discuss, and not formula (xvii). This is why the problem of high contact did not enter into my considerations.

I will not touch upon the subject of my numerical example to which Prof. Pearson devoted considerable attention. If it was chosen well—it confirms my proof; if I chose it wrong, which I am ready to admit, then it only remains to find or construct another example serving as an illustration of the general principles which I tried to find.

[*Editorial comment.* The continuance of this discussion seems to me idle. M. Wiśniewski now seeks to place upon me the statement that the correlation of two constant quantities is zero, but if the phrase on p. 369 is somewhat carelessly worded, I think the meaning is clear. If we represent by r_{xy} the correlation coefficient of x and y, i.e.

$$r_{xy} = \frac{S(x-\bar{x})(y-\bar{y})}{\{S(x-\bar{x})^2 \times S(y-\bar{y})^2\}^{\frac{1}{2}}},$$

it is needful not only for the product moment, but for one of the standard deviations to vanish in order that

$$r_{xy} = \frac{0}{0}, \text{ an indeterminate form.}$$

Of this I was fully aware. But this is not the case with which we are dealing. Neither σ_{C_x} nor σ_{C_y} is zero, but the problem is: Does the product moment of the numerator vanish and, if so, under what conditions? The conditions are shown in my paper to be (i) those under which Sheppard's corrections give a high degree of approximation to the moment-coefficients, and (ii) linear or approximately linear regressions for the variates.

In my reply to M. Wiśniewski, I showed that my formula (i) was absolutely true, although he had written of it "It is more than sure that Karl Pearson was aware of the approximative character" of this formula (i). He now, though somewhat obscurely, admits the truth of this. But he further says: "I thought that the developments, which are strictly true when applied to means of variates within given class-intervals, as formula (i), are transferred as approximations to class-marks."

This sentence seems to assert that the "mean of a variate within a given class-interval" is not a class-mark. Or, that M. Wiśniewski is using that term for a character, not exclusively

* This on the assumption that (iv) is strictly true, which is not necessarily the case.

intended by the inventor of the term*. The puzzling point is that the phrase overlooks entirely the fact that the bulk of my original memoir was devoted to the conditions under which various characters of the given class-interval could replace the mean of variates in the interval. It was found that the chief limitation was high contact of the frequency curve at both terminals. If M. Wiśniewski had really studied my memoir, how did he come to choose a frequency surface in which both the x and y marginal frequencies rose abruptly at one terminal? He now promises to find another example, or "construct one" in which they do not. It will be sufficient to discuss it when we have it before us. K.P.]

* "Quite recently Dr G. A. Jaederholm, a Swedish psychologist, wrote to me asking what was the correlation between the true quantitative value of a variate in any individual, and that individual's category or class-mark. The answer is an obvious one, but I do not know that I have seen it stated or any discussion of it given. It of course assumes that at the back of the categorical classification a true quantitative value lies.....Let $\bar{x}_s$ be the mean variate of the group of n_s individuals who fall into the sth class....Then it is as reasonable to call $\bar{x}_s$ the class-mark, as C_s, for given one, the other is fixed." [Class-marks as "healthy" or "intelligent" may be supposed to be the mean variates of their class-intervals.] The words cited above are from the first page (p. 116) of my original memoir in *Biometrika*, Vol. IX. pp. 116—139, 1913, and sufficiently indicate what the inventor of the term "class-mark" intended it to signify. It is difficult to understand how M. Wiśniewski could have read them, and then supposed my formulae to be incorrect.

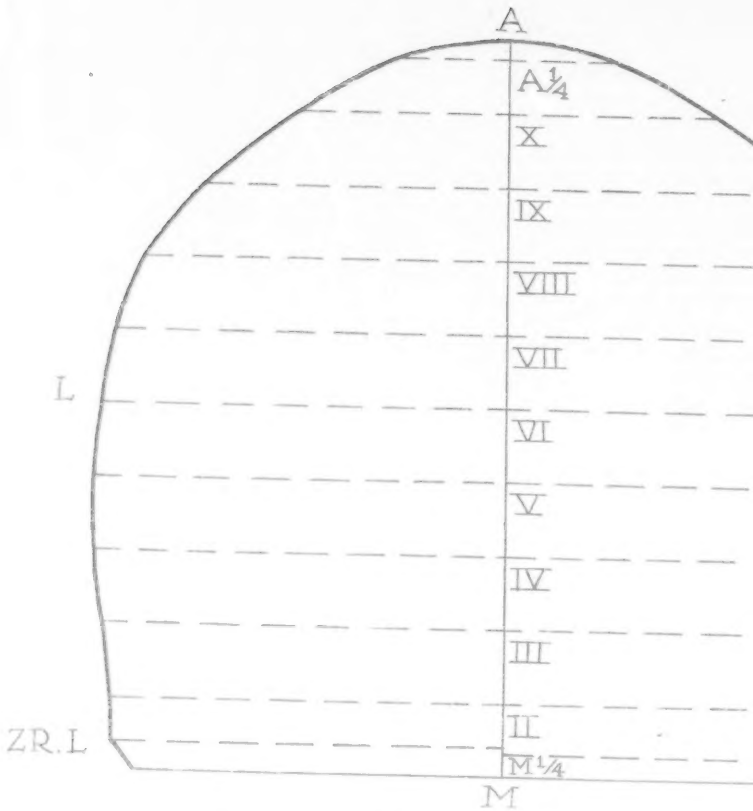
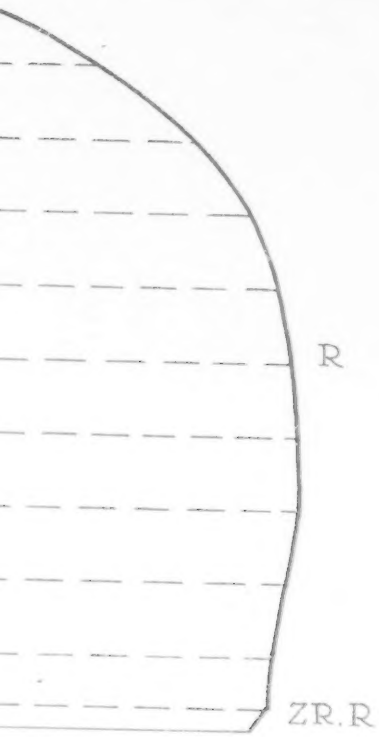


Fig. 2. New Britain Male Transverse Type Contour, based on 62 cas



, based on 62 cases.

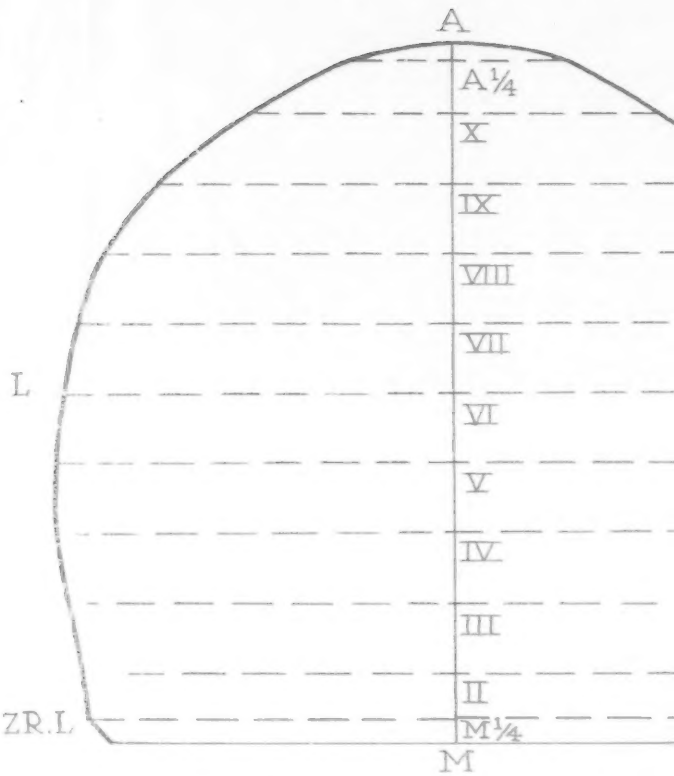
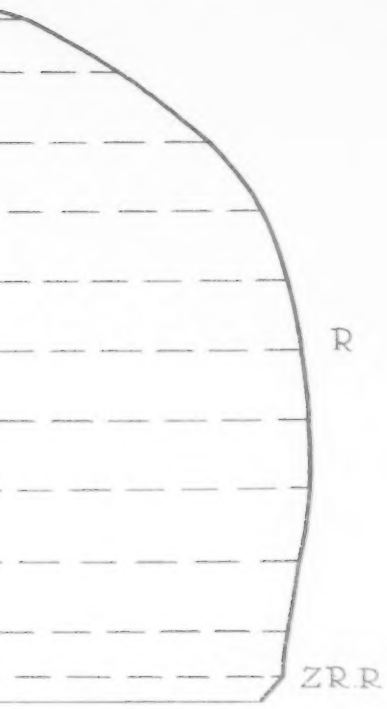


Fig. 3. New Britain Female Transverse Type Contour, based on



Contour, based on 37 cases.

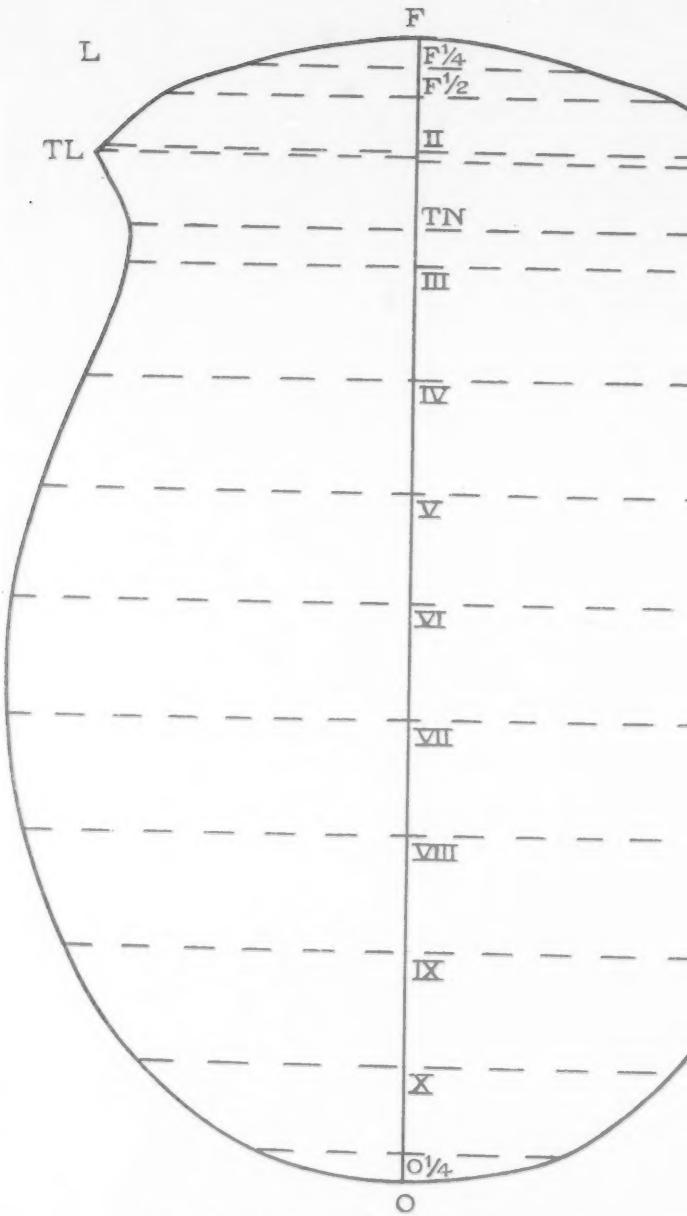
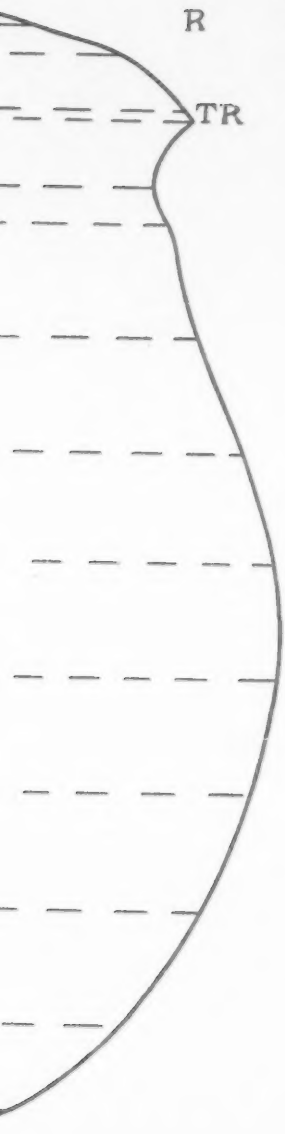


Fig. 4. New Britain Male Horizontal Type Contour, based on 62 case



based on 62 cases.

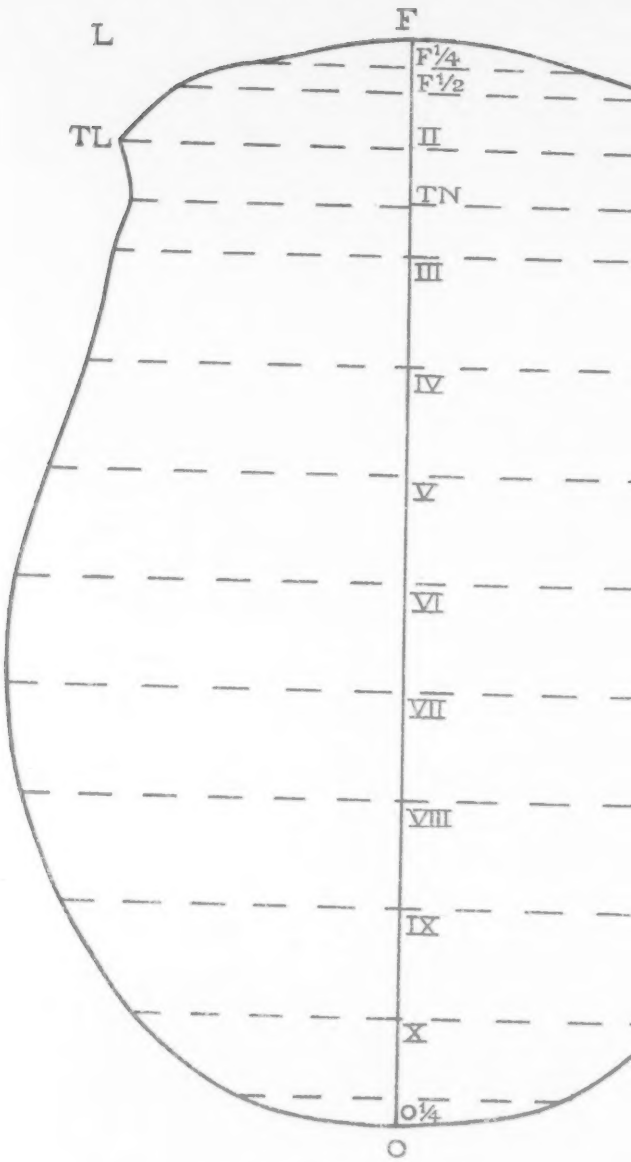
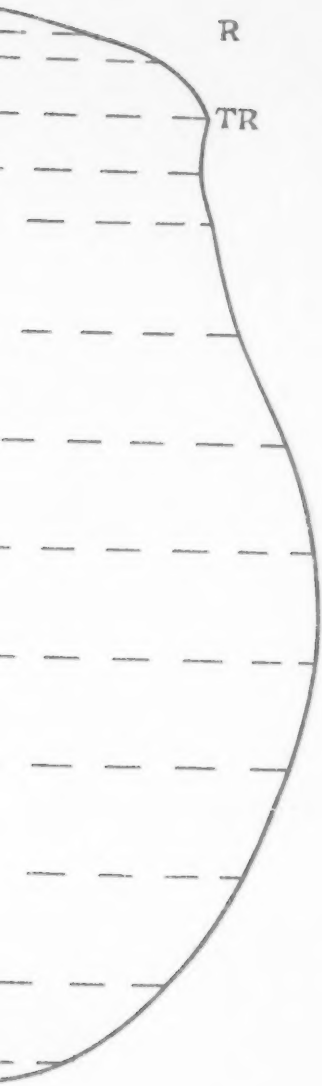
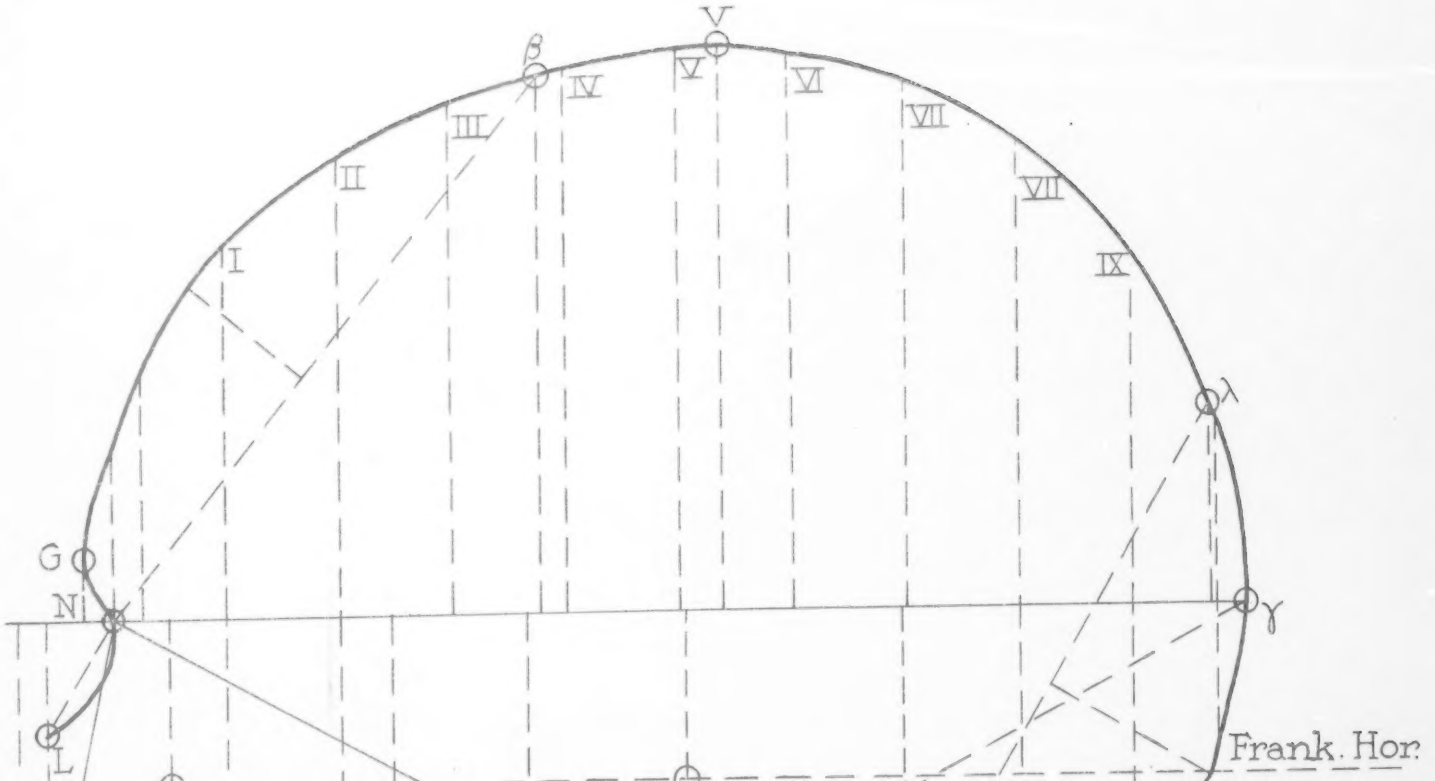


Fig. 5. New Britain Female Horizontal Type Contour, based on 3



ntour, based on 37 cases.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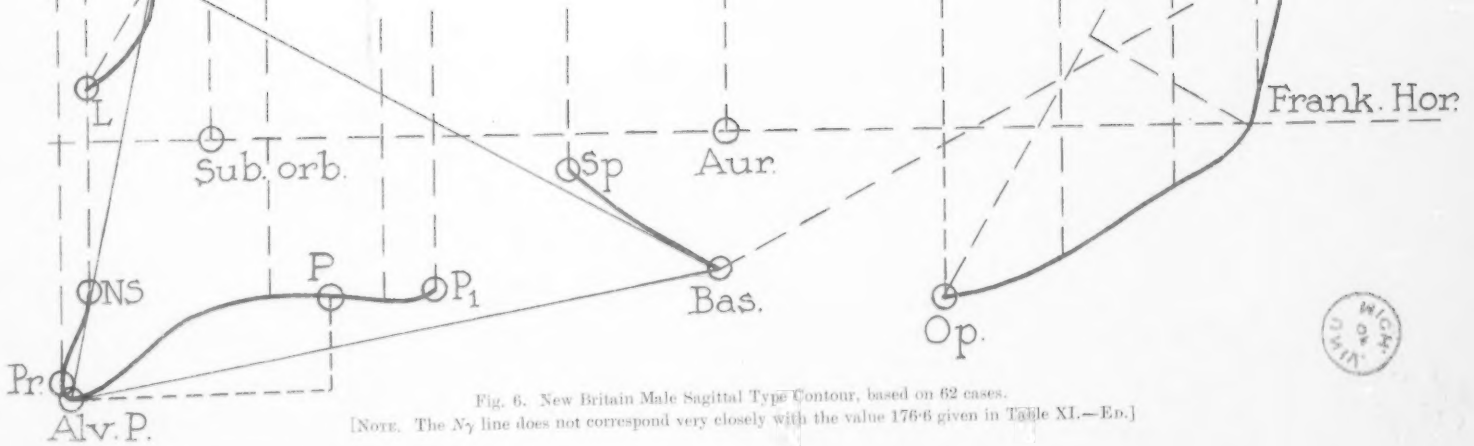
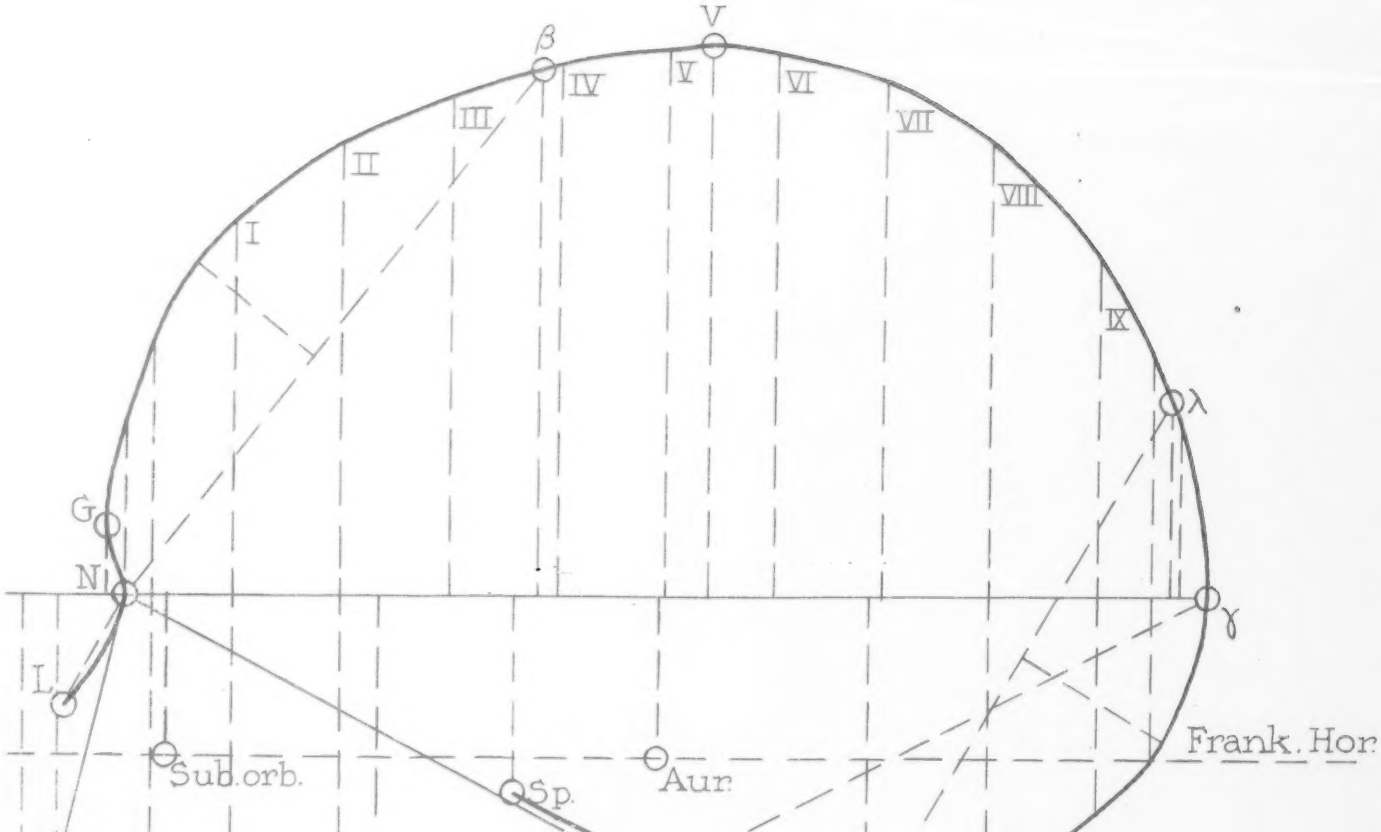


Fig. 6. New Britain Male Sagittal Type Contour, based on 62 cases.

[Note. The $N\gamma$ line does not correspond very closely with the value 176·6 given in Table XI.—Ed.]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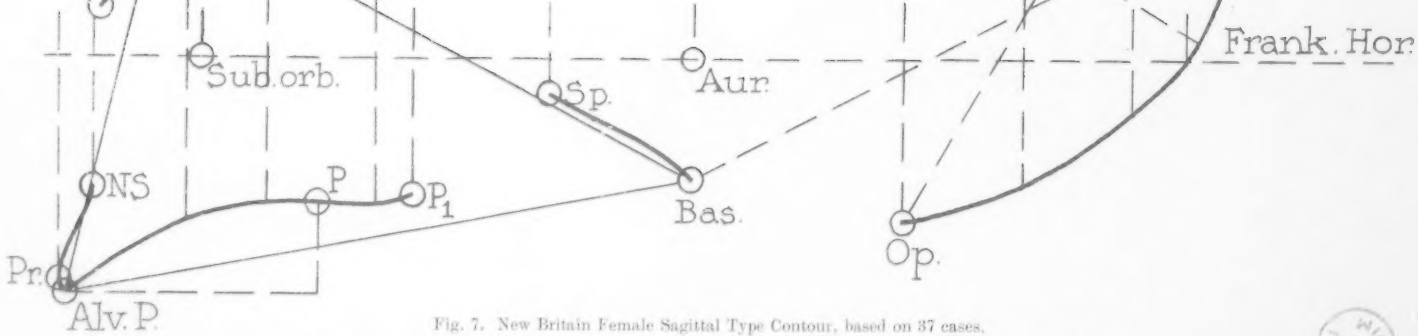

Fig. 7. New Britain Female Sagittal Type Contour, based on 37 cases.

[Note. The $N\gamma$ line does not correspond very closely with the value 168·4 given in Table XI.—Ed.]